

Università degli Studi di Trieste  
Dipartimento di Ingegneria e Architettura  
A.A. 2021-2022

# Scienza e Tecnologia dei Materiali Ceramici

## Modulo 2: Materiali Nanostrutturati

- Lezione 2 -


Vanni Lughi

[vlughi@units.it](mailto:vlughi@units.it)

040 558 3769

Dipartimento di Ingegneria e Architettura  
Università degli Studi di Trieste

5  $\mu\text{m}$

The background of the slide is a grayscale scanning electron microscope (SEM) image of a nanostructured ceramic material. The image shows a dense, porous network of interconnected fibers or particles, creating a complex, three-dimensional structure. A white horizontal scale bar is located in the bottom right corner, labeled '5 μm'.

# Esame e criteri di valutazione

## **Modalità d'esame e criteri di valutazione**

- Ricerca bibliografica su argomento a scelta
- Homework
- Valutazione finale:
  - Presentazione e discussione della ricerca bibliografica (10 punti)
  - Partecipazione e homework (5 punti)
  - Domanda a scelta dello studente (5)
  - Domanda a scelta del docente (10 punti)

## **Appelli**

- Ufficiali (previa conferma)
- Su appuntamento (collettivo)

# Previous lectures: Review

- Course “instructions”
- Definition of objectives and overview of course content

# Homework #1

Identify «hot topics» in the nanoworld. Classify them according to: societal challenges, key enabling technologies, applications

## **Societal Challenges:**

- **Health and Wellbeing**
- **Secure, green, efficient energy**
- **Food and Waste**

## **Key Enabling Technologies**

- Micro- and nanoelectronics
- Advanced Materials and Advanced Manufacturing
- Photonics
- Biotechnology (industrial)

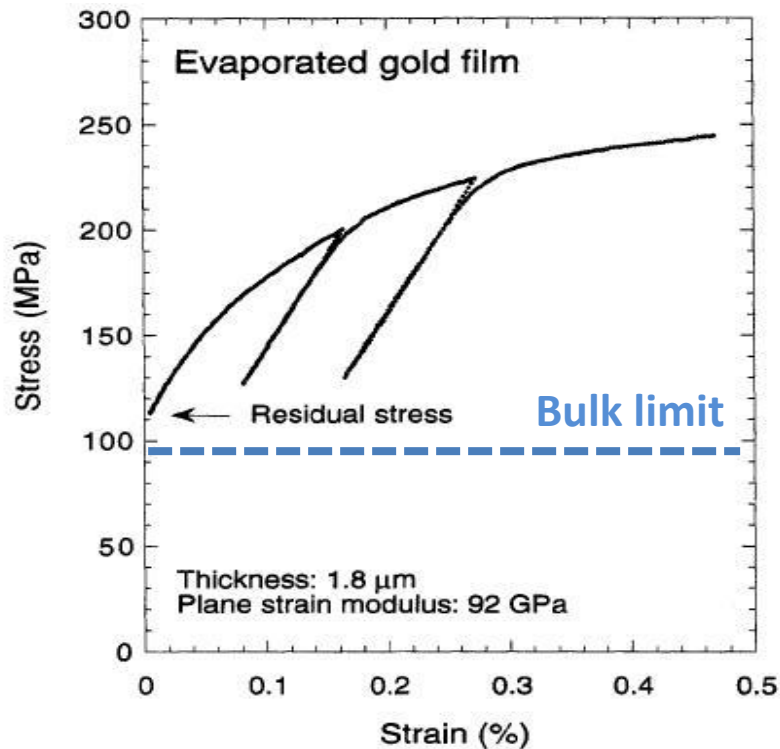
# This lecture: Content

- Examples of what is nano and what is not
- Surfaces and interfaces (see lecture notes)
  - Review of relevant thermodynamic definitions
  - Fundamentals on surface energy and chemical potential

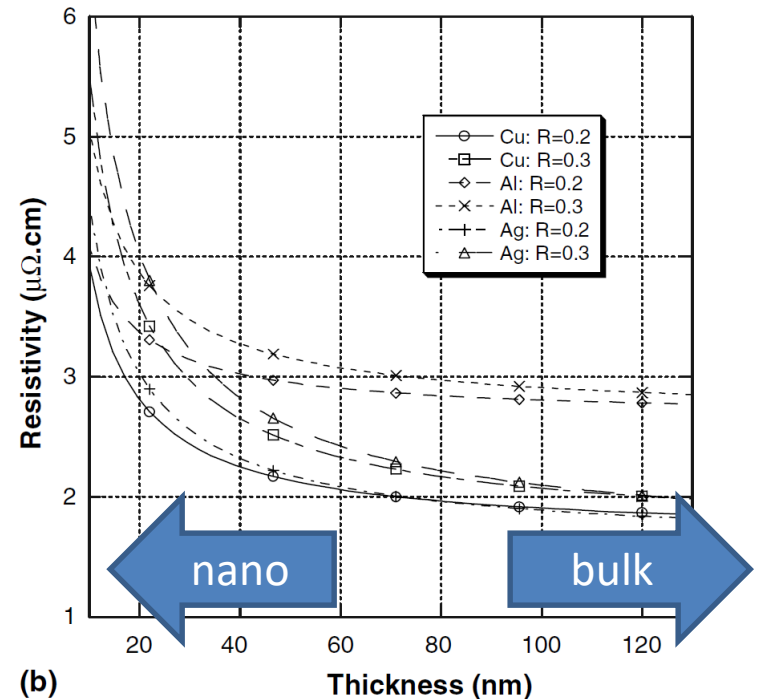
# When “nano” is actually “nano”?

- Example 1: Thin films

## Mechanical properties (e.g. strength)



## Electrical properties (e.g. resistivity)

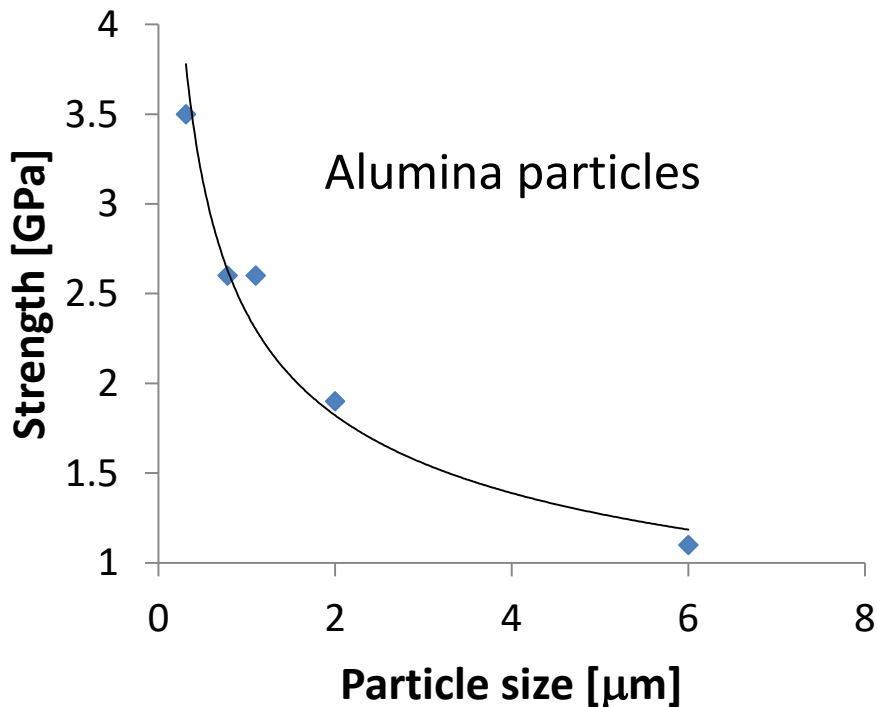


The “nano” limit is different for different materials properties

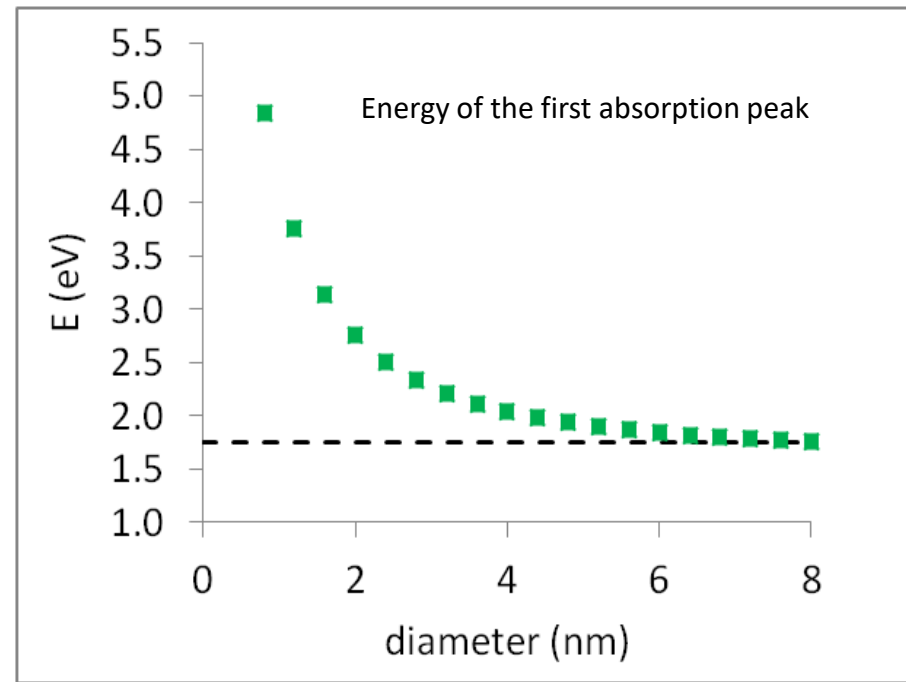
# When “nano” is actually “nano”?

- Example 2: Spherical particles

## Mechanical properties (e.g. strength)

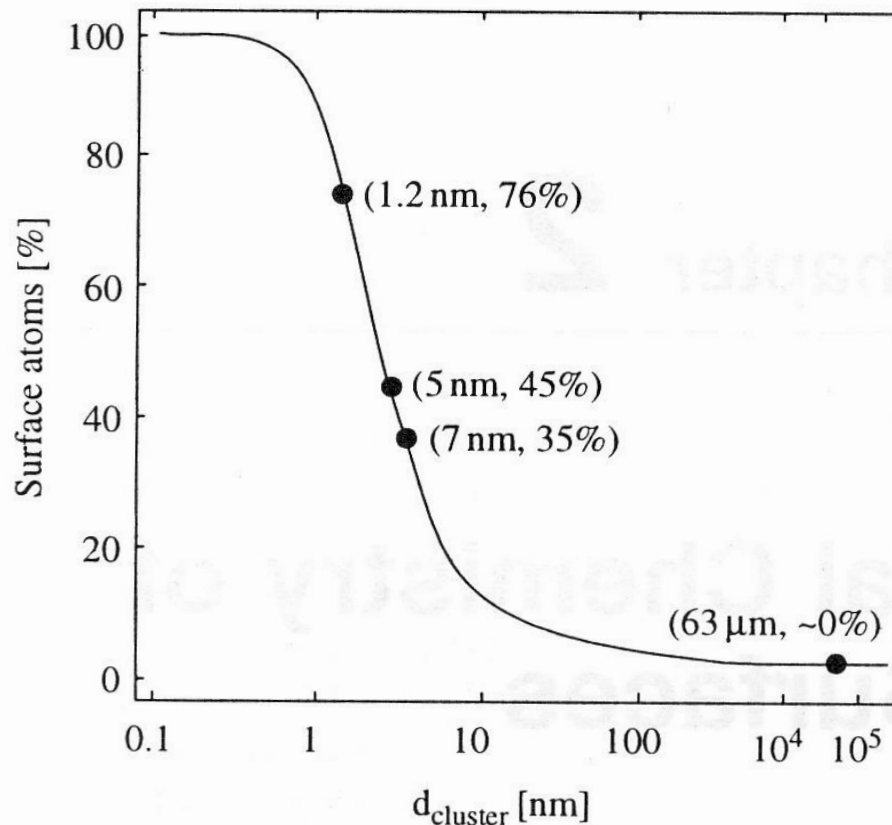


## Optoelectronic properties (e.g. color)



# Importance of Surface

- Fraction of surface atoms increases rapidly when size drops





# Importance of Surface

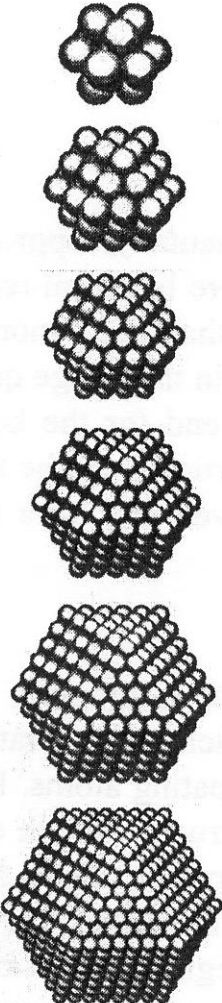
For a compact structure  
with coordination 12:

Number of surface atoms,  $S$ :

$$S = 10n^2 + 2$$

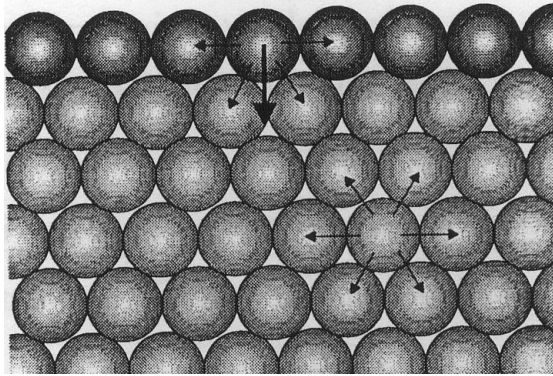
( $n$  is the number of shells)

Full-shell Clusters	Total Number of Atoms	Surface Atoms (%)
1 Shell	13	92
2 Shells	55	76
3 Shells	147	63
4 Shells	309	52
5 Shells	561	45
7 Shells	1415	35



# Importance of Surface

## Surface energy



Forces acting on bulk and surface atoms

$$\gamma = \left( \frac{\partial G_s}{\partial A} \right)_{P,T}$$

# Importance of Surface

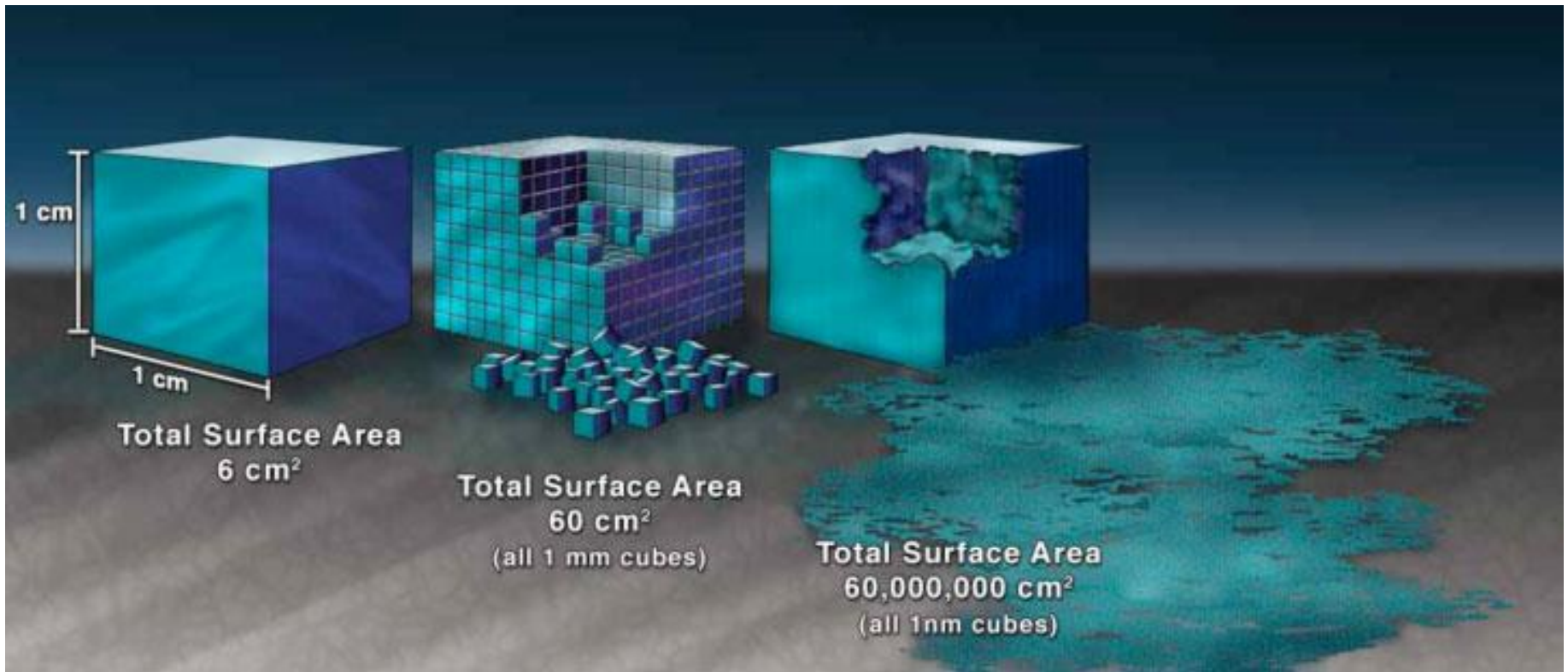
## Total surface energy increase

**Table 2.1.** Variation of surface energy with particle size.<sup>22</sup>

<i>Side (cm)</i>	<i>Total surface area (cm<sup>2</sup>)</i>	<i>Total edge (cm)</i>	<i>Surface energy (J/g)</i>	<i>Edge energy (J/g)</i>
0.77	3.6	9.3	$7.2 \times 10^{-5}$	$2.8 \times 10^{-12}$
0.1	28	550	$5.6 \times 10^{-4}$	$1.7 \times 10^{-10}$
0.01	280	$5.5 \times 10^4$	$5.6 \times 10^{-3}$	$1.7 \times 10^{-8}$
0.001	$2.8 \times 10^3$	$5.5 \times 10^6$	$5.6 \times 10^{-2}$	$1.7 \times 10^{-6}$
$10^{-4}$ (1 $\mu\text{m}$ )	$2.8 \times 10^4$	$5.5 \times 10^8$	0.56	$1.7 \times 10^{-4}$
$10^{-7}$ (1 nm)	$2.8 \times 10^7$	$5.5 \times 10^{14}$	560	170

# Importance of Surface

Total surface energy increase



# Where are we?

## 2. Design di materiali nanostrutturati

- **Fondamentali: stato solido, approfondimenti di fisica e chimica (anche sparsi durante il corso)**
- Proprietà alla nanoscala
- Attività seminariale sulla modellazione
- Esempi in cui la nanostruttura determina le proprietà e di come queste possono essere "previste":
  - quantum wells; banda intermedia; nanocompositi; opali artificiali; modificazione dell'energia superficiale mediante nanocoatings; etc.

# Importance of Surface – Properties at the Nanoscale

- Effect of size on vapor pressure

(This is a curvature effect!)

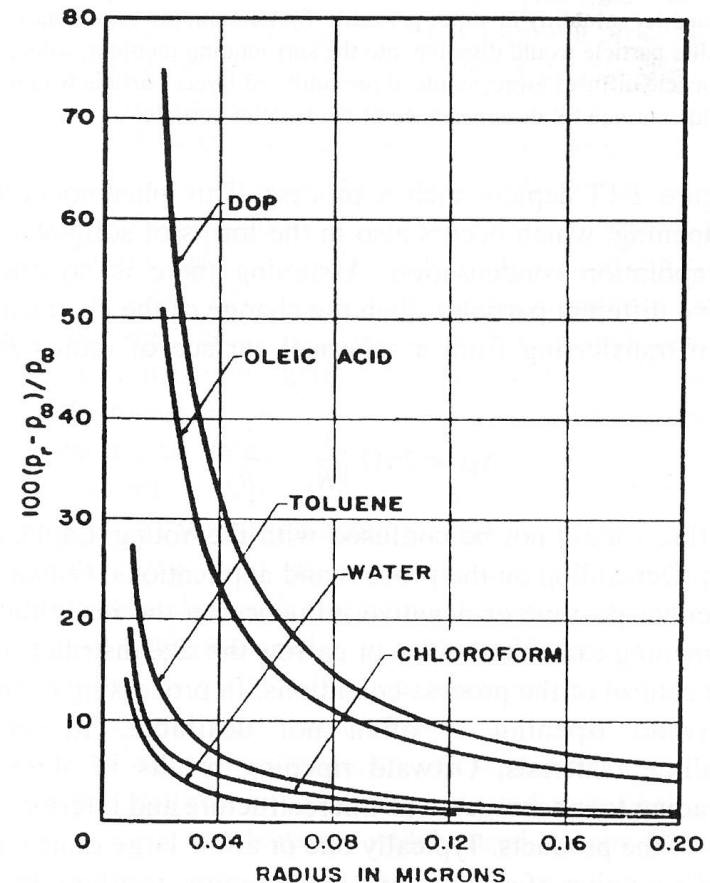
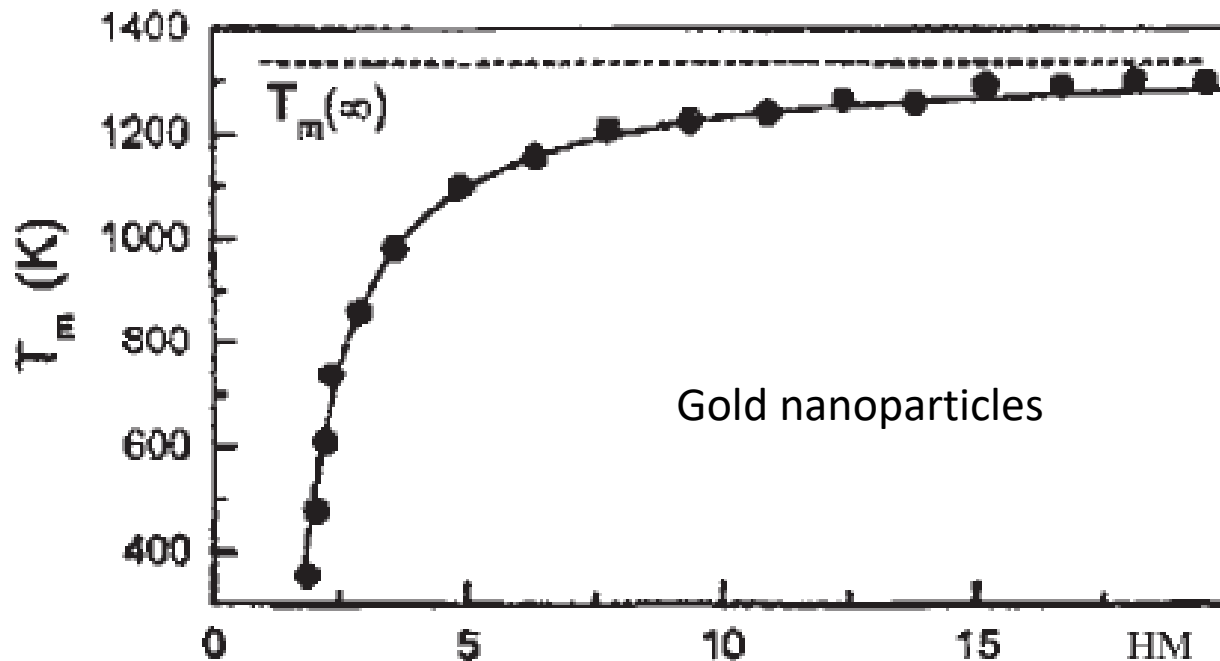


Fig. 2.12. Vapor pressure of a number of liquids as a function of droplet radius. [V.K. La Mer and R. Gruen, *Trans. Faraday Soc.* 48, 410 (1952).]

# Importance of Surface – Properties at the Nanoscale

- Melting point vs size

$$T_b - T_m = \left( \frac{2T_b}{\Delta H \rho_s r_s} \right) \left( \gamma_s - \gamma_l \left( \frac{\rho_s}{\rho_l} \right)^{\frac{2}{3}} \right)$$



# Importance of Surface – Properties at the Nanoscale

- Inhibition of phase transformation in zirconia

Driving force to transformation:

$$\Delta G_{t-m} = \Delta G_{t-m}^{CHEM} + \cancel{\Delta G_{t-m}^{STRAIN}} + \Delta G_{t-m}^{SURF}$$

where  $\Delta G_{t-m}^{SURF} = A_m \gamma_m - A_t \gamma_t$

For stress-free, undoped spherical zirconia particles:

$$\Delta G_{t-m} = \frac{4}{3} \pi r^3 (g_m - g_t) + 4 \pi r^2 (\gamma_m - \gamma_t)$$

A critical particle radius exist, for which the tetragonal phase is thermodynamically stable

$$r_c = -3 \frac{\gamma_m - \gamma_t}{g_m - g_t} \quad (5 \text{ to } 10 \text{ nm for pure zirconia powders, } 30 \text{ nm for solid ceramics})$$



# Thermodynamics of surface-intensive systems

$$G = U - TS + PV + \mu N + \gamma A$$

ad-hoc term

$$\gamma = \left. \frac{\partial G}{\partial A} \right|_{T,P,N}$$

Surface tension

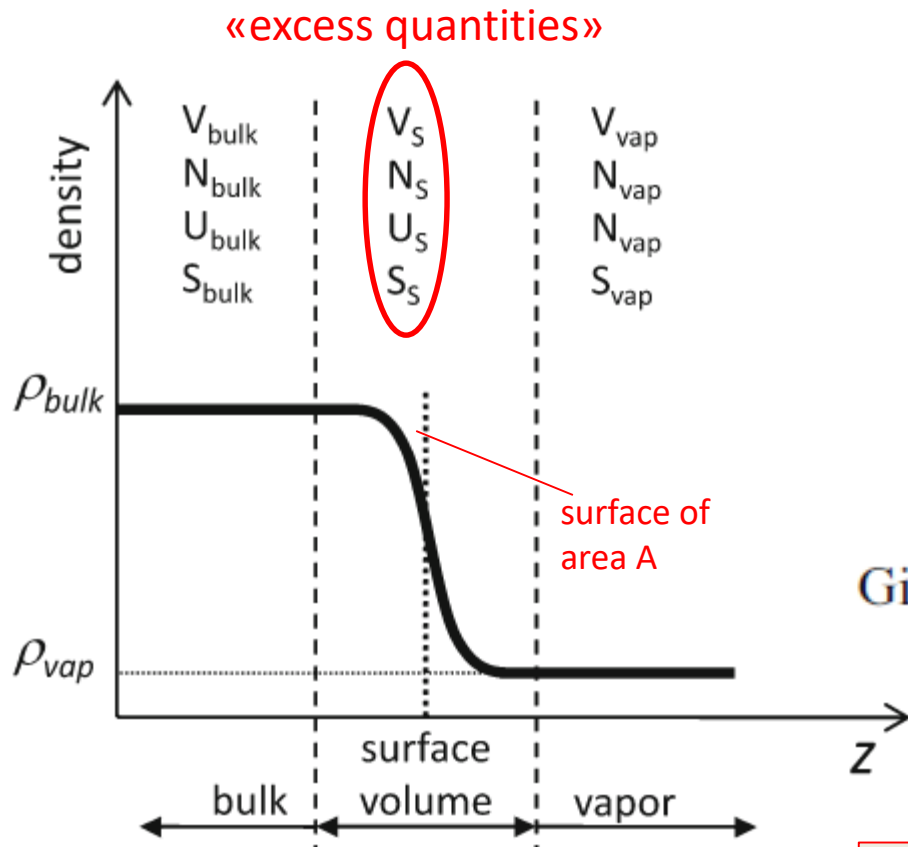
(also: surface energy, surface free energy, etc.)

$$\mu = \left. \frac{\partial G}{\partial N} \right|_{T,P,A}$$

Chemical potential

# Surface thermodynamics

## - Gibbs' approach -



$$G_S = U_S - TS_S + PV_S$$

Surface energy  $u_S = \frac{U_S}{A}$

Gibbs surface free energy  $g_S = \frac{G_S}{A}$

In general  $g_s = \gamma$   
only in pure single component systems

# Estimates of surface energy

Surface energy  $\gamma$  is of the same order of the surface tension  $F_L$ :

$$F_L \sim \gamma$$

Estimate of  $\gamma$  for a (111) surface in a compact structure leads to:

$$u_s = 0.25 L_s / N_a \text{ [J/surf. atom]}$$

$L_s$ : sublimation latent heat  
 $N_a$ : Avogadro

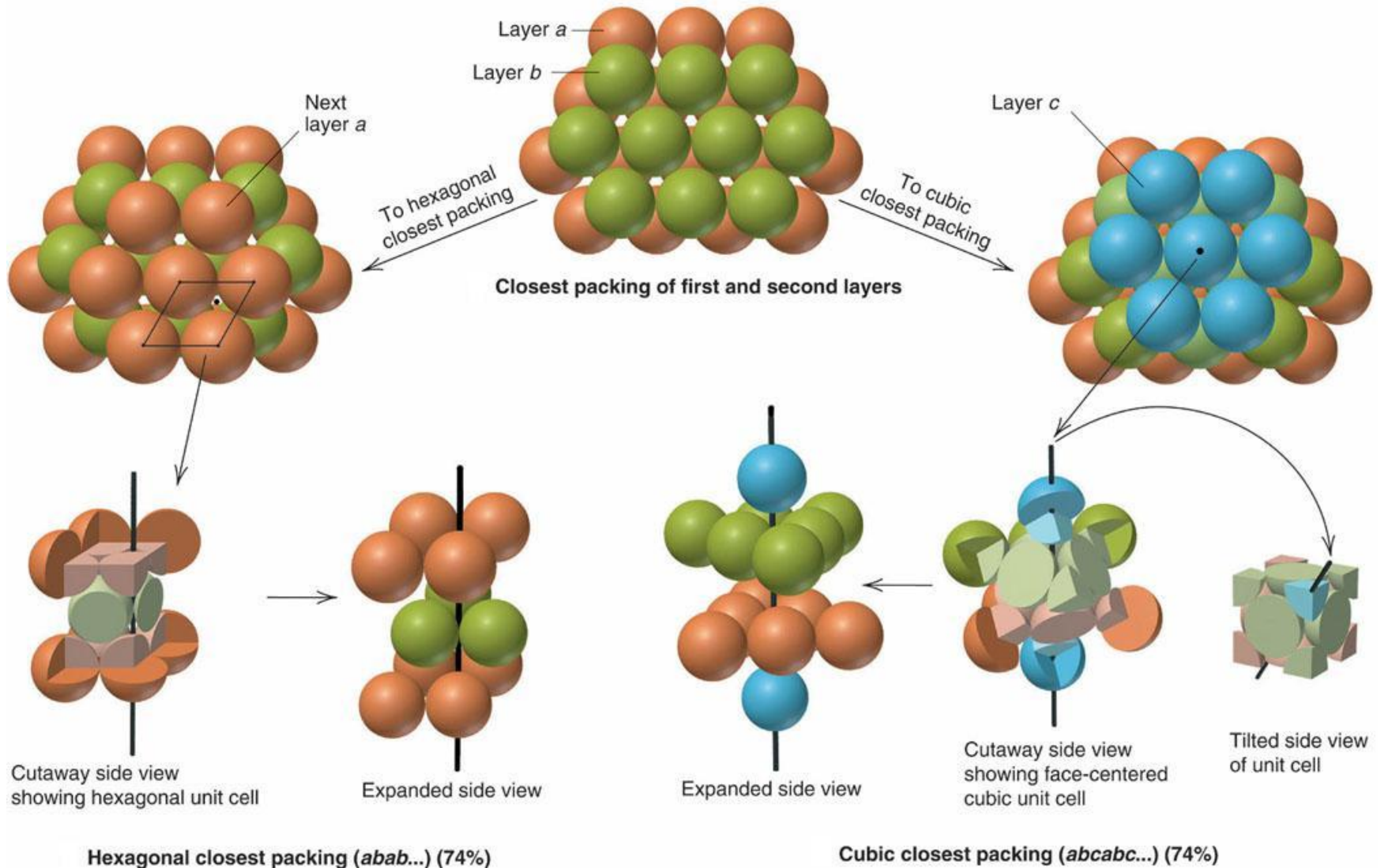
Considering that  $\gamma = u_s + Pv_s - Ts_s$

we need to take into account entropy effects:

$$\gamma = 0.15 L_s / N_a \text{ [J/surf. atom]}$$

$$\left( \frac{\partial \gamma}{\partial T} \right)_P = -S$$

# Close-packed Structures



# Corrections to surface energy estimates: Surface reconstruction

Minimization of energy - Entropy increases

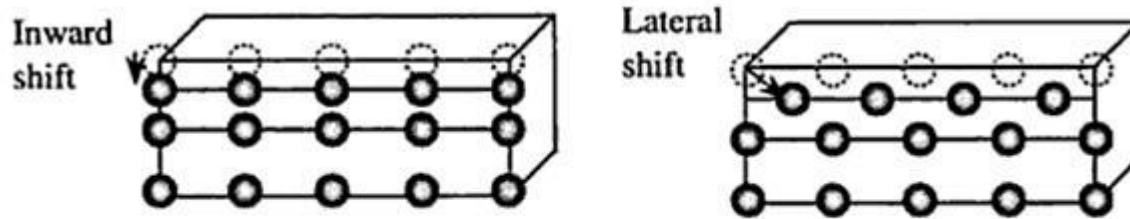
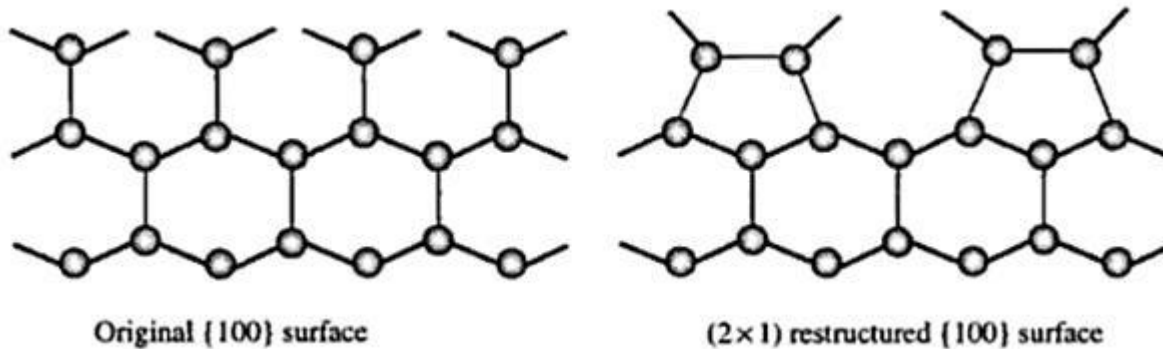
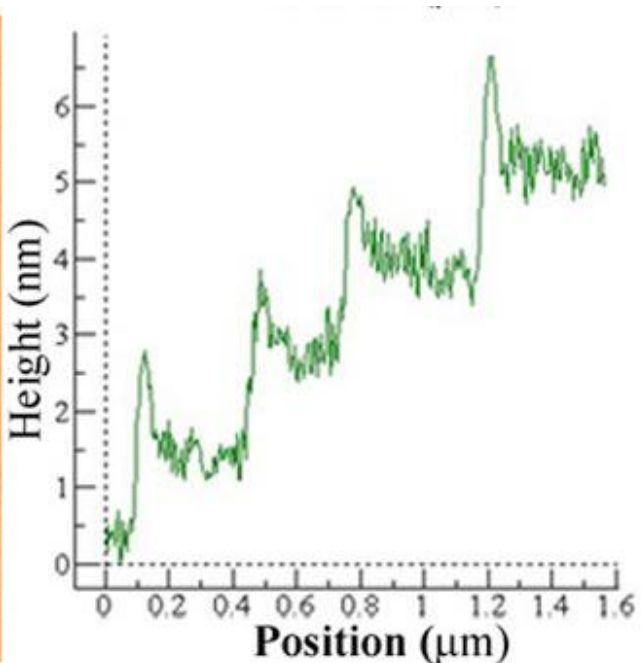
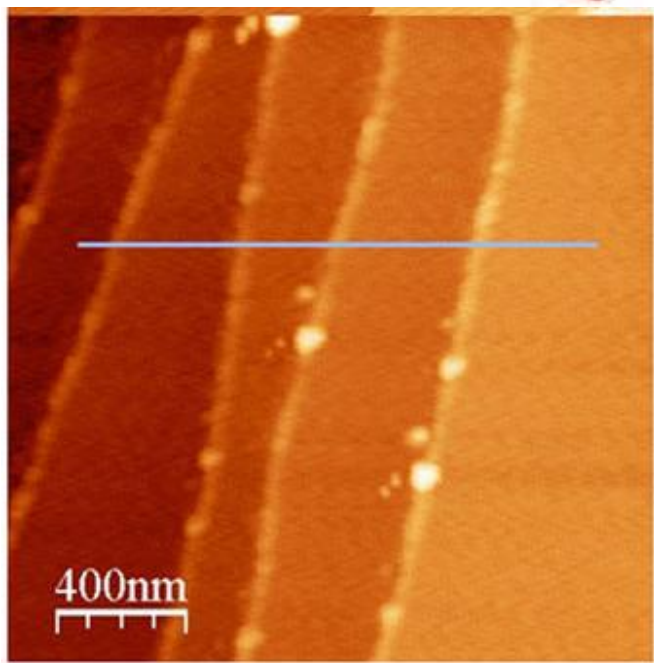
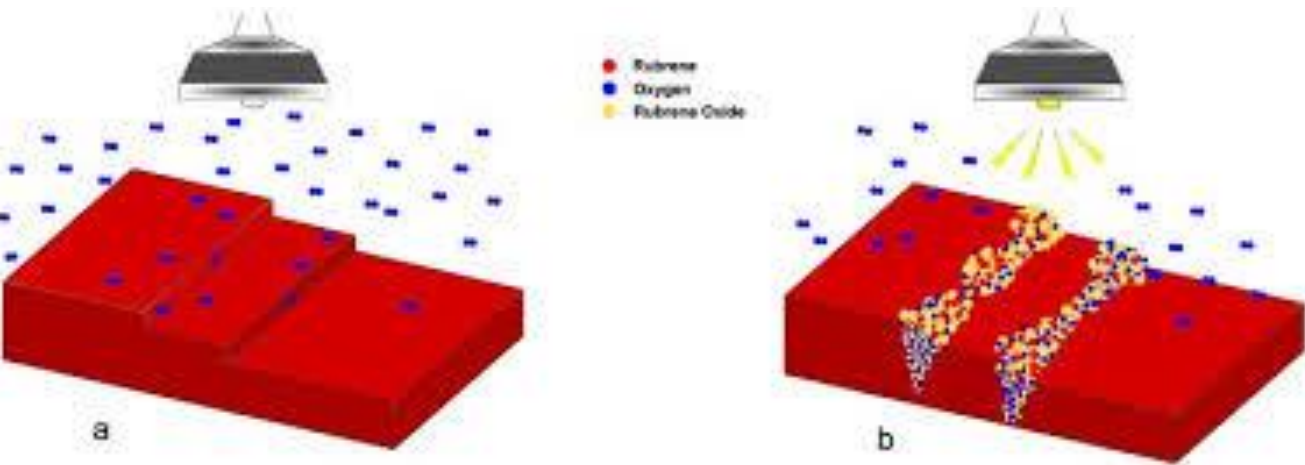


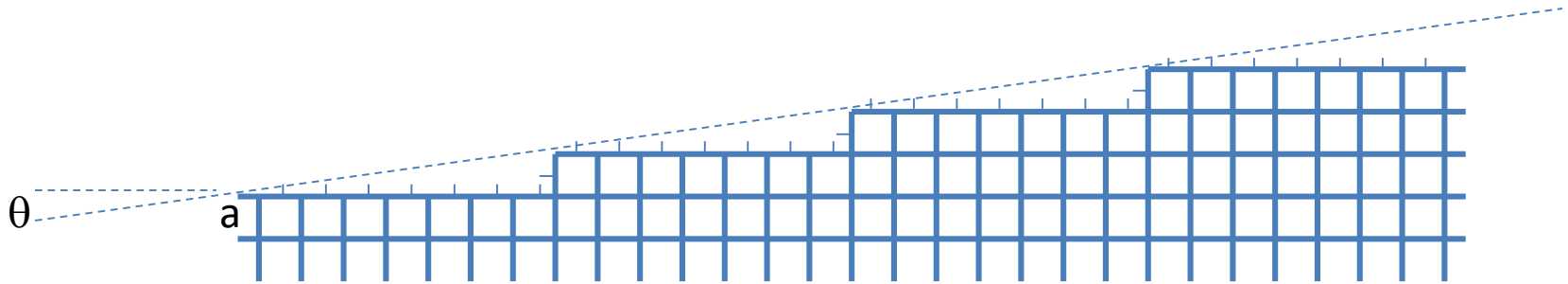
Fig. 2.4. Schematic showing surface atoms shifting either inwardly or laterally so as to reduce the surface energy.



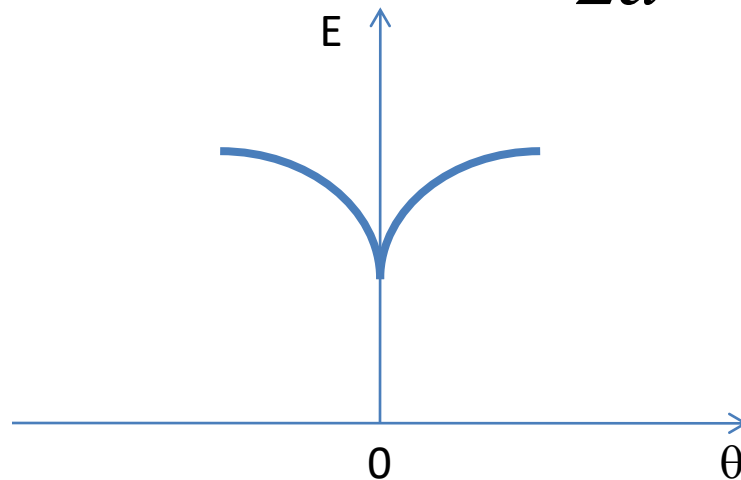
# High Index Planes (Vicinal Surfaces)



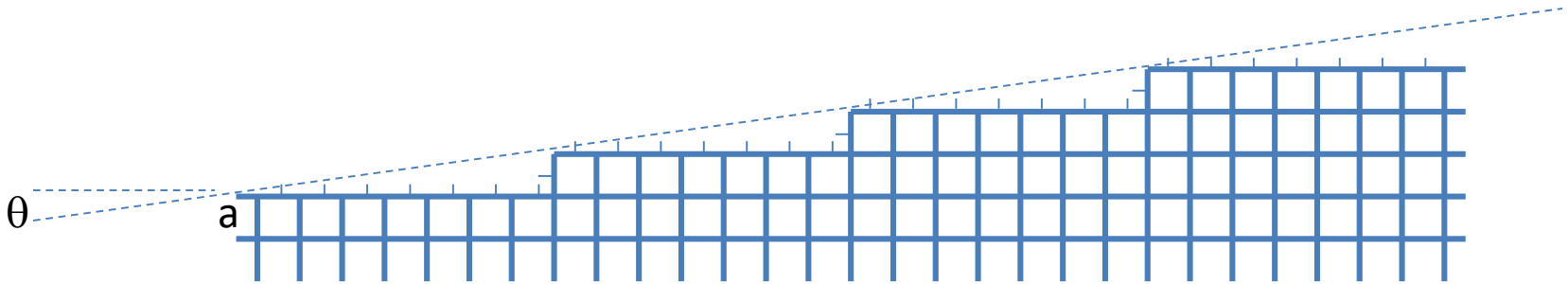
# Surface Energy of High Index Planes



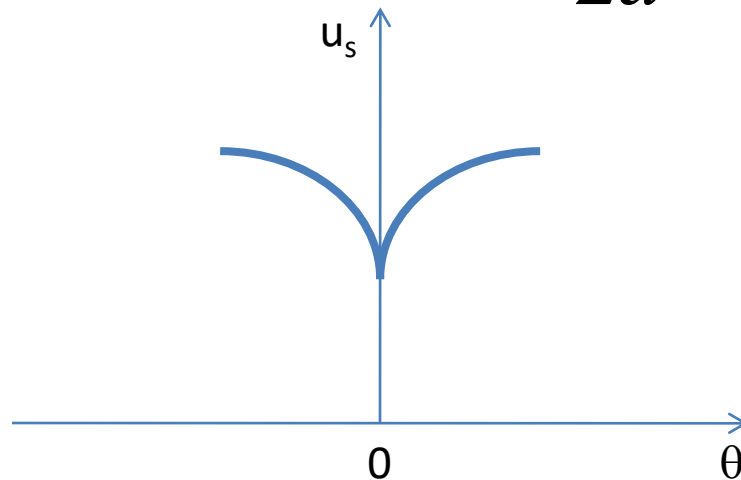
$$u_s = (\cos \mathcal{G} + \sin |\mathcal{G}|) \frac{\varepsilon}{2a^2}$$



# Surface Energy of High Index Planes



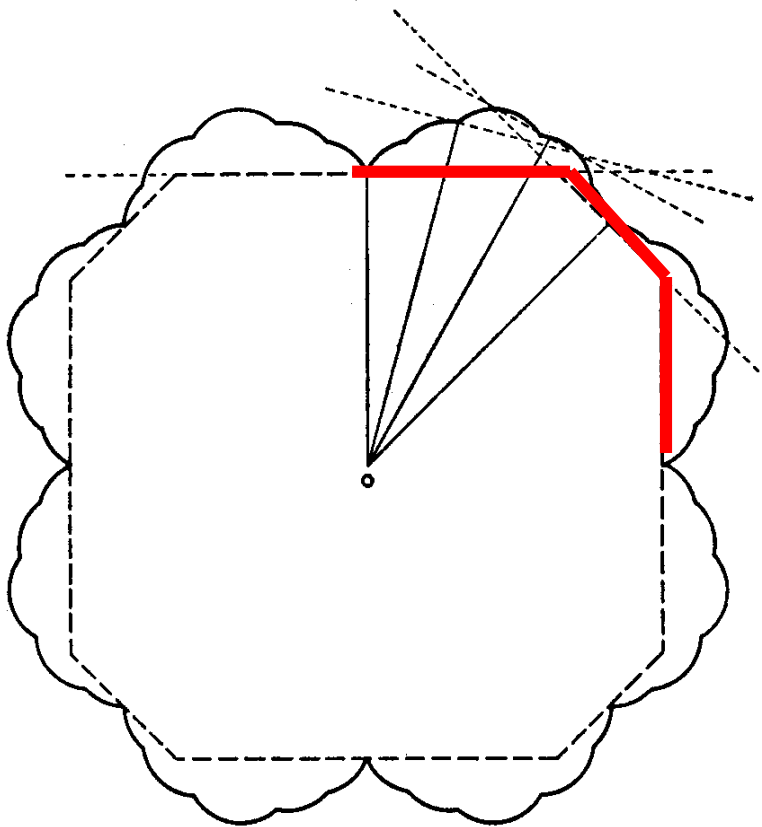
$$u_s = (\cos \mathcal{G} + \sin |\mathcal{G}|) \frac{\varepsilon}{2a^2}$$





# Wulff Construction

Equilibrium shape of an isolated crystal



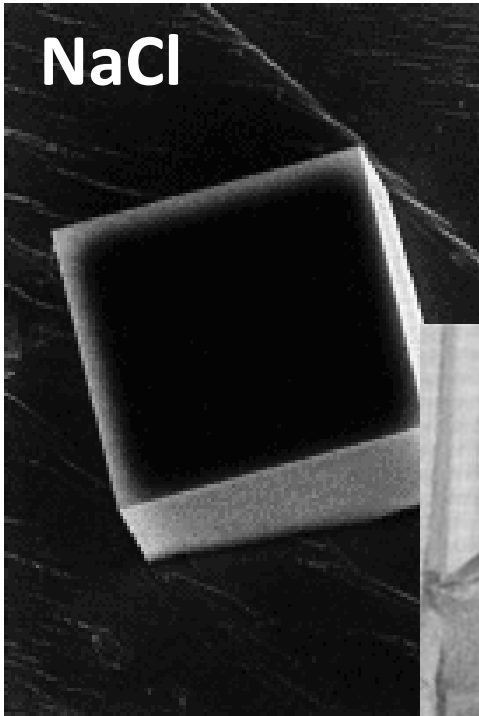
Gibb's theorem:

Minimize 
$$\sum_i \gamma_i A_i$$

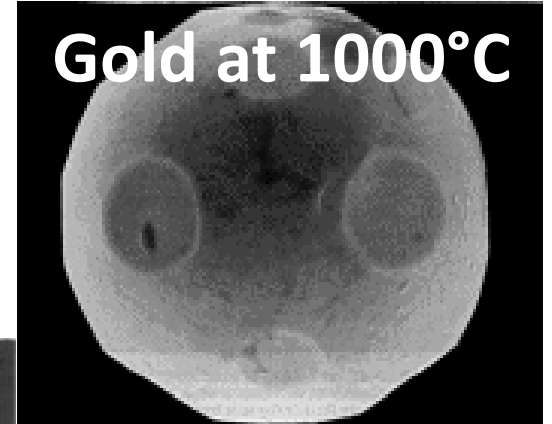
Also a way to find the surface energy when the shape of a crystal is known

# Equilibrium shape of an isolated crystal

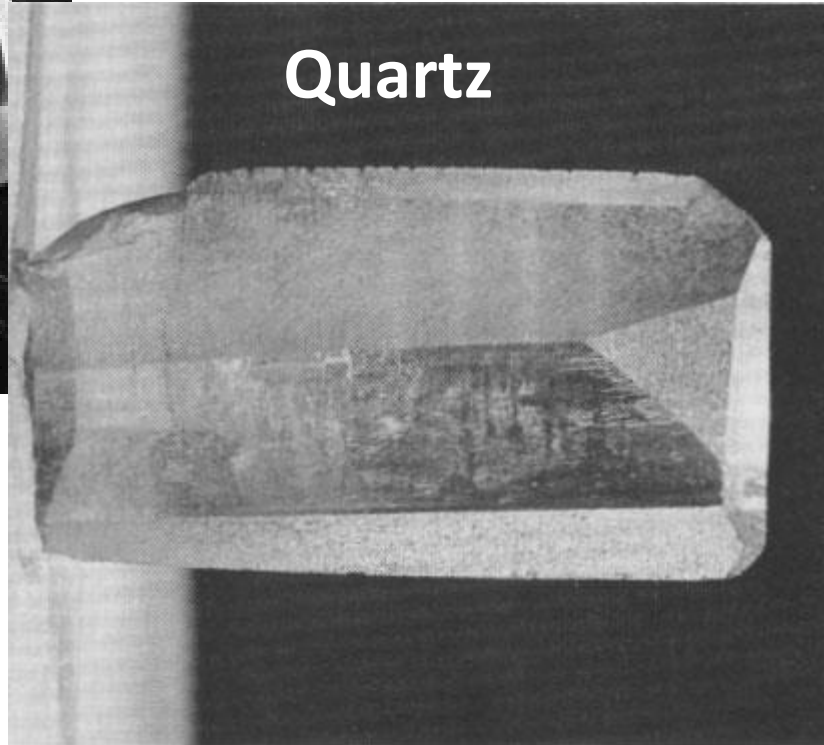
NaCl



Gold at 1000°C



Quartz



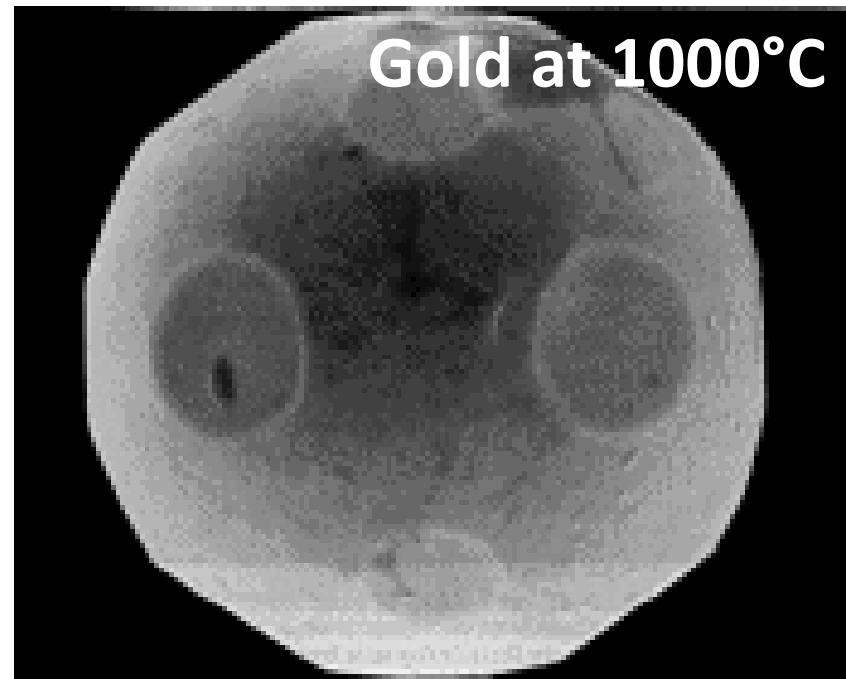
# Equilibrium shape: Roughening Temperature

- The surface energy depends on temperature :

$$\left( \frac{\partial \gamma}{\partial T} \right)_P = -S$$



Facets tend to disappear upon increasing temperature



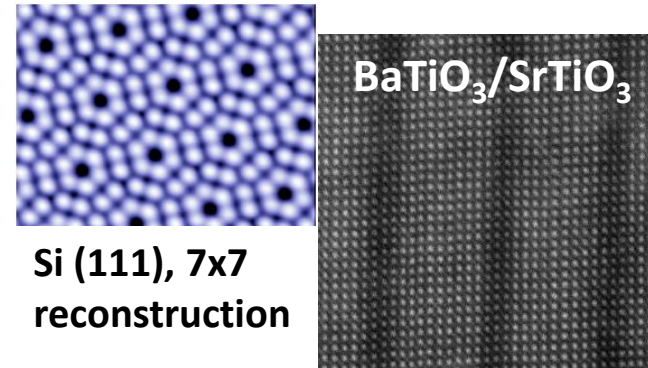
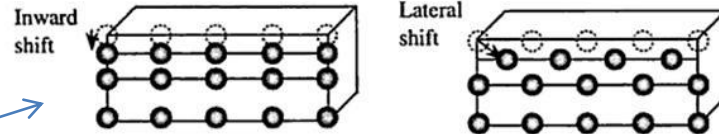
# Take-home message

- Importance of the surface
- Homegame: for a system containing a given amount of material, calculate how the surface area increases when the size of the system's element decrease

# Surface energy minimization

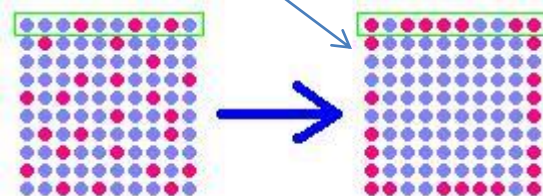
## 1. Surface mechanisms

- Surface relaxation
- Surface reconstruction  
More than one dangling bond per atom
- Chemi- or physisorption of species at the surface



- Surface segregation

Important consequences on doping of nanostructures



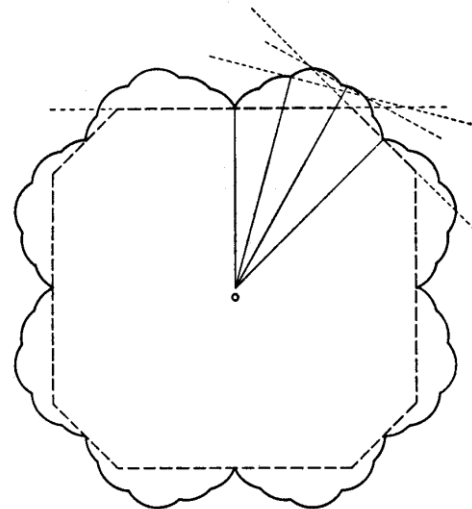
Surface mechanisms of energy minimization are of major importance in nanostructures

# Surface energy minimization

## 2. Individual structure mechanisms

- Isotropic systems (liquids, amorphous solids):  
Total surface area minimization

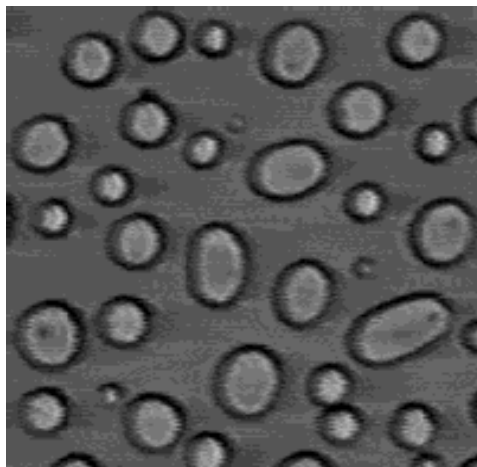
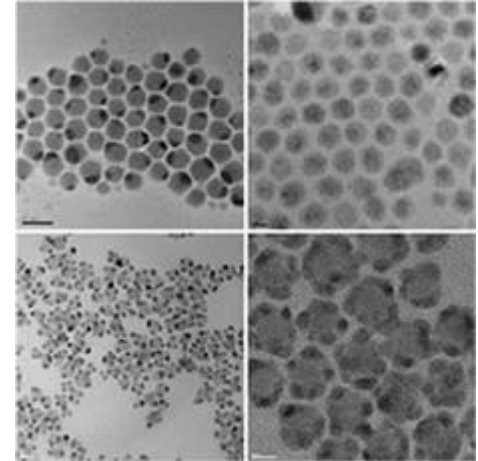
- Anisotropic systems (solid crystals): Minimize  $\sum_i \gamma_i A_i$   
➔ Wulff's theorem



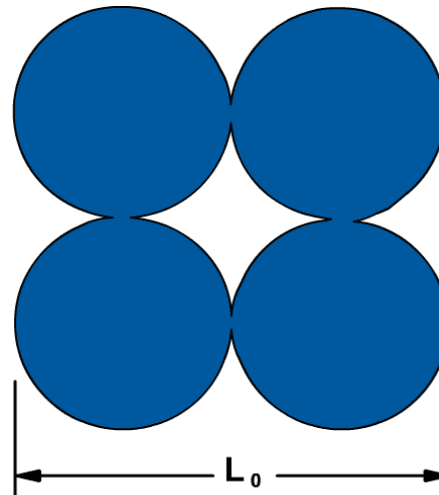
# Surface energy minimization

## 3. Overall system mechanisms

- Aggregation of individual structures
- Formation of new structures
  - Sintering
  - Ostwald ripening

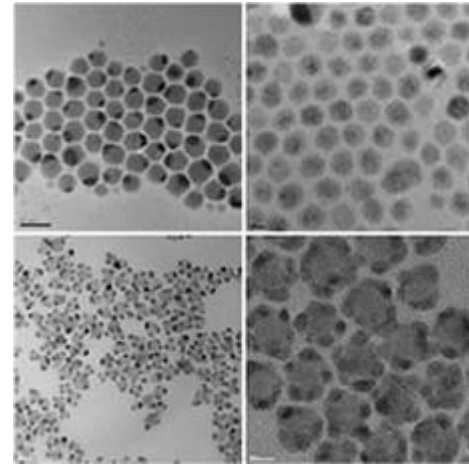


**Sintering**  
Changes in pore shape



# Agglomeration - aggregation

- Chemical bonds
- Physical bonds (Van der Waals)



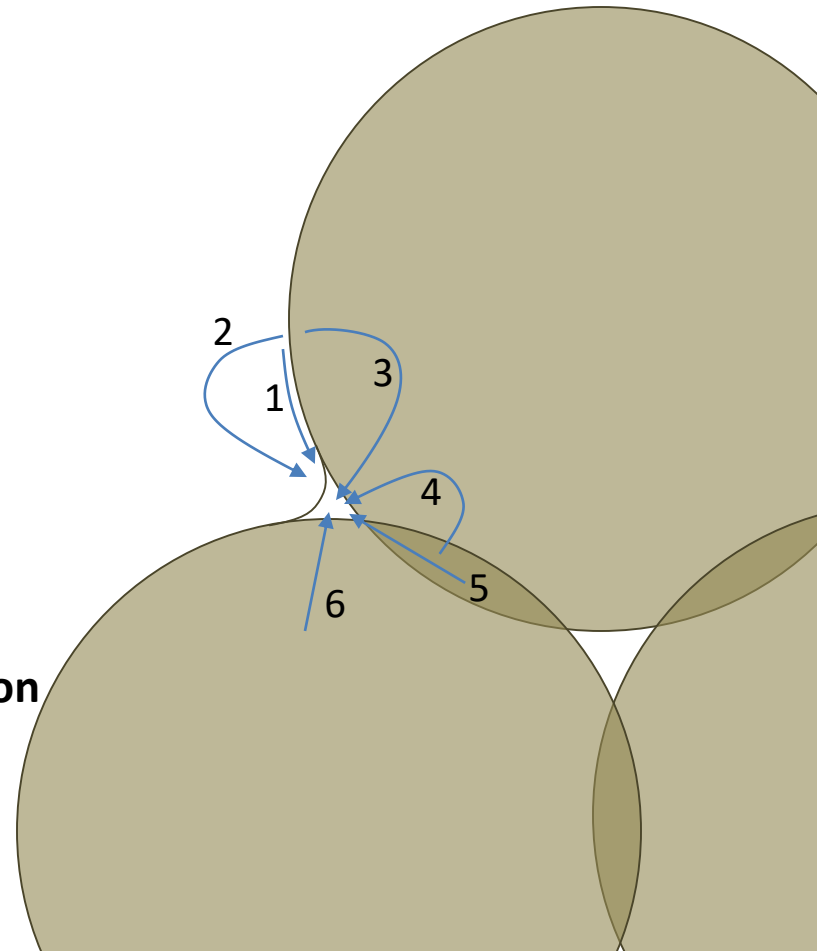


# Sintering

Mechanisms in diffusion controlled sintering:

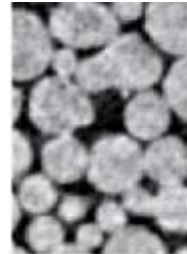
	<u>Mechanism</u>	<u>Source of matter</u>
1.	Surface diffusion	Surface
2.	Vapor transport	Surface
3.	Lattice diffusion	Surface
4.	Lattice diffusion	Grain boundary
5.	Boundary diffusion	Grain boundary
6.	Lattice diffusion	Dislocations

- All mechanisms are driven by **surface area reduction**
- All lead to **neck growth**
- Only mechanisms 4, 5, and 6 lead to **densification**

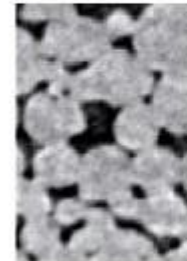
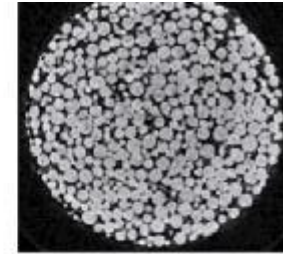


# Sintering

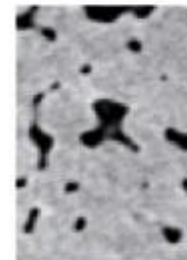
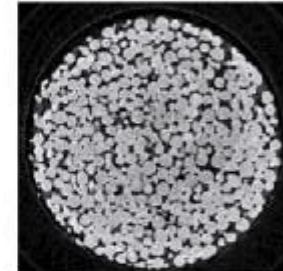
- **Stage 1:**  
Initial neck growth
- **Stage 2:**  
Large necks, cylindrical interconnected pores
- **Stage 3:**  
Isolated, spherical pores



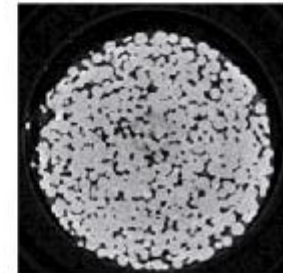
1



2



3



100  $\mu\text{m}$

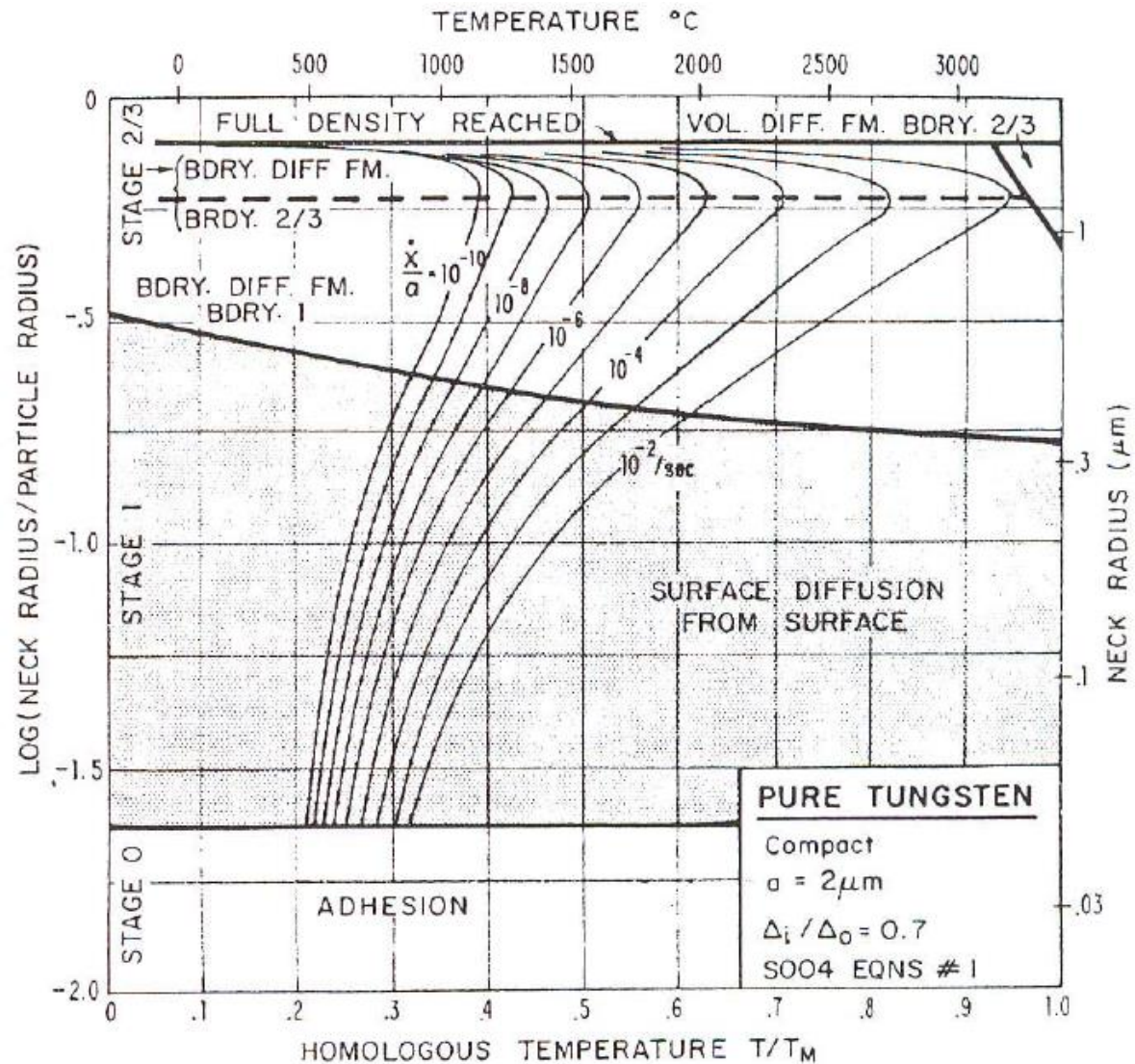
200  $\mu\text{m}$

# Sintering rate (time) diagrams

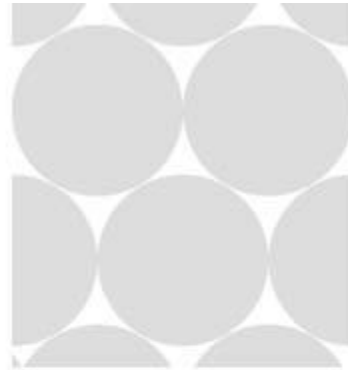
Rates are driven by the curvature difference

$$\frac{dr}{dt} = A \cdot D \cdot K^n$$

More to come!

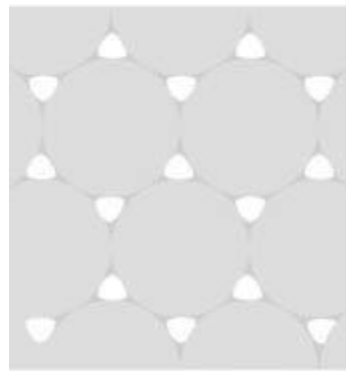


# Sintering conditions



Volume sources  
↓

Surface sources  
↓



- Low porosity
- Densification



- High porosity
- No Densification

# Ostwald ripening

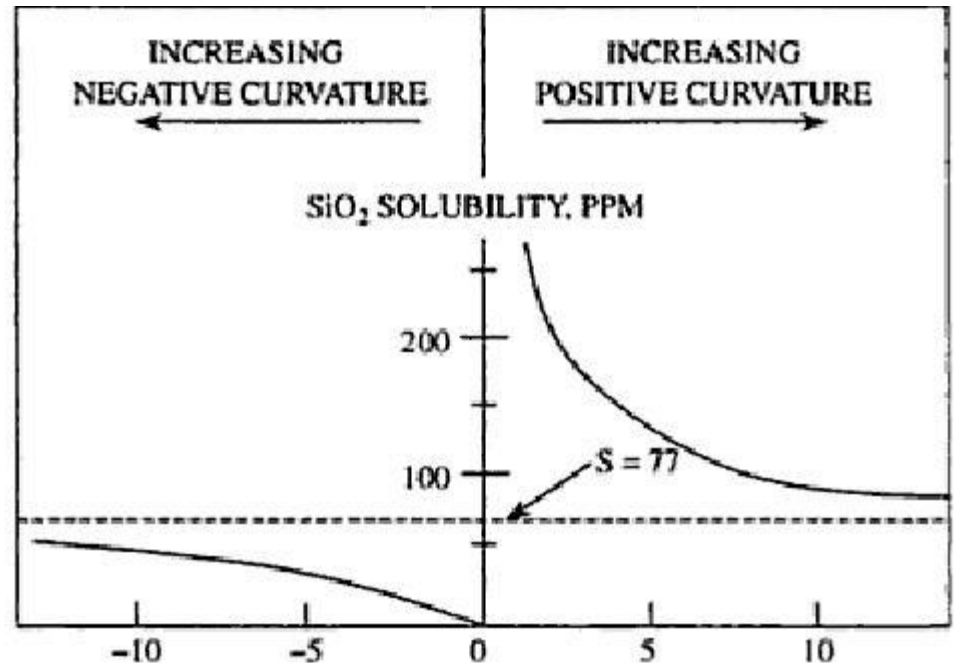
- curvature and chemical potential -

Young – Laplace equation

$$\Delta\mu = \frac{2\gamma\Omega}{r}$$

Gibbs – Thompson equation

$$k_B T \ln \frac{S_c}{S_\infty} = \frac{2\gamma\Omega}{r}$$

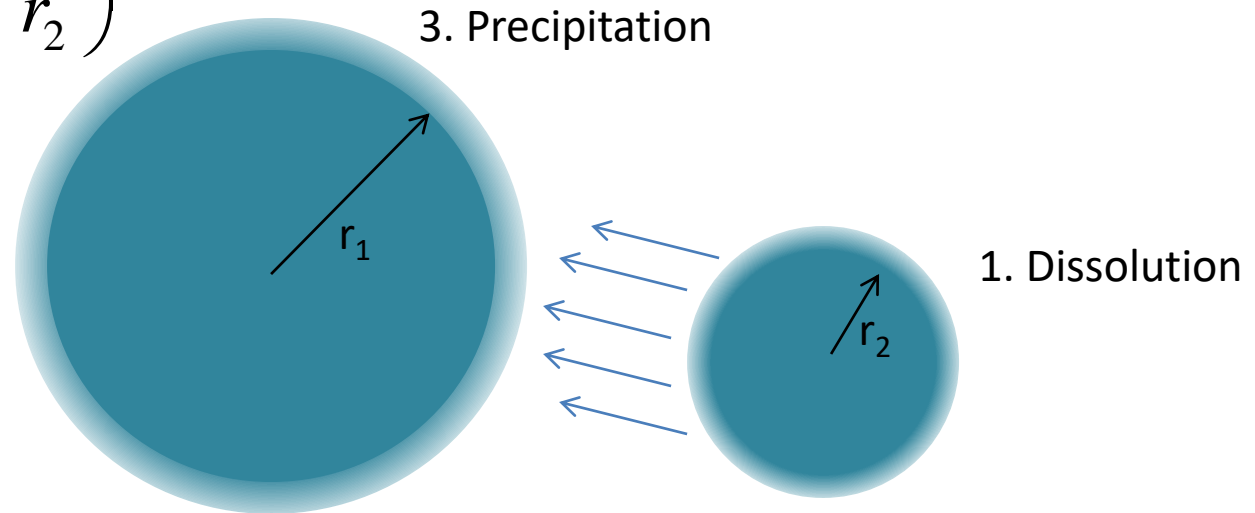


# Ostwald ripening

- curvature and chemical potential -

Driving force for Ostwald ripening

$$\Delta\mu = 2\gamma\Omega\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



# Ostwald ripening

Consequences:

- Exaggerated grain growth in sintering
- Growth of nanocrystals in solution:
  - Can either focus or broaden the size distribution