Condensed Matter Physics I
Final written test
academic year 2009/2010
July 12, 2010

(*Time: 3 hours*)

Exercise 1: *Bravais lattices with basis and structure factors*

Some superconductors are made by CuO_2 layers, that in some cases have a pattern like the one shown in Fig. 1. Let us assume a the minimum distance between two Cu atoms.

1. Describe a possible choice for a set of primitive lattice vectors and basis (write the vectors!); draw the primitive unit cell.

2. Assuming that the atomic form factors are $f_{Cu} = 2f_O$, calculate the structure factor and find the points of the reciprocal lattice where the structure factor is maximum and where it is vanishing.

3. Other compounds are characterized by the fact the the atomic positions of O are not equivalent, but they are slightly displaced in direction perpendicular to the plane by a certain quantity $\pm\delta$, as shown in Fig. 2. Specify and draw the basis vectors, primitive cell, the lattice spacing, the reciprocal lattice basis vectors in this other case.

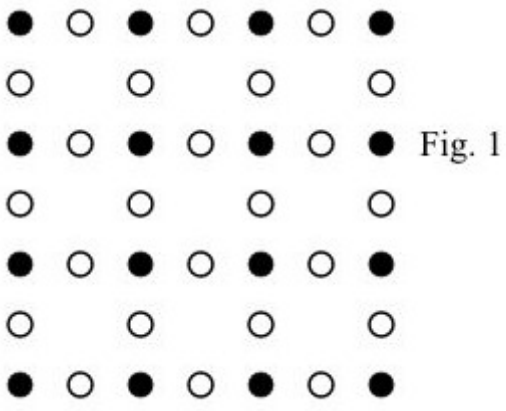

Fig. 1

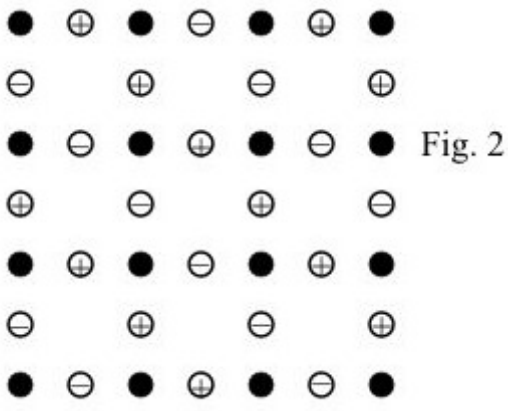

Fig. 2

Exercise 2: *Examples of Fermi gases*

1. *Electrons in 3D:* Calculate the Fermi energy for electrons in tungsten (Z=2, density 10.3 g cm^{-3}). What is the corresponding Fermi temperature (i.e., the Fermi energy expressed in temperature units)?

2. *Electrons in 1D:* Some long-chain organic molecules with mobile electrons given by donors are 1D conductors. Derive expressions for E_F for a 1D conductor with n free electrons per unit length. Evaluate E_F for an atomic spacing of 2.5 Å between donor atoms and 0.5 free electron per donor atom.

3. *Liquid ^{3}He:* Liquid ^{3}He (it liquifies at about 3 K), the light isotope of helium, has spin 1/2 and so is a fermion. Making the approximation that ^{3}He as a perfect gas and noting that its density is 0.081 g cm^3, calculate the Fermi energy E_F .

Exercise 3: Tight binding *model*

Consider s-type electrons in a one-dimensional lattice with lattice paramenter a. Make use of a *tight binding* model with *nearest and next-nearest neighbor hopping* and with negligible *overlap*. With reference to the texbook for the definitions of $\gamma(\mathbf{R})$, consider: $\gamma(\mathbf{R}_{NN})=t$ where $\mathbf{R}_{NN}$ is for nearest neighbors, $\gamma(\mathbf{R}_{NNN})=t'$ where $\mathbf{R}_{NNN}$ is for next-nearest neighbors, $\gamma(\mathbf{R})=0$ otherwise.

1. Write explicitely the expression for the energy band. (for the sake of simplicity, write $E_s + \beta \equiv E_0$, again with reference to the textbook for the definition of β)

2. Consider $t' = -t$ and make a plot of the band in the first Brillouin zone, specifying peculiar points.

3. Calculate the effective mass in the points corresponding to the maxima and minima of the band calculated in (2).

4. Make the extension to the 2d case, considering a square lattice but with *nearest neighbor hopping* only and again with negligible *overlap*. Write explicitely also in this case the expression for the energy band. (again write $E_s + \beta \equiv E_0$).

5. What is the Fermi energy in case of half-filling of the band, i.e. in case of one spin-1/2 electron per unit cell and at T=0?

6. For the case considered above (5), draw the Fermi surface ("curve", actually, in 2d) in the (k_x, k_y) plane.

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

- When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.