**Condensed Matter Physics I**
**final written test**
**academic year 2014/2015**
**July 13, 2015**

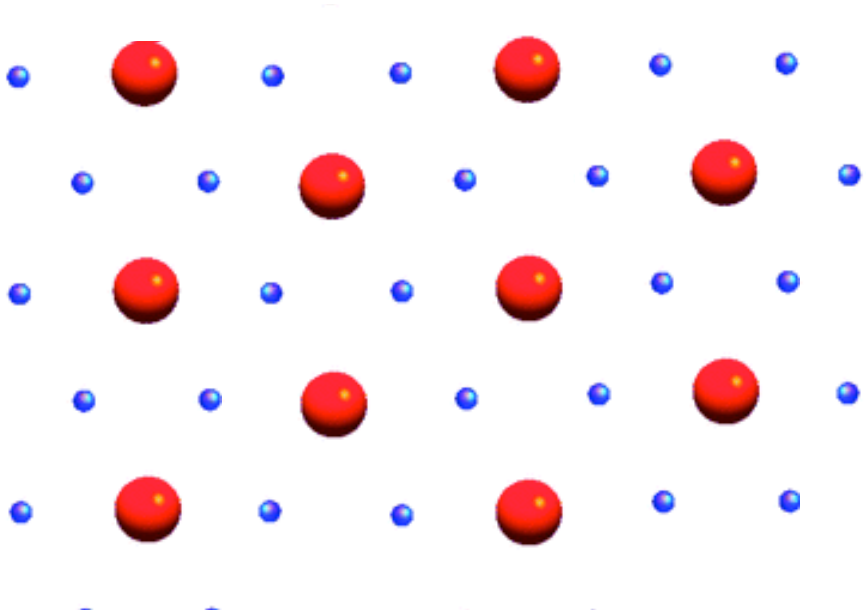
(*Time: 3 hours*)

*NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.*

## **Exercise 1**: *Crystalline structures*

*(You may solve this first exercise directly on this sheet of paper)*
The figure below shows a 2D periodic array of atoms of two different types.

1. Identify the type of Bravais lattice, write the primitive basis vectors $\{\mathbf{a}_i\}$, sketch them in the figure, together with the corresponding primitive unit cell.
2. Is this a *simple* Bravais lattice or is it a *lattice with a basis*? If it is a *lattice with a basis*: (i) how many points are in the basis? (ii) sketch them in the figure and write the corresponding vectors.
3. Write the primitive vectors $\{\mathbf{b}_i\}$ of the reciprocal lattice.
4. Write the geometrical structure factors $S(\mathbf{K})$ on a generic reciprocal lattice vector $\mathbf{K}$.
5. Specify the expression of $S(\mathbf{K})$ if all the atoms have the same atomic form factor.
6. Show that in the latter case there are some reciprocal lattice vectors $\mathbf{K}$ where $S(\mathbf{K})$ vanishes. Show that the removal of these vectors of zero structure factor from the reciprocal lattice reduces it to a new lattice. Specify which one and why.



**Exercise 2**: *Band structure of a 1D solid*

Consider a 1D band whose energy is given by $E(k) = E_0 - t\cos(ak)$.

1. Calculate explicitly the density of states $g(E)$ and make a plot. Check whether (and, in case, where) $g(E)$ has the expected van Hove singularities.
2. Calculate explicitly the group velocity $v(k)$. Calculate the effective mass $m^*(k)$ at the extrema of the band and discuss the character of the corresponding charge carriers.
3. Suppose the band is half occupied. What is $v_F$, the group velocity at the Fermi level?
4. Calculate the Fermi energy for 1 and 2 electrons per unit cell.
5. For one electron per unit cell, calculate the low-temperature specific heat (per cell)!
6. Consider always a 1D solid, but well described by a free electron picture. Calculate the Fermi energy in case of 1 and 2 electrons per unit cell. Can you comment on the conductivity of this solid? Justifying your answer.