

**Condensed Matter Physics I**  
**final written test**  
**academic year 2013/2014**  
**July 14, 2014**

(Time: 3 hours)

*NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.*

**Exercise 1: Crystalline structures**

Consider a 2D solid with primitive lattice vectors  $\vec{a}_1 = a(1, 0)$  and  $\vec{a}_2 = a\left(\frac{1}{2}, 1\right)$ .

1. Calculate the area of the primitive cell.
2. Find the primitive vectors of the reciprocal lattice  $\vec{b}_1$  and  $\vec{b}_2$ .
3. Make a *precise* drawing in the reciprocal space of the first and second Brillouin zones, and calculate their areas.
4. Consider now to have a Bravais lattice with the primitive lattice vectors as above and a basis of two atoms within the primitive cell, with  $\vec{d}_1 = (0, 0)$  and  $\vec{d}_2 = \left(\frac{a}{2}, 0\right)$  and atomic form factors  $f_1$  and  $f_2$  respectively. Calculate explicitly the structure factor  $S(\vec{K})$ .
5. In case of  $f_1 = f_2$ , on which reciprocal lattice vectors  $\vec{K}$  the structure factor is not vanishing? (Give the expression of such  $\vec{K}$  vectors)
6. To which structure in the direct space do they correspond? Comment the result.

**Exercise 2:** *Band structure of a 1D solid*

Consider a 1D band whose energy is given by  $E(k) = E_0 - t \cos(ak)$ .

1. Calculate explicitly the density of states  $g(E)$  and make a plot. Check whether (and, in case, where)  $g(E)$  has the expected van Hove singularities.
2. Calculate explicitly the group velocity  $v(k)$ . Calculate the effective mass  $m^*(k)$  at the extrema of the band and discuss the character of the corresponding charge carriers.
3. Suppose the band is half occupied. What is  $v_F$ , the group velocity at the Fermi level?
4. Calculate the Fermi energy for 1 and 2 electrons per unit cell.
5. For one electron per unit cell, calculate the low-temperature specific heat (per cell)!
6. Consider always a 1D solid, but well described by a free electron picture. Calculate the Fermi energy in case of 1 and 2 electrons per unit cell. Comment his behaviour in terms of conductivity, justifying your answer.