Condensed Matter Physics I
II test - 19 January 2016
(3 hours)

- Solve all the exercises.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Crystalline structures*

1. Prove that the ideal c/a ratio for the hexagonal close-packed structure is $\sqrt{8/3}$.
2. Sodium metal transforms from BCC to HCP at about 23 K (the martensite transition). Assume that the density remains constant, and find the lattice constant of the HCP phase, given that the cubic lattice constant is 4.23 Å and the c/a ratio is ideal.
3. For the HCP phase, calculate the *s*tructure factor.
4. Show that its value leads to a vanishing energy gap for states on the hexagonal face of the Brillouin zone for this structure in the weak potential approximation.
5. Nickel metal has FCC structure and lattice parameter $a = 3.524$ Å. What is the family of planes {hkl} with an interplanar spacing of d = 1.576 Å? Give the Miller indices, sketch a conventional cubic cell and a representative plane.

Exercise 2: *Van Hove singularities and Tight binding model*

1. Discuss the possible van Hove singularity of the electron density of states $g(E)$ in 1D. Calculate their form.
2. Consider now specifically a 1D lattice with spacing a described by the tight binding approximation, s band and nearest-neighbor hopping only. Show that the band dispersion is $E(k) = E_0 - \gamma cos(ka)$ ($\gamma > 0$). Calculate the electron density of states $g(E)$.
3. Show that the velocity is zero at the Brillouin edge and zone center.
4. Check if it has Van Hove singularities, specify for which values of k, and calculate their expression.
5. Calculate the Fermi velocity v_F and the Fermi energy E_F for 0.5, 1 and 2 electrons per unit cell.
6. Calculate the low-temperature specific heat in the case of one electron per cell.