Condensed Matter Physics I
I test - 23 November 2010
(3 hours)

(*Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.*)

Exercise: *Free electron gas*

1. Write the expression of the density of states $g(E)$ for free electron gas in 1D and 3D respectively, justifying the results. Write for both cases the electron density n in terms of an energy integral, using the density of states $g(E)$ and the Fermi function $f(E, \mu)$.

2. The behavior of the chemical potential $\mu(T)$ for small non-zero temperatures is the following: for fixed n, increasing T from 0K, $\mu(T)$ increases for a 1D Fermi gas, but decreases in 3D. Explain qualitatively these results, using the expressions written above for n (do not make use of Sommerfeld expansion).

3. Using the Sommerfeld expansion, give the explicit expression for the change of $\mu(T)$ in 1D and 3D in terms of T and with respect to $\mu(T = 0K)$ (n should not compare in such expressions).

4. From the 3D expression, evaluate numerically $\mu(T = 0K)$ and $\mu(T = 300K)$ for a K crystal, characterized by a BCC structure with lattice parameter 5.23 Å (Neglect the lattice thermal expansion).

5. Calculate the plasma frequency of K.

Exercise 2: *Crystalline structures*

1. The perovskite structure of Barium titanide is shown in the figure on the left, with Ba on the corners. Describe the Bravais lattice, and write the primitive translation vectors and the vectors of the basis, specifying which are Ba, Ti, O.

2. For each of the three atoms Ba, Ti, and O: how many and which kind of atoms are they coordinated with?

3. Calculate the structure factor S($\mathbf{K}$) in terms of the atomic form factors f_{Ba}, f_{Ti}, f_O.

4. Assume now that f_{Ba}, f_{Ti}, f_O are independent on $\mathbf{K}$ and that f_{Ba}=f_{Ti}. Can you determine for which $\mathbf{K}$ vectors the structure factor S($\mathbf{K}$) is maximum and how much is it?

5. The fluorite structure of CaF_2 is shown in the figure. Which is Ca and which is F? Justify your answer.

6. Describe the kind of Bravais lattice, specify how many atoms are in the basis, and write the primitive translation vectors and the vectors of the basis (indicating with a the side of the conventional cube show in the figure).

7. Describe the coordination of Ca and of F.

8. Calculate the structure factor S($\mathbf{K}$) in terms of the atomic form factors f_{Ca} and f_F.