Condensed Matter Physics I
final written test
academic year 2014/2015
February 25, 2015

(*Time: 3 hours*)

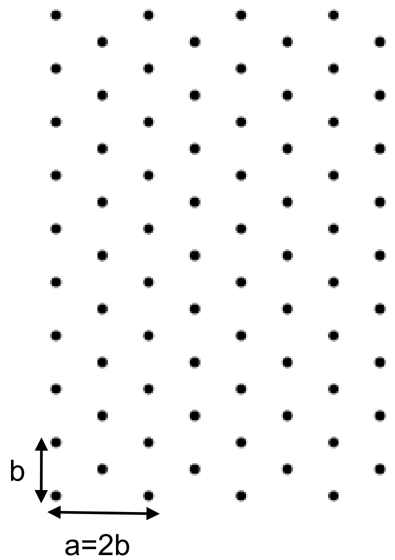
Exercise 1: *Crystalline structures*

Consider the two-dimensional lattice shown in the figure with a= 2.5 Å.

1. How you will classify it? Find its primitive translation vectors and draw its Wigner-Seitz cell. What is the area of this unit cell?

2. Draw the reciprocal lattice, give its primitive vectors specifying their k_x and k_y coordinates.

3. Draw the first Brillouin zone. What is its area?



Exercise 2: *Free-electron model*

Consider Al at room temperature. Its electron density is $n = 18.1 \cdot 10^{22}/cm^3$ and its electrical resistivity is $\rho = 2.45\ \mu\Omega \cdot cm$.

1. Find its electron relaxation time τ and electron mean free path ℓ (consider the Sommerfeld model).

2. Consider AC conductivity. At which frequency w the conductivity $\sigma(\omega)$ will be 1/10 of its zero-frequency value?

3. Give an estimate of the thermal conductivity of Al.

Exercise : *Semi-classical model of the electron dynamics and Bloch oscillations*

Consider the semi-classical model of the electron dynamics. We focus on the one-dimensional tight-binding model with the dispersion relation $E(k) = -2t\cos(ka)$, where t is the nearest neighbor hopping constant and a the lattice constant (for simplicity we consider only one band).

1. Consider an applied uniform electric field E. Calculate analytically $k(t)$ and $x(t)$. Show that E does not accelerate the electrons but lets them oscillate around some fixed position.

2. Give the expression of the amplitude of the oscillations in real space as a function of the applied electric field E.

3. Give the expression of the period of the oscillations as a function of the applied electric field E and calculate how big a field must be applied in Cu in order that a cycle can finish before the electron scatters. Consider for Cu a relaxation time τ of 21×10^{-14} sec and a lattice spacing of 0.361 nm.

4. For "normal" electric fields and typical relaxation times, in which case we could observe Bloch oscillations? *(Hint: Which other parameter of the physical system does enter in the expression of the period of oscillations?)*

5. We now add a small damping term, so that the rate of change of the quasi-momentum is given by
$$\hbar\dot{k} = F_{ext} - \frac{m\dot{r}}{\tau}$$
where τ is the relaxation time. Which is $k(t)$? Can this damping term lead to a vanishing of the oscillations and thus to a stationary solution?

6. What would the corresponding condition be and how would the stationary solution look like?

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

- When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.