DENSITY OF STATES

Aim: calculate a global quantity (e.g.: total energy density) from contributions which depend on $\vec{k}$ only through $\mathcal{E}$:

spin degeneracy

$$\boxed{f = \frac{F_{TOT}}{V} = \frac{1}{V}\sum_{\vec{k}} F\left(\mathcal{E}(\vec{k})\right) \overset{(3D)}{=} \frac{2}{(2\pi)^3}\int_{\vec{k}\text{ space}} F\left(\mathcal{E}(\vec{k})\right) d\vec{k}}$$

Examples:

$F_{TOT} = E_{TOT}$, $F\left(\underbrace{\mathcal{E}(\vec{k})}\right) = \begin{cases} \mathcal{E}(\vec{k}) & \text{in case of } T = 0K,\ k \leq k_F \\ \underbrace{\mathcal{E}(\vec{k})}\ f_{FD}\left(\underbrace{\mathcal{E}(\vec{k})}\right) & \text{in case of } T \neq 0K \end{cases}$

dependence on $\vec{k}$

The $\int$ in $\vec{k}$-space can be transformed into an $\int$ in $\mathcal{E}$, through a function $g(\mathcal{E})$ which depends only on the functional dependence of $\mathcal{E}$ on $\vec{k}$ and on the dimension of the $\vec{k}$ space, (IMPORTANT RESULT!) NOT on F!

$$\boxed{f = \frac{1}{V}\sum_{k} F\left(\mathcal{E}(\vec{k})\right) = \int F(\mathcal{E})\, g(\mathcal{E})\, d\mathcal{E}}$$

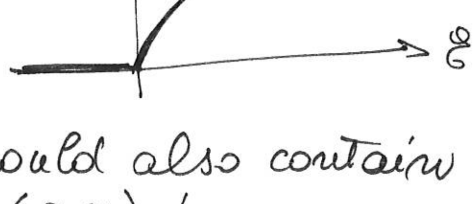
3D case (with and spin degeneracy): $d\vec{k} = k^2\, dk\, d\Omega$

$$\begin{aligned} f &= \frac{1}{4\pi^3}\int F\left(\mathcal{E}(\vec{k})\right) d\vec{k} \\ &= \frac{1}{4\pi^3}\int F\left(\mathcal{E}(\vec{k})\right) k^2\, dk\, d\Omega \\ &= \frac{4\pi}{4\pi^3}\int F\left(\mathcal{E}\cancel{(\vec{k})}\right)\sqrt{\frac{2m\mathcal{E}}{\hbar^2}}\,\frac{m}{\hbar^2}\, d\mathcal{E} \\ &= \int F(\mathcal{E})\underbrace{\frac{m}{\pi^2\hbar^2}\sqrt{\frac{2m\mathcal{E}}{\hbar^2}}}_{= g(\mathcal{E}) \text{ for } \mathcal{E} \geq 0}\, d\mathcal{E} \end{aligned}$$

free e^-: $\mathcal{E} = \dfrac{\hbar^2 k^2}{2m}$

$\hookrightarrow d\mathcal{E} = \dfrac{\hbar^2}{m} k\, dk$

and $\begin{cases} k = \sqrt{\dfrac{2m\mathcal{E}}{\hbar^2}} \\ \mathcal{E} \geq 0 \end{cases}$!



$F\left(\mathcal{E}(\vec{k})\right)$ is totally generic! It could also contain $f_{FD}\left(\mathcal{E}(\vec{k})\right)$!

DIMENSION OF $g(\mathcal{E})$: $\left[\frac{1}{V}\right]\left[\frac{1}{\mathcal{E}}\right]$