Ex. 2

see AM Ch 9 p 161 Fig 9.5

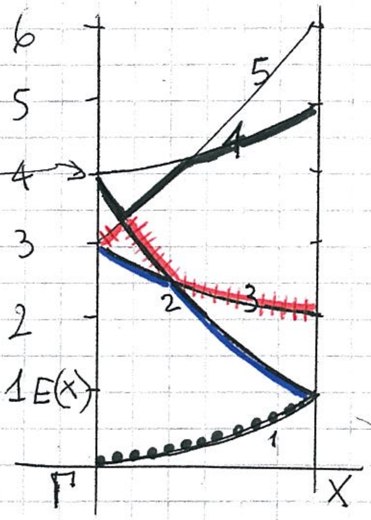
1) free el. FCC $\rightarrow$ rec lattice: BCC with side $\frac{4\pi}{a}$:

$\vec{b}_1 = \frac{2\pi}{a}(\bar{1}\,1\,1)$

$\vec{b}_2 = \frac{2\pi}{a}(1\,\bar{1}\,1)$

$\vec{b}_3 = \frac{2\pi}{a}(1\,1\,\bar{1})$

(2) also from transl by $\frac{4\pi}{a}(100)$



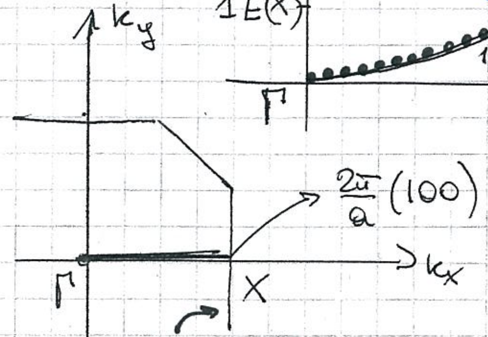
$\vec{b}_2+\vec{b}_3 = \frac{2\pi}{a}(2\,0\,0)$

$\vec{b}_1+\vec{b}_2 = \frac{2\pi}{a}(0\,0\,2)$

$\vec{b}_1+\vec{b}_3 = \frac{2\pi}{a}(0\,2\,0)$

$\vec{b}_1+\vec{b}_2+\vec{b}_3 = \frac{2\pi}{a}(1\,1\,1)$

$\vec{b}_2-\vec{b}_1 = \frac{2\pi}{a}(2\,\bar{2}\,0)$



section of the truncated octahedron

$\vec{k} \in [\Gamma, X]$: $\vec{k} = \frac{2\pi}{a}(k,0,0)$ with $k \in [0,1]$

$$E(X) = \frac{\hbar^2|\vec{k}_X|^2}{2m} = \frac{\hbar^2}{2m}\frac{4\pi^2}{a^2} = \frac{2\hbar^2\pi^2}{ma^2}$$

Branch(1): $E(k) = \frac{\hbar^2}{2m}|\vec{k}|^2 = E(X)\cdot k^2 \Rightarrow k=0: \frac{E}{E(X)} = 0$, $k=1: \frac{E}{E(X)} = 1$

Branch(2): $E(k) = \frac{\hbar^2}{2m}|\vec{k}-(\vec{b}_2+\vec{b}_3)|^2 = E(X)\cdot|(k-2,0,0)|^2 = E(X)\cdot(k-2)^2$

$k=0 \rightarrow \frac{E}{E(X)} = 4$

$k=1 \rightarrow \frac{E}{E(X)} = 1$

Branch(3): $E(k) = \frac{\hbar^2}{2m}|\vec{k}+\vec{b}_1|^2 = E(X)\cdot|(k-1,1,1)|^2 = E(X)(k^2+3-2k)$

$\Rightarrow k=0: \frac{E}{E(X)} = 3$; $k=1 \rightarrow \frac{E}{E(X)} = 2$

Idem for $\vec{k}-\vec{b}_2$, $\vec{k}-\vec{b}_3$, $\vec{k}-(\vec{b}_1+\vec{b}_2+\vec{b}_3)$

Branch(4): $E(k) = \frac{\hbar^2}{2m}|\vec{k}-(\vec{b}_1+\vec{b}_2)|^2 = E(X)|(k,0,2)|^2 = E(X)\cdot(k^2+4)$

Idem for $\vec{k}-(\vec{b}_1+\vec{b}_3)$ $\Rightarrow k=0: \frac{E}{E(X)} = 4$; $k=1 \rightarrow \frac{E}{E(X)} = 5$

Branch(5): $E(k) = \frac{\hbar^2}{2m}|\vec{k}-\vec{b}_1|^2 = E(X)|(k+1,\bar{1},\bar{1})|^2 = E(X)\cdot(k^2+3+2k)$

Idem for $\vec{k}+\vec{b}_2$, $\vec{k}+\vec{b}_3$ $\Rightarrow k=0: \frac{E}{E(X)} = 3$; $k=1 \rightarrow \frac{E}{E(X)} = 6$