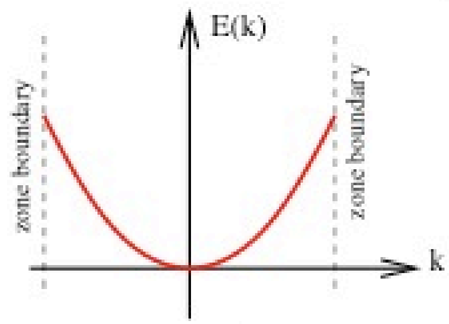
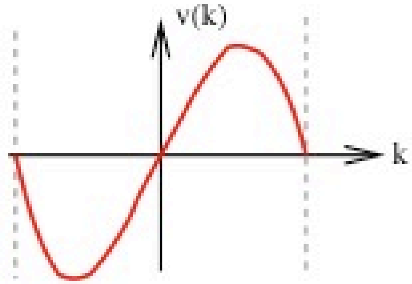
Motion in a uniform E field





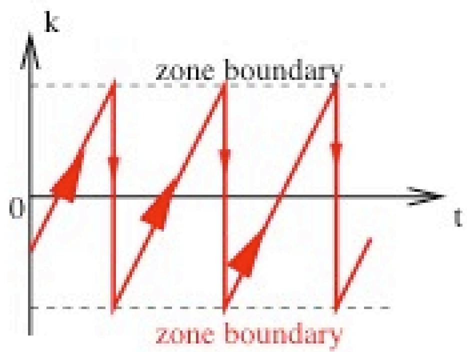
$$\hbar \frac{d\mathbf{k}}{dt} = -e\mathbf{E}$$

$$\mathbf{k}(t) = \mathbf{k}(0) - \frac{e\mathbf{E}}{\hbar} t$$

without collisions or for $t << \tau$

with collisions $\mathbf{k}$ saturates at

$$\mathbf{k}_{avg} = -\frac{e\mathbf{E}}{\hbar} t_{avg} = -\frac{e\mathbf{E}}{\hbar} \tau$$



without collisions or for $t << \tau$

electron velocity oscillates → electron motion is oscillatory

Bloch oscillations

But: if the band is filled an applied electric field cannot change k
→ no current is induced by an applied electric field

Motion in a uniform H field (i)

velocity $$\mathbf{v}_n = \dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial E_n}{\partial \mathbf{k}} \quad \leftarrow$$

equation of motion $$\hbar\dot{\mathbf{k}} = -e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_n \times \mathbf{H}\right)$$

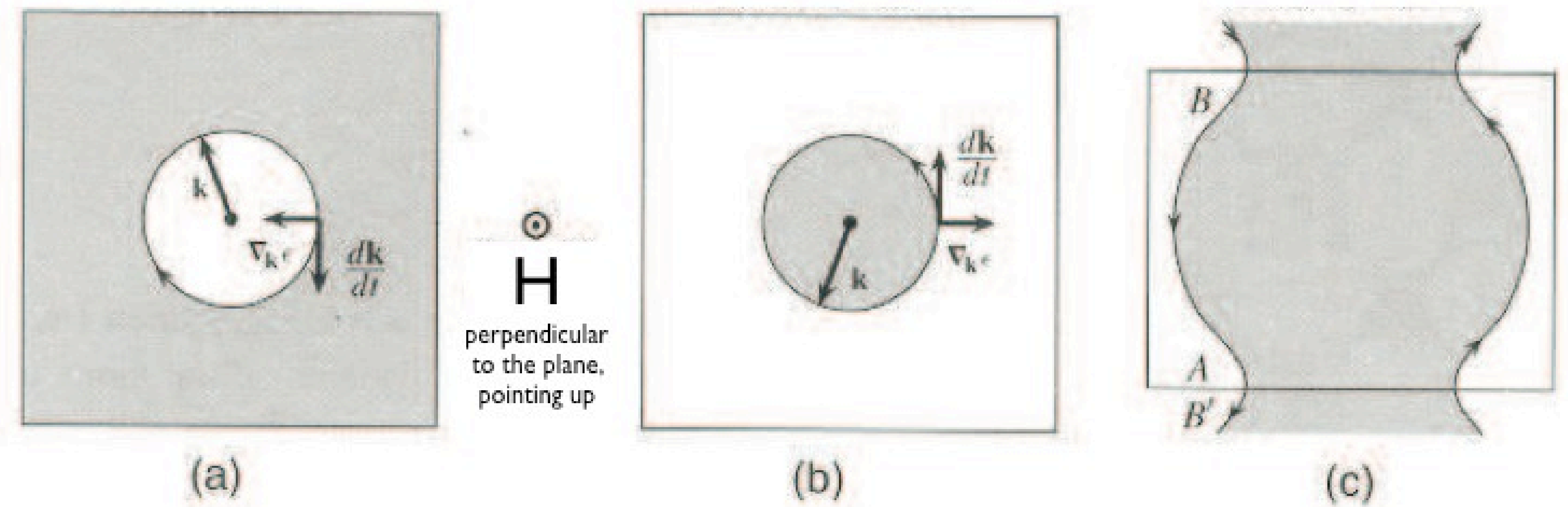
$$\Rightarrow \quad \hbar\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar c}\frac{\partial E_n}{\partial \mathbf{k}} \times \mathbf{H}$$

$$\Rightarrow \quad \mathbf{k} \text{ evolves } \perp \text{ to } \frac{\partial E_n}{\partial \mathbf{k}} \text{ and } \mathbf{H}:$$

electrons in a static magnetic field move on a curve of constant energy on a plane normal to **H**

(an electron on the Fermi surface will move in a curve on the Fermi surface)

Motion in a uniform H field (ii)



holelike orbit

electronlike orbit

open orbit

clockwise motion, as expected for a positively charged particle

anticlockwise motion, as expected for a negatively charged particle

Motion in a uniform H field (iii)

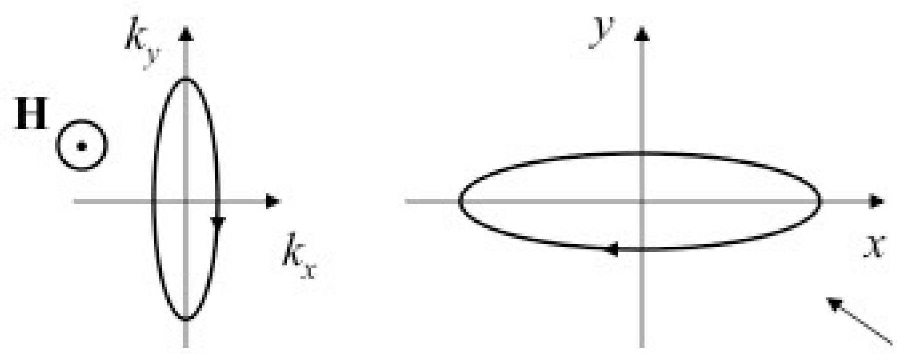
real space orbit vs *k*-space orbit

From the eqs. of motions it follows:

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar c}\frac{d\mathbf{r}_\perp}{dt} \times \mathbf{H} = -\frac{eH}{\hbar c}\frac{d\mathbf{r}_\perp}{dt} \times \hat{\mathbf{H}}$$

(where $\mathbf{r}_\perp$ is the projection of $\mathbf{r}$ on a plane $\perp\mathbf{H}$, and $\hat{\mathbf{H}} = \mathbf{H}/H$)

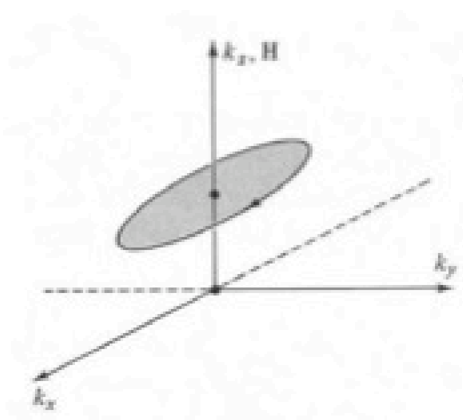
i.e. $\mathbf{r}$ and $\mathbf{k}$ evolve following orbits $\perp$ one to the other:

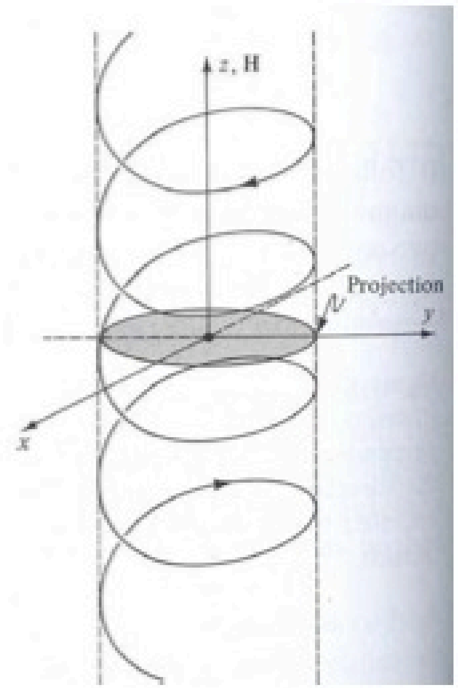


Motion in a uniform H field (iv)

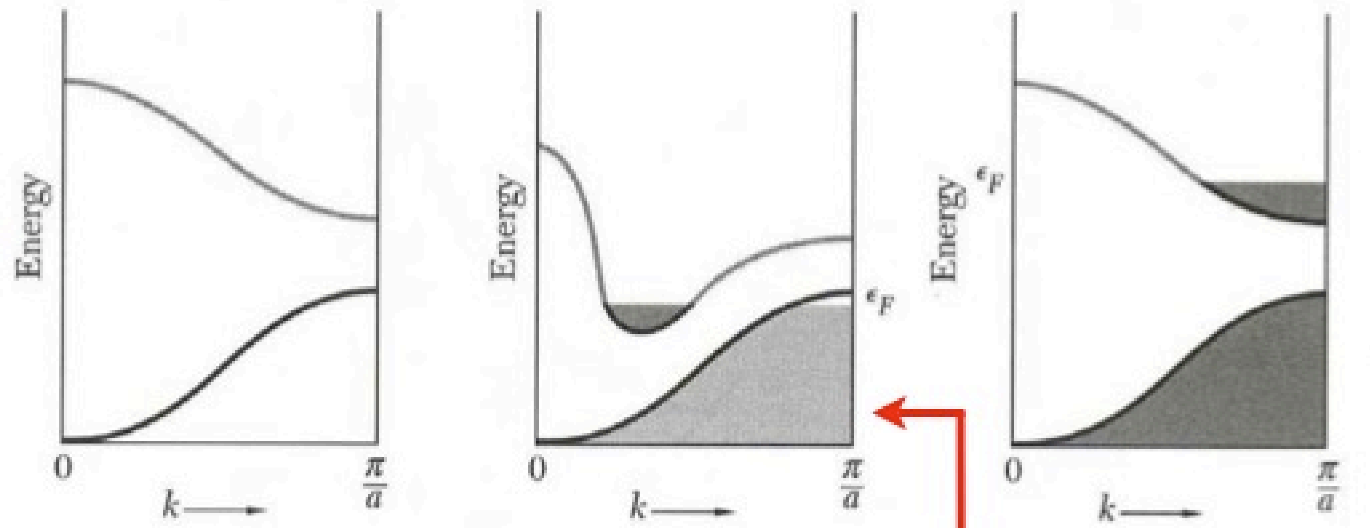
3D: the <u>projection of the real space orbit</u> in a plane perpendicular to the field is the k-space orbit rotated through 90^0 about the field direction

and scaled by the factor $l_H^2 = \dfrac{\hbar c}{eH}$





metals and insulators



insulator or semiconductor

metal or semimetal

metal

a certain number of bands are completely filled, all other remains empty

a configuration with a band gap can arise only if number of electrons per primitive cell is even

some bands are partly filled

(this is case for an odd number of el.; could be also with an even number of electrons but in presence of a band crossing)