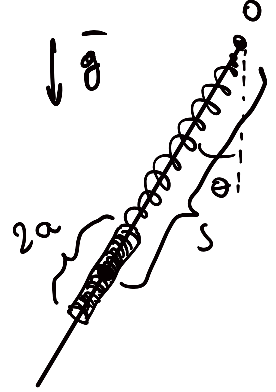

lungh. a riposo = 0

$$V = -mgs\cos\theta + \frac{1}{2}k(s-a)^2$$

1) Lagrangiana e matrice cinetica

$$T = \underbrace{\frac{m}{2}\left(\dot s^2 + s^2\dot\theta^2\right)}_{\text{en cin del c.m.}} + \frac{1}{2}I\dot\theta^2$$

$$I = \int_{-a}^{a} r^2 \rho\, dr = \left.\frac{r^3\rho}{3}\right|_{-a/2}^{a/2} = 2\frac{a^3\rho}{3} = \frac{ma^2}{3}$$

$$L = \frac{m\dot s^2}{2} + \frac{1}{2}\left(ms^2 + I\right)\dot\theta^2 + mgs\cos\theta - \frac{1}{2}k(s-a)^2$$

2) Eq. Lagrange

$$\frac{d}{dt}\frac{\partial L}{\partial \dot s} = m\ddot s \qquad \frac{\partial L}{\partial s} = ms\dot\theta^2 + mg\cos\theta - k(s-a)$$

$$\ddot s = s\dot\theta^2 + g\cos\theta - \frac{k}{m}(s-a)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = \frac{d}{dt}\left[(ms^2+I)\dot\theta\right] = 2ms\dot s\dot\theta + (ms^2+I)\ddot\theta \qquad \frac{\partial L}{\partial\theta} = -mgs\,\mathrm{sen}\,\theta$$

$$\ddot\theta = -\frac{ms\left(g\,\mathrm{sen}\,\theta + 2\dot s\dot\theta\right)}{ms^2 + I}$$

3) Coord. cicliche? No, nessuna.
Se sì, cost. del moto.

4) Pti equil. e stabilità

$$V = -mgs\cos\theta + \frac{1}{2}k(s-a)^2$$

$$\frac{\partial V}{\partial s} = -mg\cos\theta + k(s-a) \rightarrow s = \frac{mg\cos\theta}{k} + a$$

esiste se $ka \le mg$

$$\cos\theta = -\frac{ka}{mg} < 0 \rightarrow \frac{\pi}{2} < \theta < \pi$$

$$\frac{\partial V}{\partial \theta} = mgs\,\mathrm{sen}\,\theta \rightarrow s = 0,\quad \theta = 0, \pi$$

$\theta = 0$ $\quad s = a + \frac{mg}{k}$

$\theta = \pi$ $\quad s = a - \frac{mg}{k}$

esiste se $ka \ge mg$

$$(s,\theta) = \left(a + \frac{mg}{K}, 0\right) \quad \left(a - \frac{mg}{K}, \pi\right) \quad \left(0, \pm\arccos\left(-\frac{Ka}{mg}\right)\right)$$

$$\partial^2 V = \begin{pmatrix} K & mg\,\text{sen}\,\theta \\ mg\,\text{sen}\,\theta & mgs\cos\theta \end{pmatrix}$$

$$\partial^2 V\Big(\underbrace{0, \pm\arccos\left(-\tfrac{Ka}{mg}\right)}_{\pm\theta_0}\Big) = \begin{pmatrix} K & mg\,\text{sen}\,\theta_0 \\ mg\,\text{sen}\,\theta_0 & 0 \end{pmatrix}$$

$\text{Tr} = K > 0$ **INSTAB.**

$\det = -mg\,\text{sen}^2\theta_0 < 0$ quando esiste

$$\partial^2 V\left(a + \frac{mg}{K}, 0\right) = \begin{pmatrix} K & 0 \\ 0 & mg\left(a + \frac{mg}{K}\right) \end{pmatrix}$$

STAB. (autovalori positivi)

$$\partial^2 V\left(a - \frac{mg}{K}, \pi\right) = \begin{pmatrix} K & 0 \\ 0 & -mg\left(a - \frac{mg}{K}\right) \end{pmatrix}$$

INSTAB. quando esiste (un autovalore negativo)

5) Linearizzazione attorno pto equil. stabile. e freq. piccole osc.

$$L = \frac{m}{2}\dot{s}^2 + \frac{1}{2}(ms^2 + I)\dot{\theta}^2 + mgs\cos\theta - \frac{1}{2}K(s-a)^2$$

$s_0 = a + \frac{mg}{K}$ $\qquad s = s_0 + \delta s$

$\theta = 0$

$$L = \frac{m}{2}\delta\dot{s}^2 + \frac{1}{2}(ms_0^2 + I)\dot{\theta}^2 + mg(s_0 + \delta s)\left(1 - \frac{\theta^2}{2} + \dots\right) - \frac{1}{2}K(\delta s + \overbrace{s_0 - a}^{mg/K})^2$$

$$\frac{m}{2}\delta\dot{s}^2 + \frac{1}{2}(ms_0^2 + I)\dot{\theta}^2 - mgs_0\frac{\theta^2}{2} + \cancel{mg\,\delta s} - \frac{1}{2}K\delta s^2 \cancel{- K\delta s\left(\frac{mg}{K}\right)}$$

$$\omega_s^2 = \frac{K}{m}$$

$$\omega_\theta^2 = \frac{\cancel{m} g s_0}{\cancel{m} s_0^2 + I/m} = \frac{g\left(a + \frac{mg}{K}\right)}{\left(a + \frac{mg}{K}\right)^2 + \frac{a^2}{3}} = \frac{g\left(a + \frac{mg}{K}\right)}{\frac{4}{3}a^2 + \frac{2amg}{K} + \left(\frac{mg}{K}\right)^2}$$

$$= \frac{g}{\underbrace{a + \frac{mg}{K} + a\,\frac{a/3}{a + mg/K}}_{\frac{mg}{K} + a\left(\frac{4a/3 + mg/k}{a + mg/k}\right)}}$$