Game Theory

Basic Concepts

- *Game* – a situation in which strategic behaviour is an important part of decision making
- *Players* – the decision makers in game
- *Actions & Strategies* – a player's plan of actions in a game
- *Payoffs* – the rewards enjoyed by a player at the end of a game
- Is Cooperation allowed?
- Information and common Knowledge

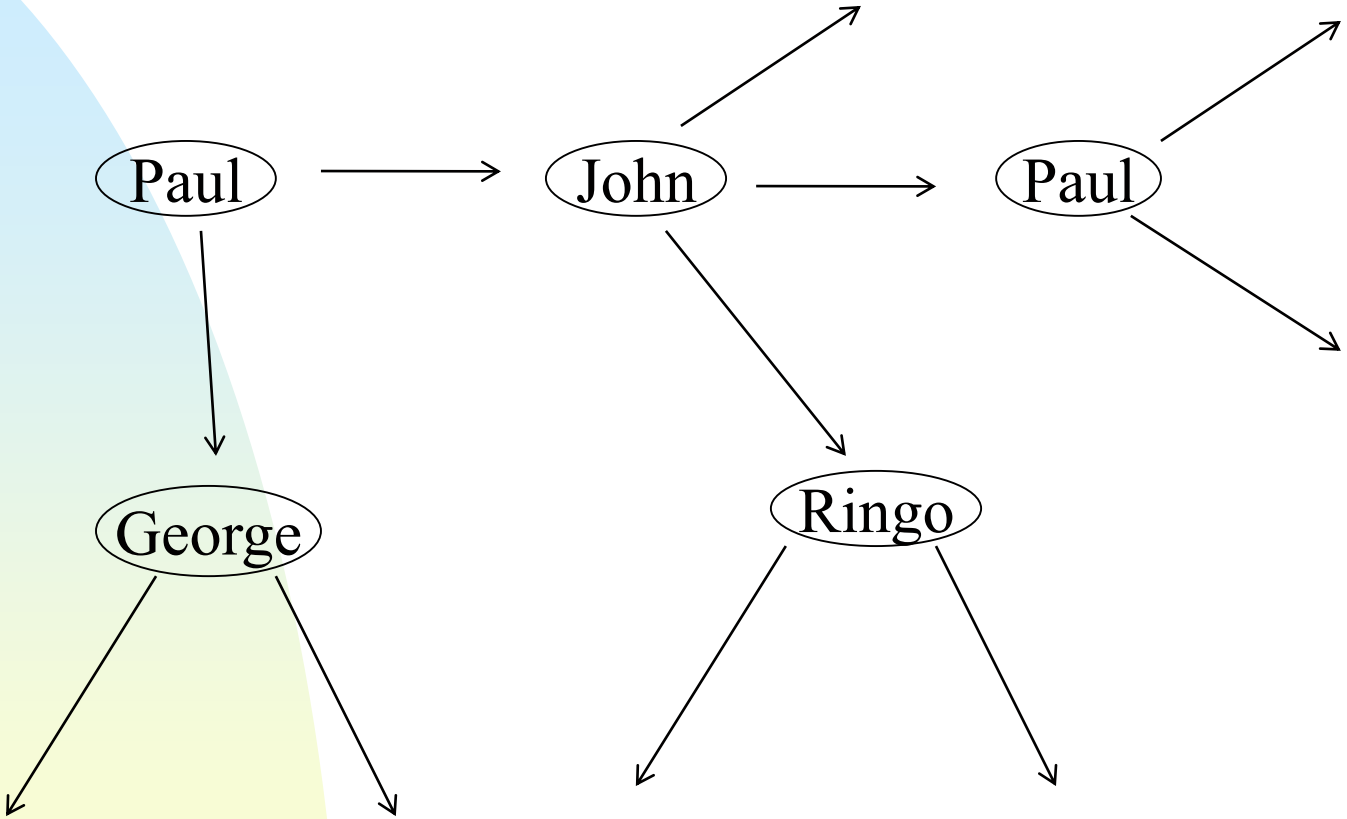
Simultaneous v. Sequential Move Games

- Games where players choose actions simultaneously are simultaneous move games.
  - Must anticipate what your opponent will do right now, recognizing that your opponent is doing the same.

Simultaneous v. Sequential Move Games

- Games where players choose actions in a particular sequence are sequential move games.
  - Examples: Chess, Bargaining/Negotiations.
  - Must look ahead in order to know what action to choose now.
  - Many sequential move games have deadlines/ time limits on moves.
- Many strategic situations involve both sequential and simultaneous moves.

Sequential game



Strategies

- A ***strategy*** must be a "comprehensive plan of action", a decision rule or set of instructions about which actions a player should take
- It is the equivalent of a memo, left behind when you go on vacation, that specifies the actions you want taken in every situation which could conceivably arise during your absence.
- Strategies will depend on whether the game is one-shot or repeated.

One-Shot versus Repeated Games

- One-shot: play of the game occurs once.
  - Players likely to not know much about one another.
  - Example - tipping on your vacation
- Repeated: play of the game is repeated with the same players.
  - Indefinitely versus finitely repeated games
  - Reputational concerns matter; opportunities for cooperative behavior may arise.

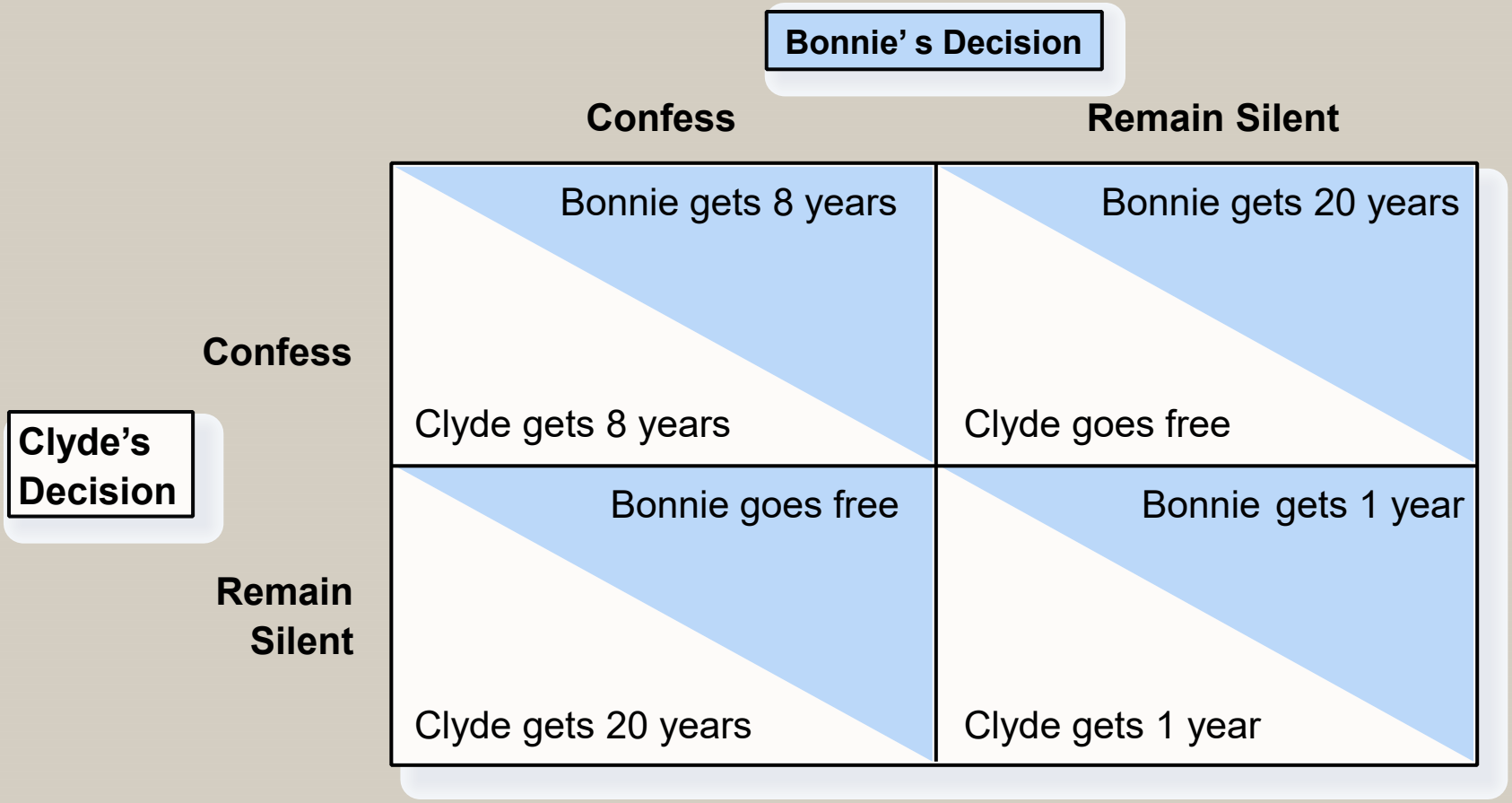
Information

- Players have *complete information* if they know exactly all the rules of the game, with payoffs and strategies available to all the other players
- If it is also known what moves others have made before them, than information is *perfect* (as in sequential games e.g. in Chess)

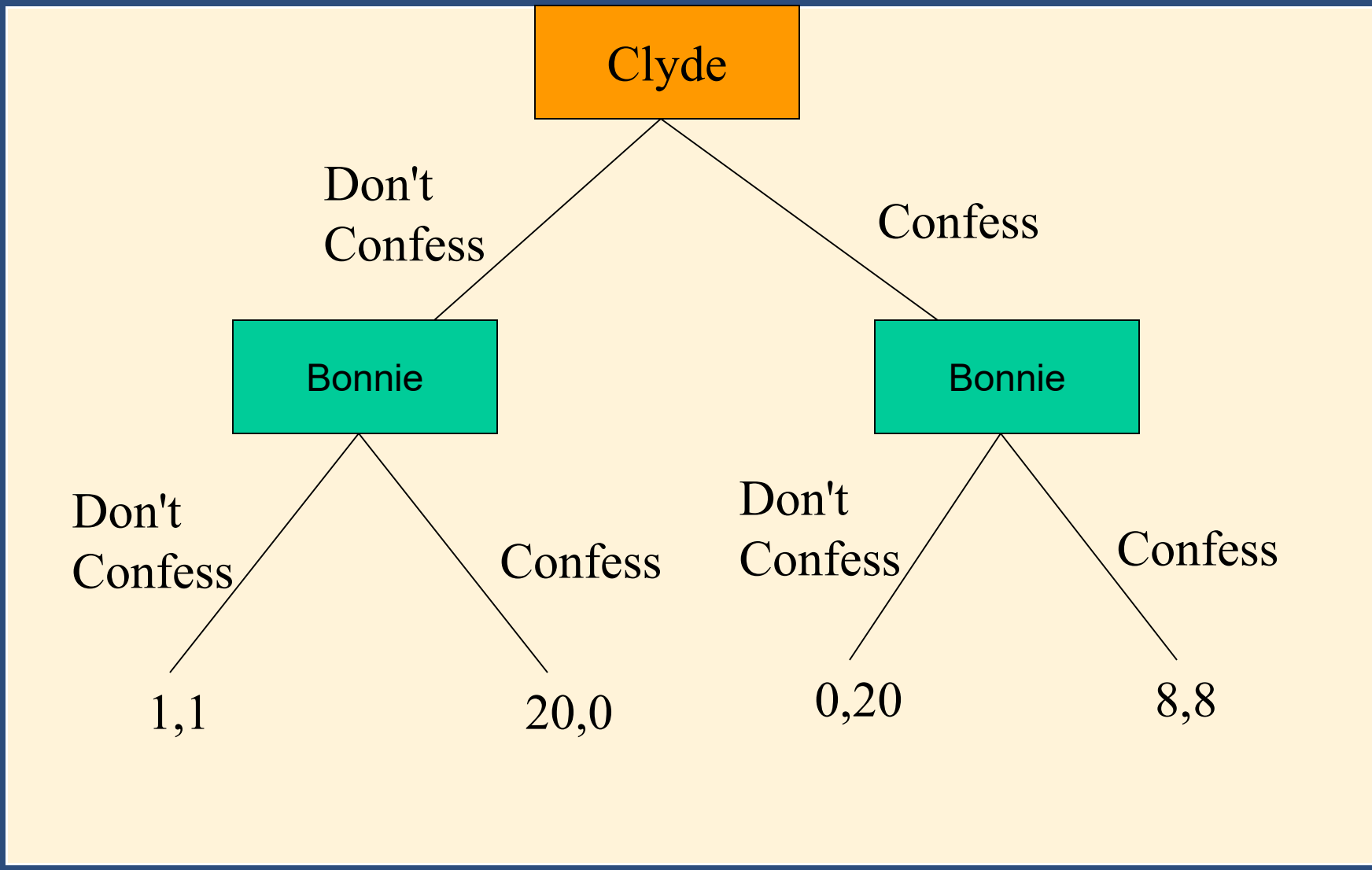
Game description

- *A standard or Normal form* – relates summary strategy to payoffs (simultaneous moves)
- *Extensive form or Game tree* – a graphical representation of individual moves by players

The Prisoners' Dilemma



Prisoners' Dilemma Game in “Extensive” Form

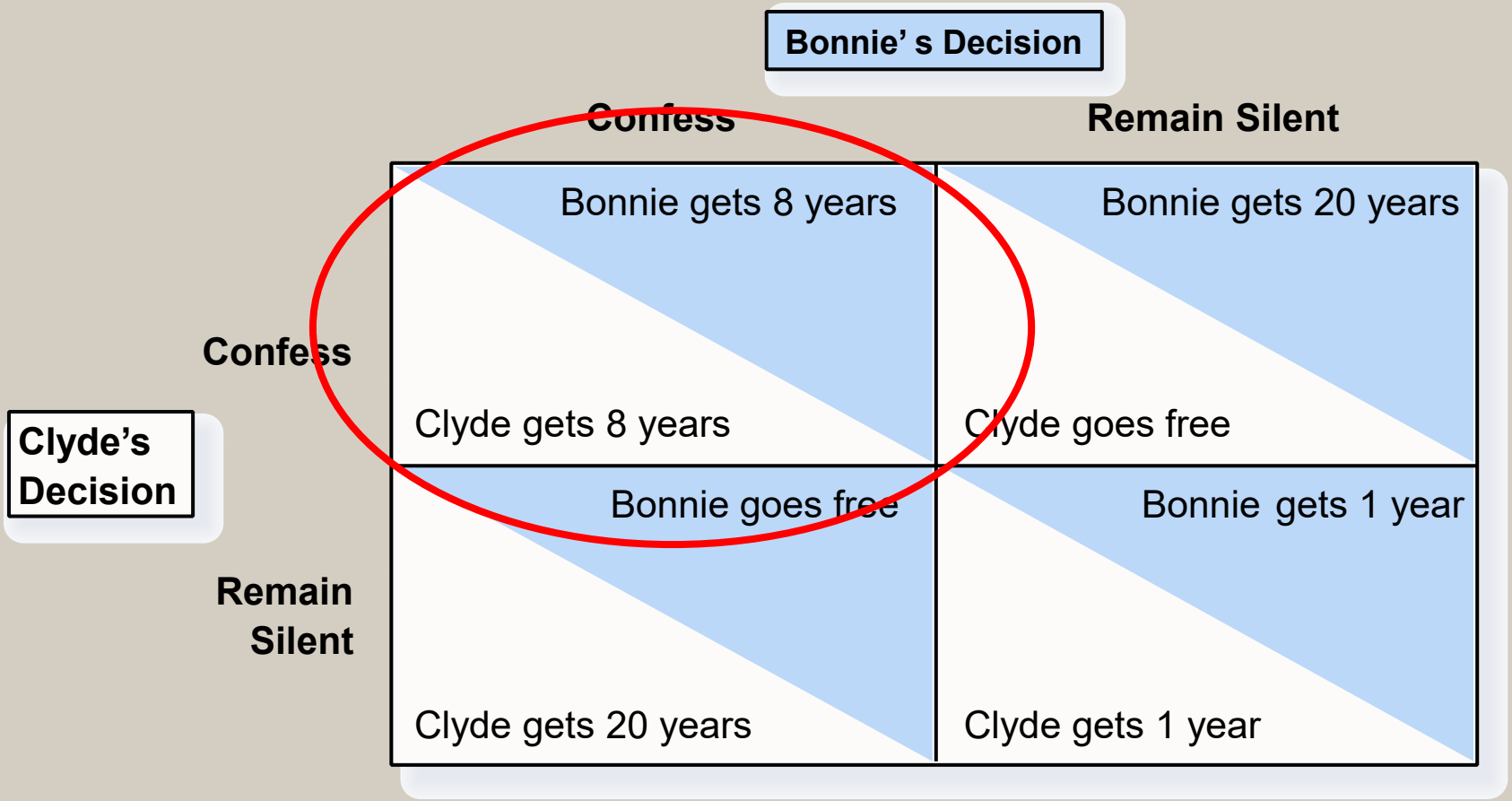


Prisoner dilemma

Dominant strategy – a strategy that works at least as well as any other one, no matter what any other player does

In the prisoner dilemma such a dominant strategy is….

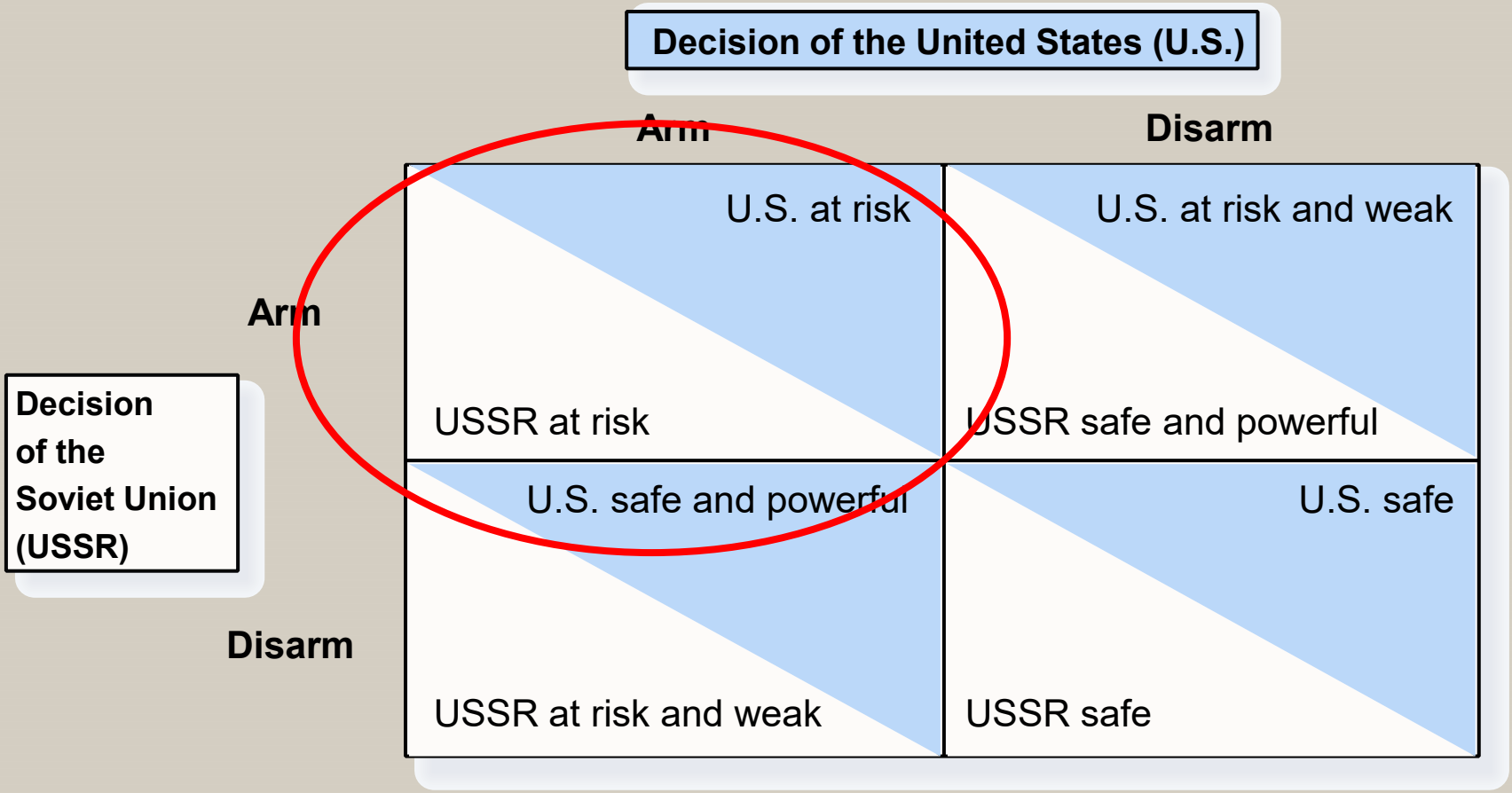
to confess



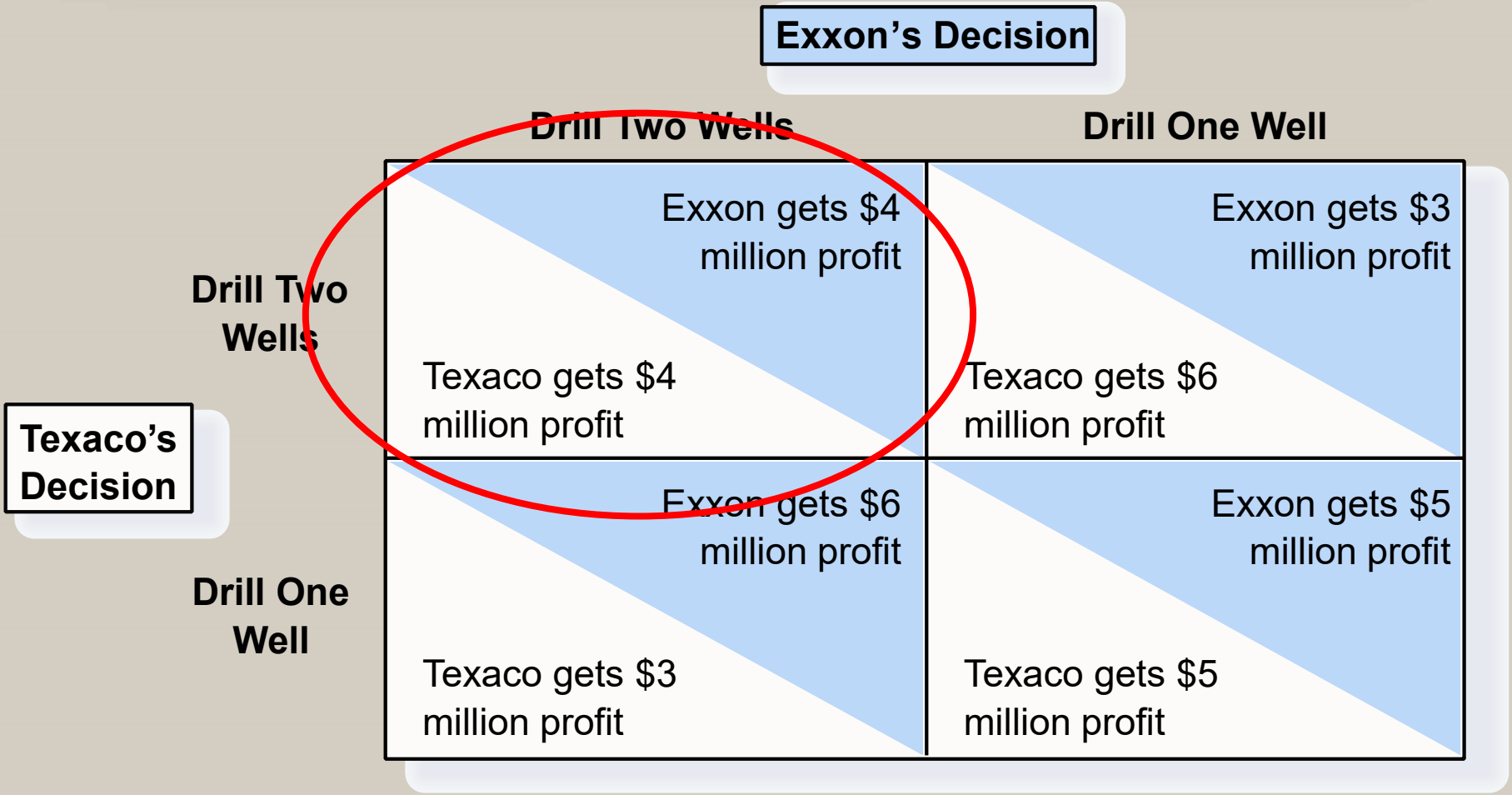
We don't need a solution concept but the less the better

- *Can you apply this framework to other issues?*

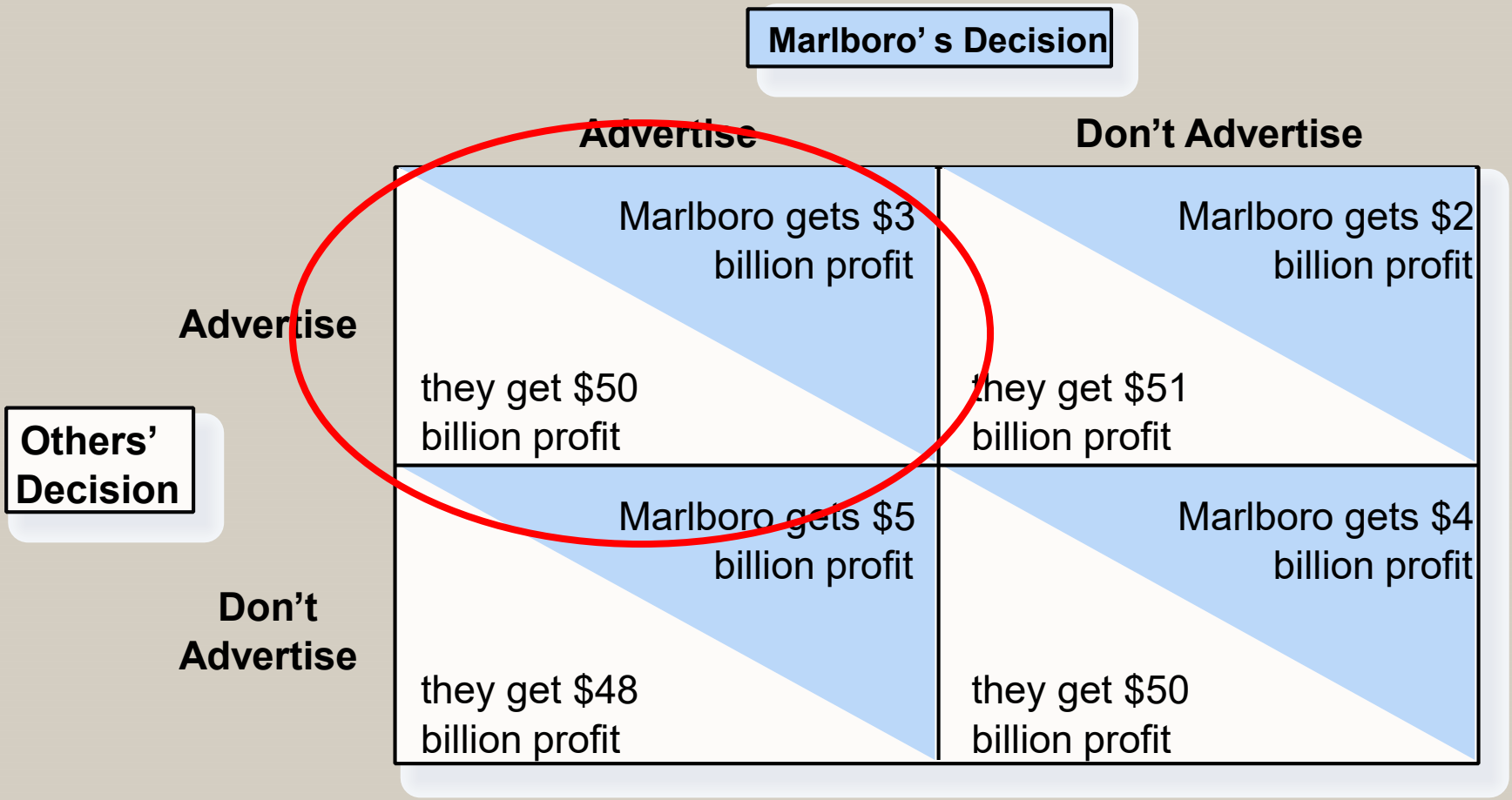
An Arms-Race Game



A Common-Resource Game



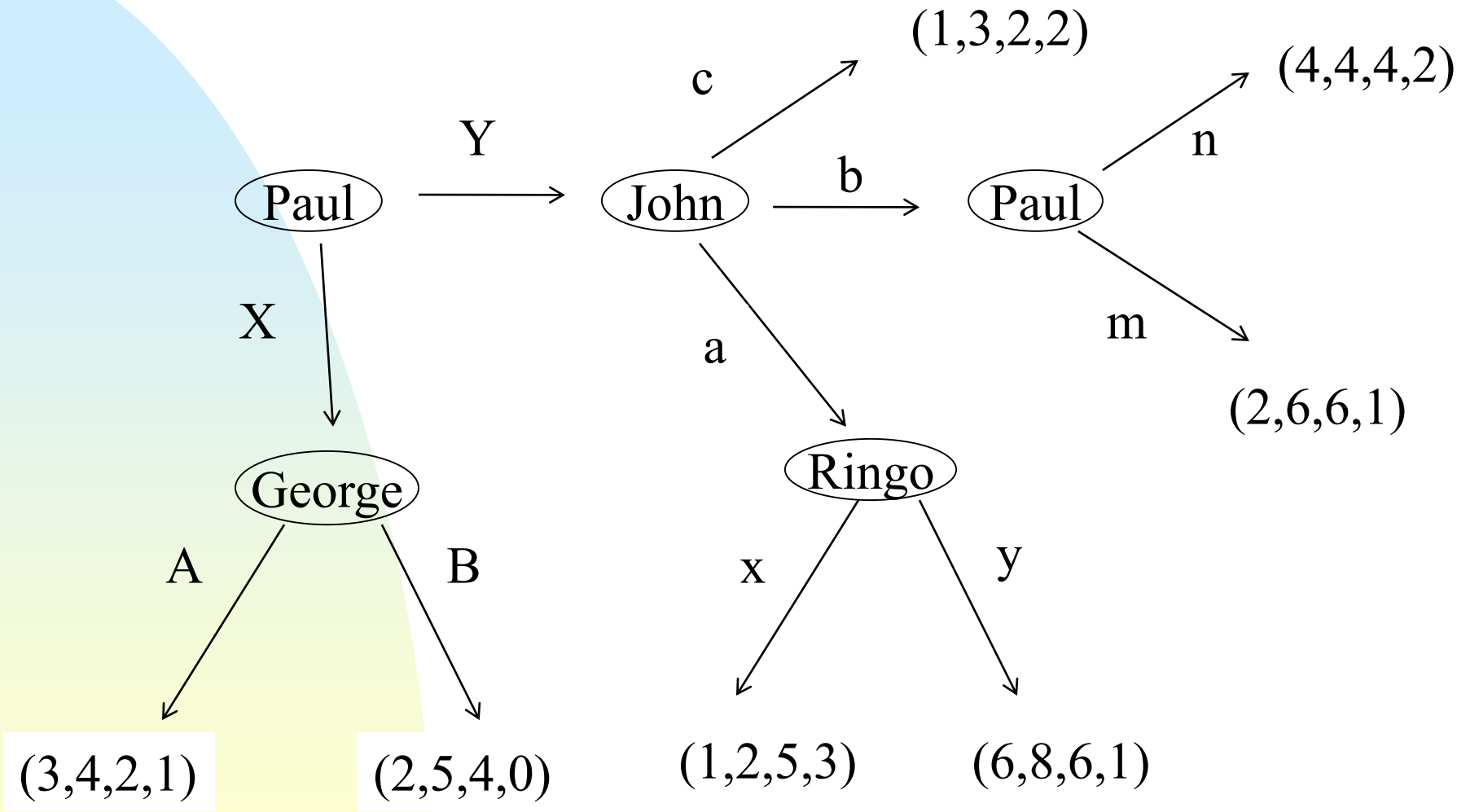
An Advertising Game



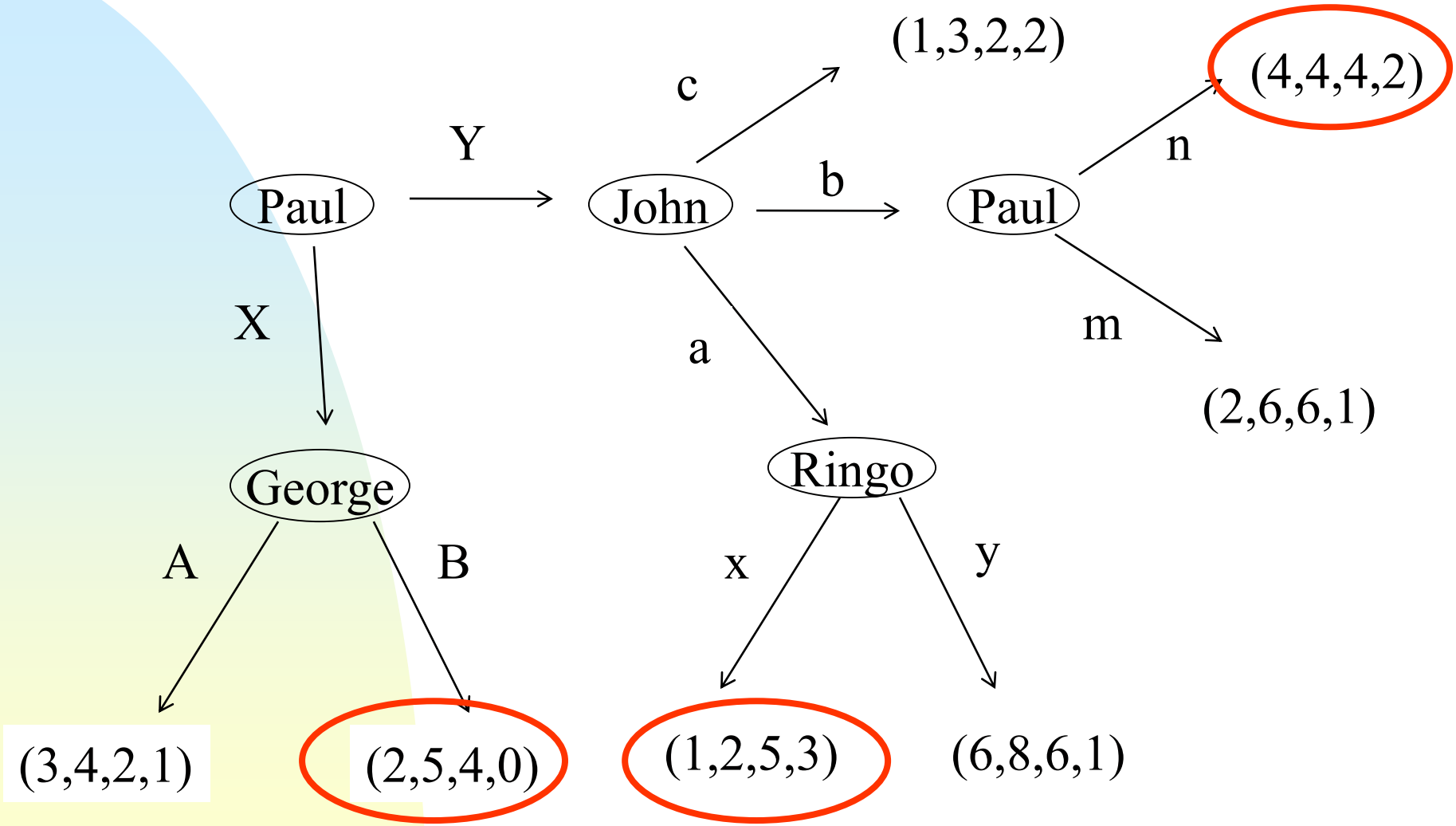
A dominant strategy

- Can be recognized in sequential games when you PRUNE the tree
- So you can solve the game!
- In the Beatles game payoffs refer to players in the very same order

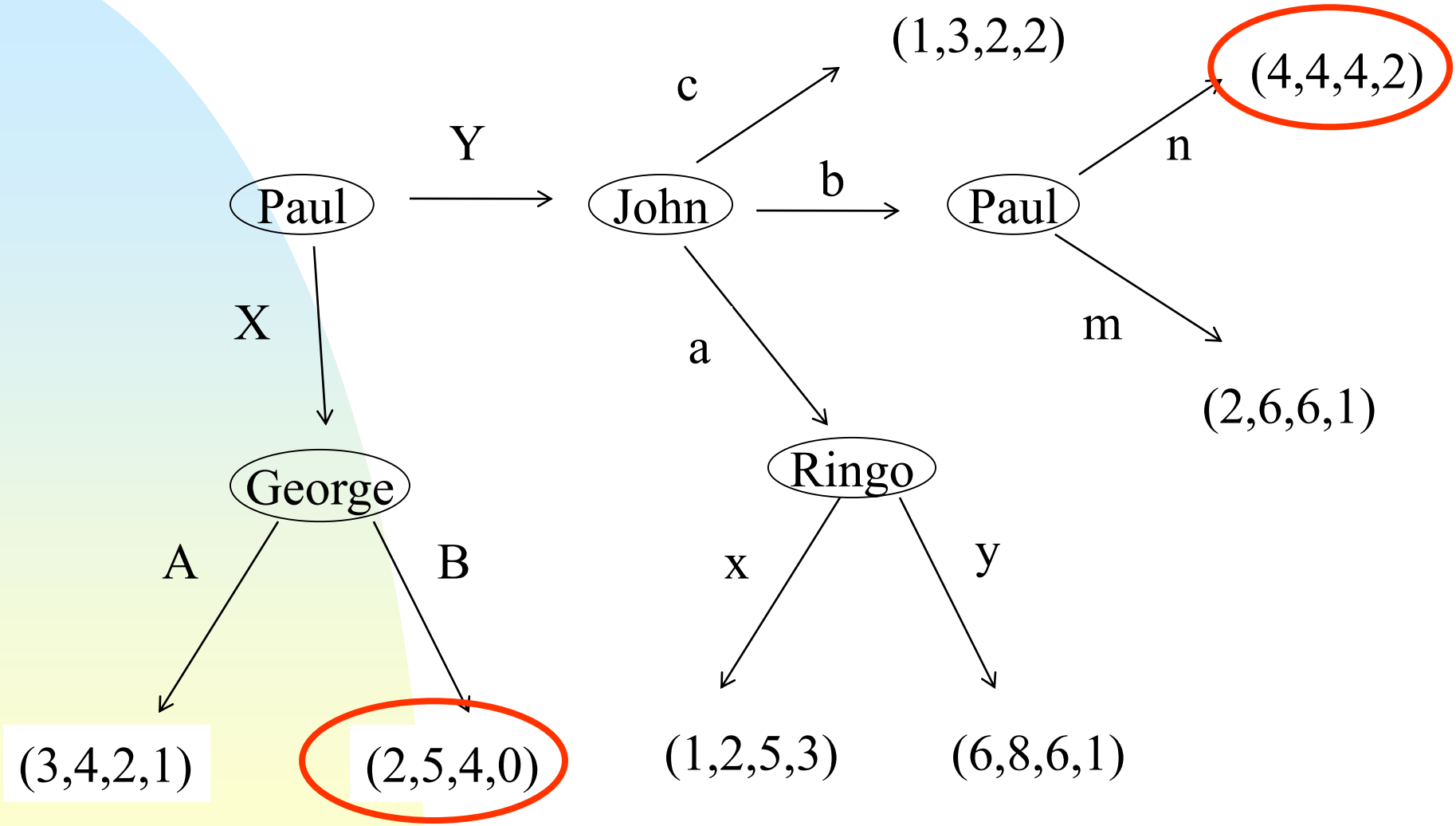
Paul, George, John and Ringo



Paul, John, George and Ringo



Paul, John, George and Ringo



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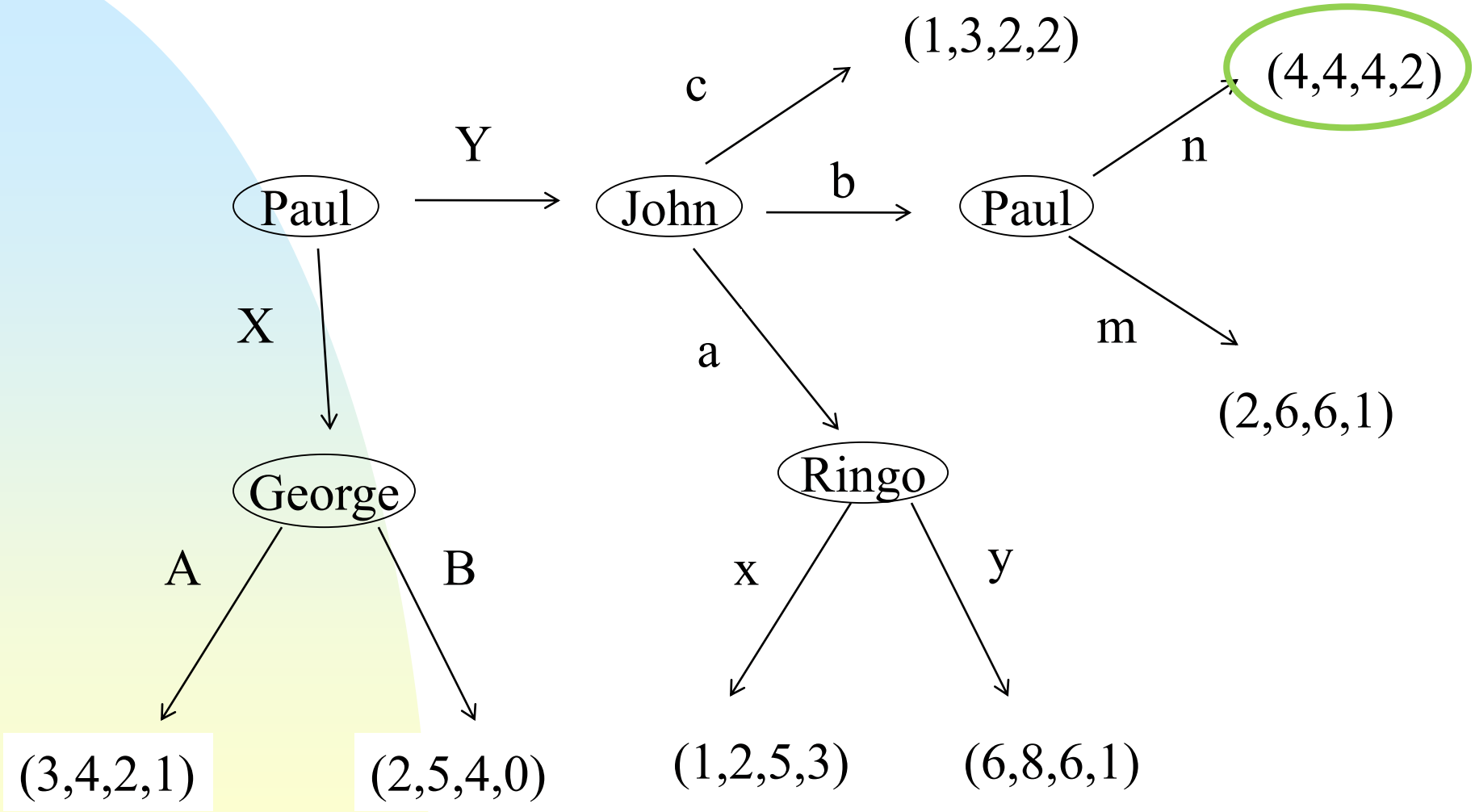


Dominance can reduce problems

		JAN		
		Old pros	Art museum	Cafeen
SAM	Old pros	6,4	4,3	4,2
	Art museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

Sam will never go to Cafeen as it is dominated by Old Pros

		JAN		
		Old pros	Art museum	Cafeen
SAM	Old pros	6,4	4,3	4,2
	Art museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

You can erase it! What's next? As Jan knows that….

		JAN		
		Old pros	Art museum	Cafeen
SAM	Old pros	6,4	4,3	4,2
	Art museum	2,1	5,5	2,2

Jan will never go to cafeen too (dominated by Art Museum)

		JAN		
		Old pros	Art museum	Cafeen
SAM	Old pros	6,4	4,3	4,2
	Art museum	2,1	5,5	2,2

Now? Dominance does not work any more….

		JAN: Old pros	JAN: Art museum
SAM	Old pros	6,4	4,3
SAM	Art museum	2,1	5,5

What about me and you?????

	talk	video	silent
stay	6,7	3,5	0,5
leave	5,4	5,3	6,5

Column 1 dominates column 2

	talk	video	silent
stay	6,7	3,5	0,5
leave	5,4	5,3	6,5

So you can erase column 2! Then?

	talk	silent
stay	6,7	0,5
leave	5,4	6,5

I need a device since there is no dominant strategy

Nash equilibrium - If each player has chosen a strategy and no player can benefit by changing his strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium

2 NASH equilibria!

	teach	silent
stay	6,7	0,5
leave	5,4	6,5

2 nash equilibria too….

		JAN: Old pros	JAN: Art museum
SAM	Old pros	6,4	4,3
SAM	Art museum	2,1	5,5

Nash equilibrium

The Nash equilibrium can be seen as the only sustainable outcome of rational negotiation in the absence of externally enforceable agreements:

There is no free riding!

Thus, **in equilibrium, no one has an incentive to change his strategy given the strategy choices of all others.**

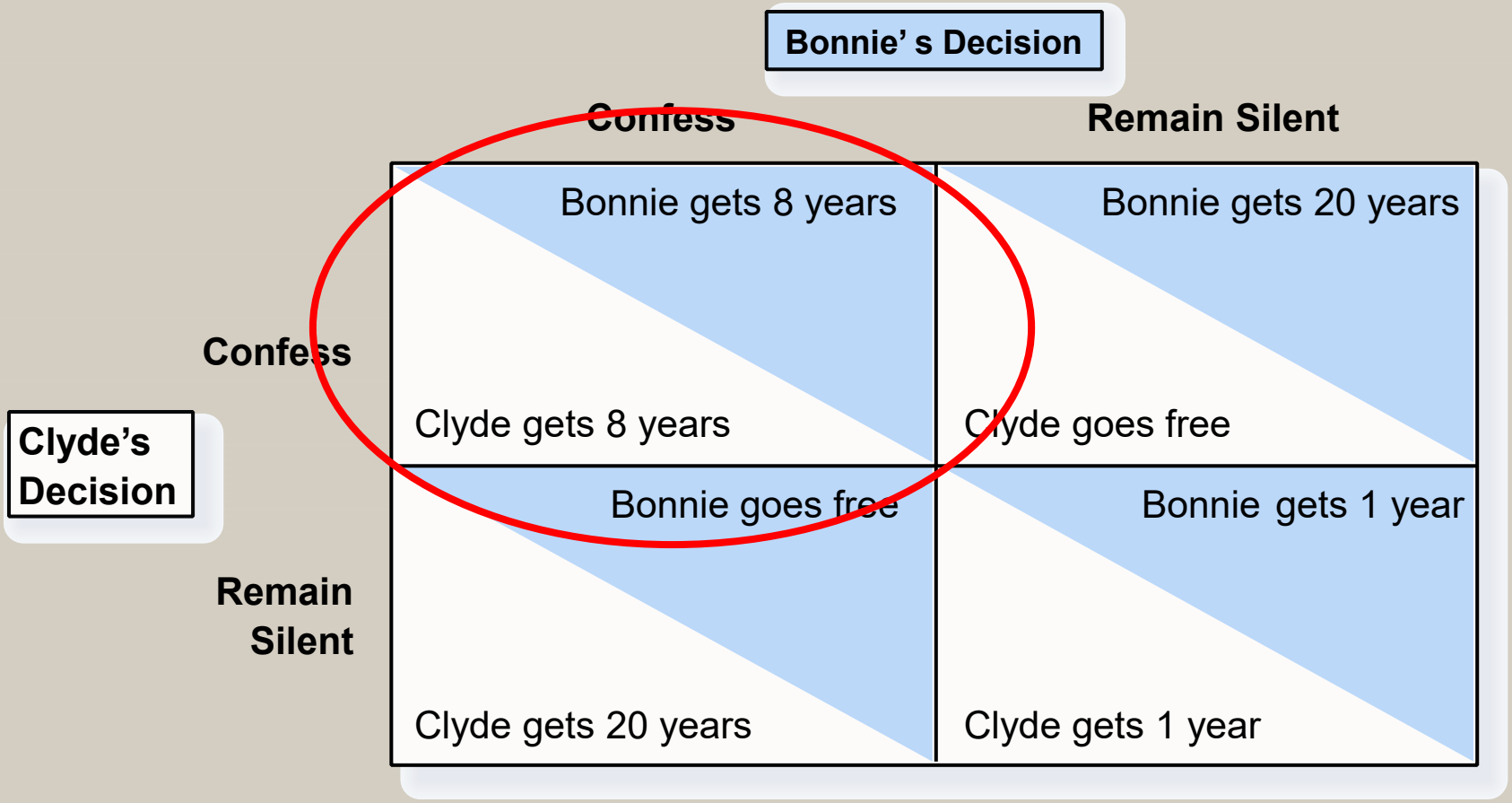
Nash Equilibrium

- In equilibrium, each player is playing the strategy that is a "best response" to the strategies of the other players, that is the strategy that yields the highest payoff given the strategies of the other players.

It nonetheless requires that other players will use the same information in a rational way and everybody is optimizing his/her behaviour

Did you watch the movie “ a beautiful mind”?

to confess is a NASH equilibrium too



Nash Equilibrium

- Nash Equilibrium does not mean:
  - The best possible outcome. Equilibrium in the one-shot prisoners' dilemma is for both players to confess ,while they prefer to walk away as soon as possible
  - Nash is not easy to accept…

MARKETS WITH ONLY A FEW SELLERS

- Characteristics of an Oligopoly Market
  - Few sellers offering similar or identical products
  - Interdependent firms
  - Best off cooperating and acting like a monopolist by producing a small quantity of output and charging a price above marginal cost

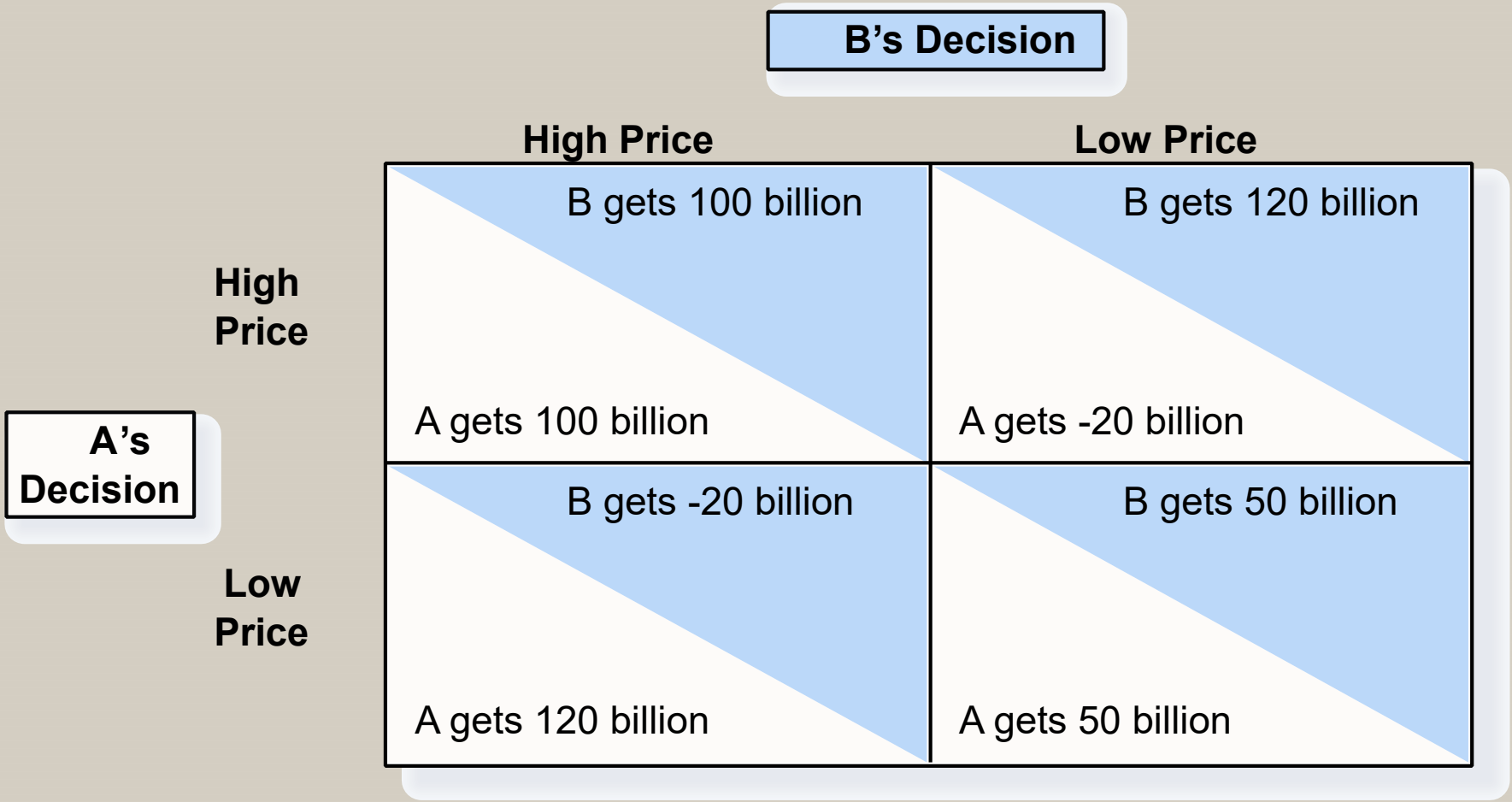
Competition, Monopolies, and Cartels

- The oligopolists may agree on a monopoly outcome.
  - *Collusion*
    - An agreement among firms in a market about quantities to produce or prices to charge.
  - *Cartel*
    - A group of firms acting in unison.
- Although oligopolists would like to form cartels and earn monopoly profits, often that is not possible. Antitrust laws prohibit explicit agreements among oligopolists as a matter of public policy.

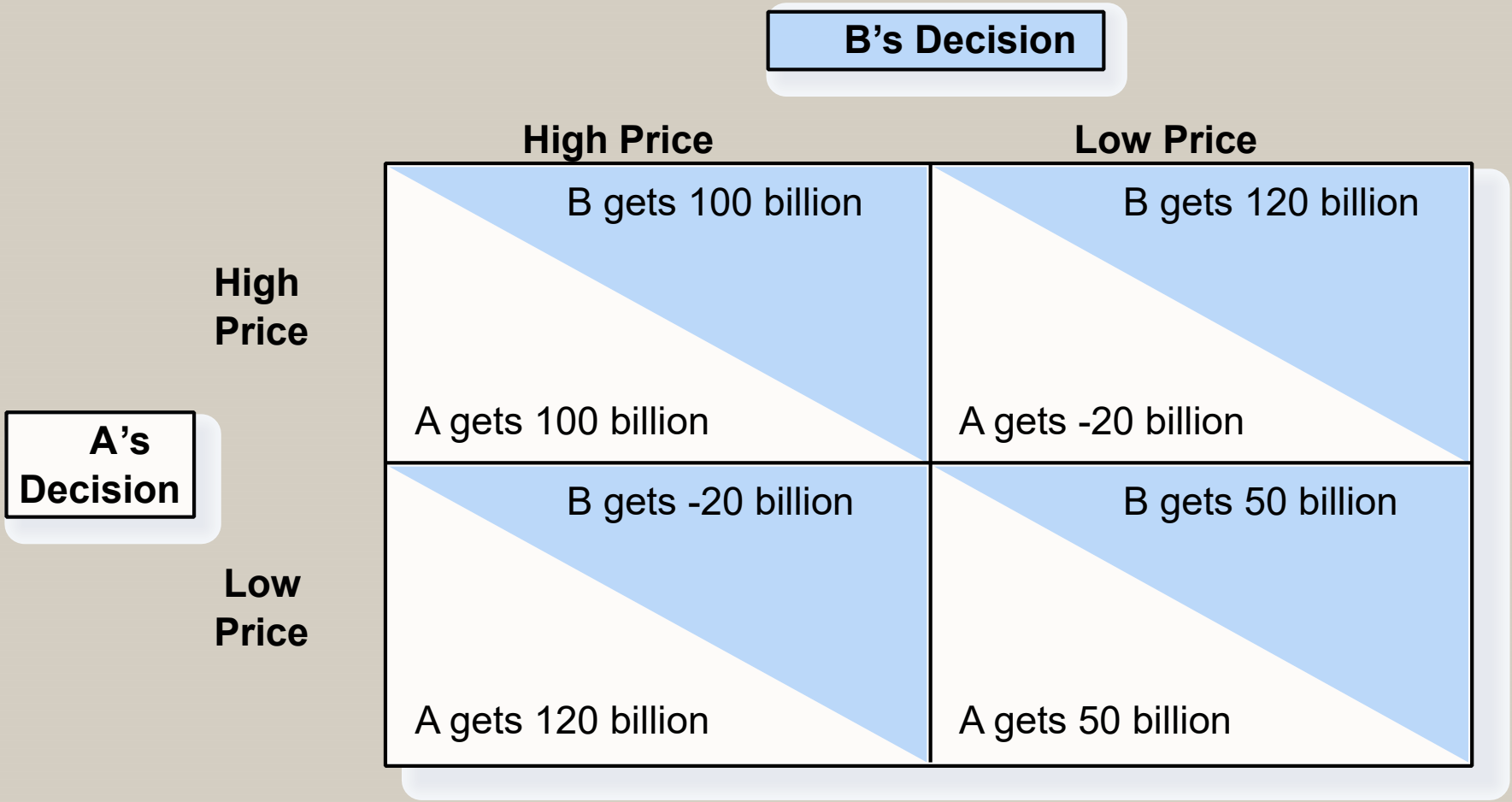
Cartels

- Is the oil market a competitive one?
- The Organization of Petroleum Exporting Countries (OPEC) was formed in 1960 with five founding members Iran, Iraq, Kuwait, Saudi Arabia and Venezuela. By the end of 1971 six other nations had joined the group: Qatar, Indonesia, Libya, United Arab Emirates, Algeria and Nigeria.
- What about Russia?

Low price (=low profit) is a NASH equilibrium too. Do you like it?



What about playing this game twice? Today and next year?



Is cooperation feasible?

- Folk theorem states that cooperation (high price) is viable when this game will go on forever
- When you don't know when the game will be over maybe you can maintain high prices strategy forever

But if you know when

This game will be over:

will you cooperate the very last time you will see your opponent?

and the time before???

TIT FOR TAT

- Axelrod (1984) asked several economists, mathematicians experts in social sciences to provide a strategy for an iterated prisoner's game, i.e. without knowing when it stops
- These strategies were matched and the game played long enough to see emerging the winning strategy: TIT FOR TAT
- You start cooperating, but you retaliate (next time) if your opponent defects now. If he'll cooperate again then you will cooperate too.

Is it strange?

BitTorrent peers use tit-for-tat strategy to optimize their download speed.

More specifically, most BitTorrent peers use a variant of Tit for two Tats which is called *optimistic unchoking*.

BitTorrent peers have a limited number of upload slots to allocate to other peers.

Is it strange?

Consequently, when a peer's upload bandwidth is saturated, it will use a tit-for-tat strategy. Cooperation is achieved when upload bandwidth is exchanged for download bandwidth. Therefore, when a peer is not uploading in return to our own peer uploading, the BitTorrent program will *choke* the connection with the uncooperative peer and allocate this upload slot to a hopefully more cooperating peer.

Is cooperation feasible?

- If deadline is known a rational player will defect at the end, and working back he/she will defect the time before too
- And the time before and before
- This is backward induction the can be used to solve the repeated Prisoners' dilemma and restore the standard solution
- but cooperation can emerge if will be together for a LONG (UNKNOWN) TIME

Another way to enforce cooperation is: increase options!

B's decision

A's dec.	Large price	low price	Large price but promise to match opponent's low price
Large price	100,100	-20,120	100,100
Low price	120,-20	50,50	50,50
Large price but promise to match opponent's low price	100,100	50,50	100,100

Large price is weakly dominated

A's dec.	B's decision: Large price	low price	Large price but promise to match opponent's low price
Large price	100,100	-20,120	100,100
Low price	120,-20	50,50	50,50
Large price but promise to match opponent's low price	100,100	50,50	100,100

Large price is weakly dominated and a (good) new NASH equilibrium

B's decision

A's dec.		low price	Large price but promise to match opponent's low price
Low price		50,50	50,50
Large price but promise to match opponent's low price		50,50	100,100