Integrated mathematics and physics

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Those teachers interested in a correlated or functional approach to mathematics will enjoy reading about an effort to teach physics and mathematics in the same course at the Putney School.

INTERMEDIATE ALGEBRA and physics can be taught together as an integrated course. The mathematics does not become just an adjunct of the science. The physics does not appear merely as an incidental application. The two can flow along together and both be strengthened by the process.

We have been teaching such a course for the past two years. Our "math-physics" is taught for two years in the tenth and eleventh grades. Extending the study of these subjects over a two-year period seems to encourage a more thoughtful and careful development of the essential ideas and skills, even though no more time is actually devoted to them. Additional advantages accrue, since the study of algebra is continued uninterrupted for three grades, and plane geometry is commonly taken the same year as algebra, so the important correlations between the two can be readily pointed out.

We have been guided by two general principles in formulating a plan for the course. First, we attempt, as far as practicable, to follow an inductive line of development from specific observation, to physical law, to mathematical abstraction. Second, we believe that the concept of a function is of primary importance—not only because it aids the solution of physical problems, but, more significantly, because it facilitates the learning of later mathematics.

To implement these ideas in the classroom, we begin by posing a laboratory problem and devising with the class the apparatus and technics needed to investigate it. When we have collected the set of data, we analyze the results in search of a verbal statement to explain the observations, then formulate it in algebraic form. The general statement can, of course, be used for predicting what would occur in other similar situations. If the process stopped here, it would be a limited matter of learning the art of formulation and computation—an important and useful part of mathematics, but only the beginning.

To make full use of our empirically derived law, it must be rearranged, or simplified, or somehow changed to solve a variety of problems. In answer to these demands, we can present the rules of algebraic operation in a motivated setting.

But, beyond this, we proceed to a higher level of abstraction, discussing similar expressions and functions—some applied to the physical world, others treated as pure mathematics. Finally we break away from the concrete basis and view the algebraic statement as an entity of powerful significance in its own right. In time, we can make clear that mathematics is not an outgrowth of natural events, but a collection of rules and procedures that can be useful in describing occurrences in the realm of matter and energy.

Consider two examples of how these methods can be applied:

The relationship between pressure and depth in a liquid is an ideal introduction to linear functions. A simple pressure gage, such as a thistle tube or funnel connected with flexible tubing to a U-tube manometer, is immersed to measured depths in a tank of water, and pressure readings are recorded. At this point we can help our students gain a new level of understanding by discussing the data, not as numbers that have to be "written up," but as a problem of interpretation in which mathematics holds the key.

Out of such a discussion comes a sense of the uniform nature of the change; by immediately graphing the information, this uniformity becomes closely associated with linearity. In this one exercise we meet problems such as: scale, the handling of approximate measurements, and the formation of a linear function from pairs of values.

From this starting point we can go as far as we wish in both areas. We can draw from physics numerous other illustrations of linear relationships. With firm roots in the everyday world, we can investigate fully the general properties of the straight line—its slope, intercepts, etc. Here, too, are clearly used the several ways of expressing a function.

For the second example, let us show how the alliance with physics can bring an exciting meaning to complex numbers. Of course, the most common use of complex numbers in more advanced physics is to describe vectorially the action of alternating currents. Although this is probably too involved for most high school students, a far simpler application is readily available.

If the study of forces and motion is delayed until near the end of the course, it is possible to examine vectors at a time when the students have enough conceptual background to handle "imaginaries" and the concomitant simple trigonometry. There are many ways in which the vector notion may be introduced; addition of nonparallel forces is probably the simplest, leading to the representation of distances, velocities, etc. One extremely rewarding sideline is the study of aircraft navigation, in which the manipulation of vectors plays a vital role.

If we begin with a laboratory exercise in which two concurrent forces and their vector sum are actually measured (by spring balances, for example), the concepts of magnitude and direction become grounded in observable fact. The next step toward increasing generality is graphical treatment of various problems of addition and subtraction, with particular attention to the resolution of a vector into orthogonal components. This latter operation is then transferred to the Cartesian coordinate plane, in which framework the algebraic form of complex numbers can readily be presented.

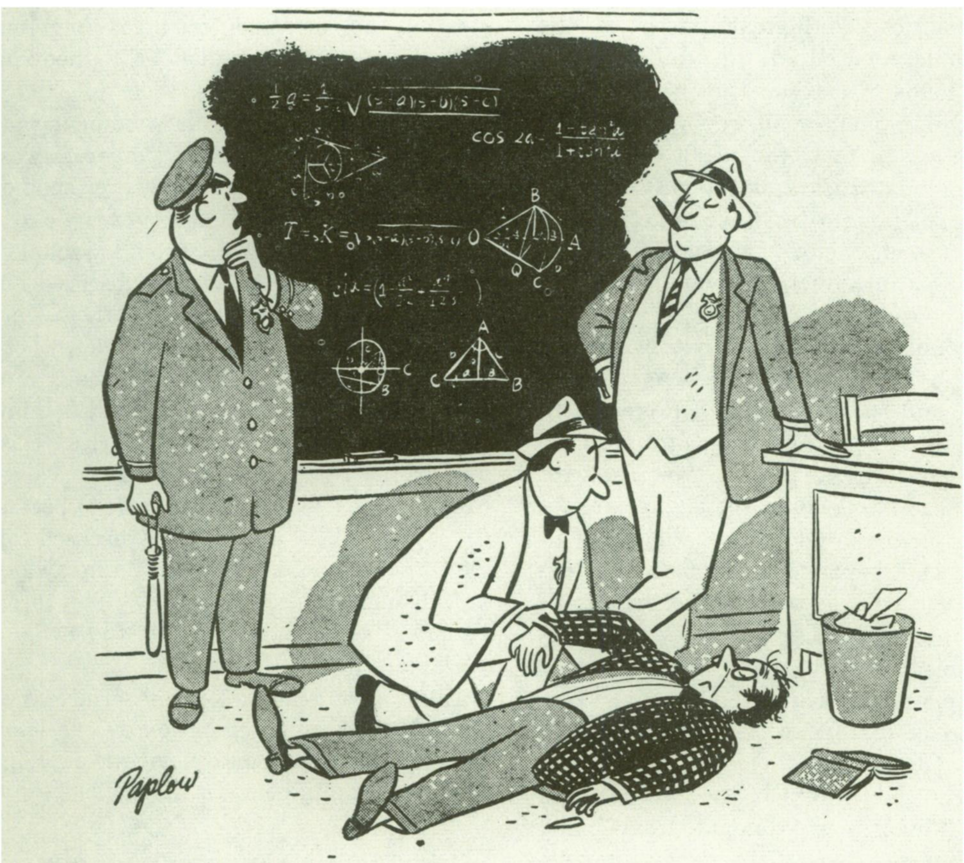
The $x+iy$ notation seems a more understandable approach for intermediate algebra students, and it can be used to demonstrate the fundamental operations. Since only right triangles need be involved, the resultant vector can be determined by elementary trigonometric means. And most significant, the work will not be just manipulating "imaginaries," because it is always possible to jump back to the world of forces and movement.

It is not essential to be tied to concrete experience. The experience started with a geometric interpretation and was symbolized by means of algebra. Now it is possible to lead our students further. At least, we can give a clear meaning to the roots of $x^2+1=0$. We can certainly show how to graph complex roots, and many students will want to pursue this further.

It may be argued that we have picked two especially suitable topics to support the thesis that the integration of mathematics and physics is desirable. This is true to some extent and therein lies a great challenge to the ingenuity of the teacher. The integration is not easily made, and there have been few previous attempts that can serve as a guide for new efforts.

There are other benefits in this plan for the cause of mathematics teaching in general. It is well known that engineering has long been a quantitative art. It is becoming increasingly apparent that valid descriptions of physical reality must be stated in mathematical terms. Yet, there is a discernible trend in science teaching to minimize the numerical aspects and to be satisfied with verbal generalizations. This tendency would seem to be carrying our students further away from the goal of gaining knowledge useful in a technological society.

The elimination of mathematics in science courses and the weakened position of mathematics in the curriculum are often justified on the ground that a numerical approach is too abstract, too "difficult" for our new high school population. We believe that the difficulty in both can be alleviated by encouraging a constant interplay between the concrete applications in physics and mathematical reasoning. Math-physics, a relatively unexplored area, may offer an opportunity for both algebra and physics to regain their stature in the modern curriculum.



"Maybe he knew too much."