Classroom Evidence of Teachers' PCK of the Interplay of Physics and Mathematics

Yaron Lehavi, Esther Bagno, Bat-Sheva Eylon, Roni Mualem, Gesche Pospiech, Ulrike Böhm, Olaf Krey and Ricardo Karam

Abstract The interrelations between Physics and Mathematics caught the attention of the physics education research community. Focusing mainly on students and teachers competency, the research in physics education (PER) found that learners, at different ages and levels, lack the ability to construct the mathematical models of physical processes or to describe the physical meaning of mathematical constructs. Mathematical knowledge was also found to reflect on the quality of explanations of physical phenomena. (Clement et al. 1981; Cohen et al. 1983; Rozier and Viennot in International Journal of Science Education 13:159–170, 1991; Rebmann and Viennot 1994; Bagno et al. in Physics Education 43(1):75–82, 2007; Redish and Smith in Journal of Engineering Education 97(3):295–307, 2008; Baumert et al. 2010; Zuccarini and Michelini 2014). The approach that underlines our study adopts the view that the context of physics teaching invites investigating the interplay between physics and mathematics. This "Phys-Math" interplay is regarded as a complex two ways track by which the knowledge and understanding of physics is constructed by learners. Our multi-national group examines this subject from various perspectives: history and philosophy of science as well as its instruction in different levels from high school to university (Eylon et al. 2010; Pospiech and Matthias 2011; Lehavi et al. 2013; Pospiech et al. 2014, 2015). The present study

Y. Lehavi (✉) · E. Bagno · B.-S. Eylon · R. Mualem
Weizmann Institute of Science, Jerusalem, Israel
e-mail: yarlehavi@gmail.com

Y. Lehavi
The David Yellin Academic College of Education, 37 Antignus St., Jerusalem, Israel

G. Pospiech
Technical University, Dresden, Germany

U. Böhm
Gymnasium Coswig, Dresden, Germany

O. Krey
Martin Luther University Halle-Wittenberg, Halle, Germany

R. Karam
University of Copenhagen, Copenhagen, Denmark

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follows our previous research in which we addressed, through interviews, the "Phys-Math" PCK of expert high school physics teachers from Israel and Germany (Lehavi et al. 2013, 2015; Pospiech et al. 2015). Here we report on a study which follows this research by analysing data collected from classes. The data was collected by videotaping physics lessons at middle school level. The videotapes were analysed, looking specifically for incidents in which Phys-Math interplay is evident.

1 Introduction

Although Physics and Mathematics can be regarded as autonomous disciplines, Physics, since its modern evolution, is considered to be heavily interrelated with Mathematics. In addition to their historical and philosophical perspectives, the "Phys-Math" interrelations caught also the attention of the physics education research community. In the past, mathematics in physics education was mainly examined within the context of problem solving (Bagno et al. 2007; Redish and Smith 2008). Research has found that learners, at different ages and levels, lack the ability to construct the mathematical models of physical processes or to describe the physical meaning of mathematical constructs. Researchers reported on students' difficulties such as in constructing equations from situations described in words (Clement et al. 1981) or in describing the physical meaning of formulae (Bagno et al. 2007). Rozier and Viennot (1991) pointed at students' difficulties in addressing multivariable problems. Rebmann and Viennot (1994) discussed the difficulty of many university physics students in applying and interpreting algebraic sign conventions consistently. Some researchers pointed out that there is blending of conceptual and formal mathematical reasoning during the mathematical processing stage (Kuo et al. 2013; Hull et al. 2013). With regard to teachers, Karam and Krey (2015) addressed the understanding and explaining of equations in physics teacher education. They attempt to bring teachers to realize the role that equations play in the formulation of theories as providing explanations for physical phenomena rather than serving as calculating tools to solve problems or for describing in a concise manner experimental regularities.

Recently, it was suggested that the whole context of physics teaching invites interplay between physics and mathematics (Eylon et al. 2010) and that a distinction should be made between a technical approach, which involves an instrumental (tool-like) use of mathematics, and a structural one, focused on reasoning about the physical world mathematically (Karam 2014). This view considers the overlap between Mathematics and Physics to be a sub-area of physics education which is characterized by its own Pedagogical Content Knowledge (PCK) and deserves research of its own. This has been the goal of a bi-national research conducted in Israel and Germany that examines the views of expert high school physics teachers with regard to the "Phys-Math" interplay and the measures they take to implement it (Lehavi et al. 2013, 2015; Pospiech et al. 2015). The teachers reflected in interviews on the importance of the "Phys-Math" interplay and provided examples of how they practice it in their teaching. In order to characterize teachers' PCK of the Phys-Math interplay, we employed the PCK model suggested by Magnusson et al. (1999) which was adapted by Etkina (2010) to physics education.

According to Magnusson et al. teachers' PCK assists them in fostering the following goals:

a. Help students develop the 'science process' skills
b. Represent a particular body of knowledge
c. Transmit the facts of science
d. Facilitate the development of scientific knowledge by confronting students with contexts to explain that challenge their naïve concepts
e. Involve students in investigating solutions to problems
f. Represent science as inquiry
g. Constitute a community of learners whose members share responsibility for understanding the physical world, particularly with respect to using tools for science.

A central construct in the model is teachers' 'orientations toward science teaching' which impacts different facets of their knowledge and views. Figure 1 represents Etkina's representation of relationships between this construct and several interrelated facets of knowledge with regard to: scientific content, students, assessments and successful teaching strategies (Etkina 2010).

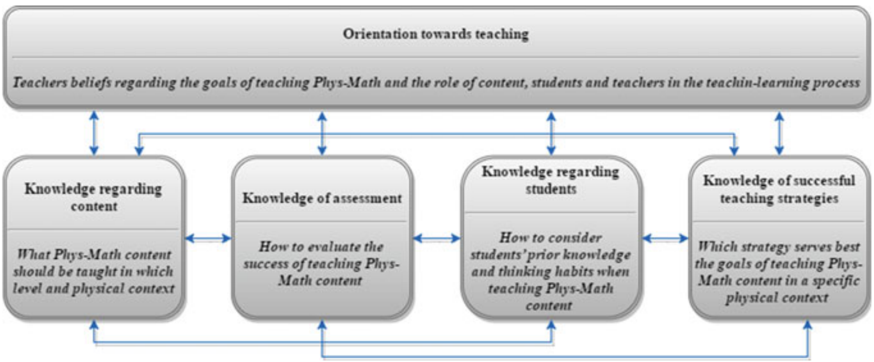

Fig. 1 Aspects of the Pedagogical Content Knowledge (PCK) framework from Magnusson et al. (1999) adapted by Etkina (2010)

2 The Bi-National Research on the "Phys-Math" PCK

2.1 Previous Findings

We limited our previous analysis of the teacher interviews in the bi-national research to the following components of Magnusson's framework:

a. The content of the Phys-Math interplay
b. Teachers' knowledge of successful teaching strategies.

Our findings revealed that teachers practice the use of Phys-Math interplay in order to foster different teaching goals and in doing so they employ different "patterns" that follow different "steps" between physics and mathematics and within each domain. Each of these patterns serves different teaching goals in the general PCK framework as can be seen in the following table (Lehavi et al. 2013, 2015) (Table 1).

The starting and ending point of all the found patterns is the physics domain, and they relate theory with experiments in different ways. The examples provided by the teachers cover different content areas of the physics curriculum. In addition, the teachers mentioned the role of Phys-Math content knowledge in deductive reasoning, in relating experiment and theory, in constructing students' broad view of physics and in problem solving. They described how they practice various teaching strategies that they employ within the Phys-Math interplay. This, together with the Phys-Math patterns, clearly fit into the PCK framework categories: orientation towards teaching, knowledge regarding content and the knowledge of successful teaching strategies.

Table 1 Phys-Math patterns, teaching goals and teaching practices (note close relations to goals a-f in the Magnusson PCK model)

Pattern	The teaching goal	The teaching practices
A. Exploration	To demonstrate how phys-math is used to explore the behavior of physical systems	Exploring within math ramifications for the physical system: borders (of validity, of approximation), extreme cases, etc.
B. Construction	To demonstrate how phys-math is used in constructing a model for physical systems	Constructing and developing (from experiments or from first principles) mathematical tools to describe and analyse physical phenomena
C. Broadening	To demonstrate how phys-math can be used in broadening the scope of a physical context	Adopting a bird's-eye view and employing general laws of physics, symmetries, similarities and analogies
D. Application	To demonstrate how phys-math provides aid in problem solving	Employing already known laws and mathematical representations in problem solving

Interestingly, our study revealed that the practice of employing different patterns of the Phys-Math interplay can distinguish master teachers from other expert teachers. The master teachers were very clear on rendering students *aware* of various aspects of the Phys-Math interplay and addressed the deep relations between physics and mathematics in philosophical and historical perspectives. They were highly aware of the various patterns of the interplay and the teaching methods for each pattern. Thus, our previous study supports the claim that teaching orientations play a critical role in distinguishing the quality of teaching (Abell 2007).

3 The Present Study: Phys-Math in Classrooms

In the present study we took a step beyond what is *described* by teachers as their interpretation of the Phys-Math interplay in the context of physics education, and investigated what is *actually* performed by them in the classroom. Based on actual scenarios of teaching we investigated how teachers' PCK with regard to the interplay between Physics and Mathematics may be manifested in their actual teaching.

The data was collected by videotaping physics lessons at grade 9 (the end of middle school level in Israel). The videos were scrutinized, looking for occasions in which a Phys-Math interplay was manifested. Our analysis was comprised of two steps: An independent analysis of each occasion by at least three researchers and a group discussion by the researchers. In this analysis the researchers were asked to relate to several aspects regarding the PCK on the Phys-Math interplay such as the above mentioned Phys-Math patterns, how a–f in the Magnusson et al. framework were manifested (or missing) in the scenario, as well as additional aspects that came up in the scenario.

We shall present here two examples and first interpretations based on the video evidence. One example is extracted from a post-lesson meeting of a teacher with a guide. The lesson and the meeting were both video-recorded. The second example is based on a video of a classroom discussion. Both examples are focused on the definition of speed.

Example 1: "Math may screen physics understanding" a post-lesson discussion (Teacher = T; Guide = G): The guide and a group of teachers are watching together the video from the teacher's lesson.

G: "… I am asking about the teaching strategy by which you define speed."
T: "First of all I will approach their [students] intuition, to see their understanding from everyday life what speed is. I want to change their view, … to explain them that speed is the change in distance versus time, for example the change in the position of an object versus time.
G: "I am interested in the method you employ in order to change their everyday intuition to the physical one."

T: “They will say that speed is how fast you move. This is from everyday experience.”
G: “But this is not what they said…”

(Both are looking at the classroom video).

G: “You asked ‘what is speed’ and the student replied: ‘x divided by t’. And then you said: ‘Right. It is the change in position versus time’…. What is the difference between what the student said and what you have said? What is the student’s difficulty which is reflected by his answer? How do you respond to this difficulty?”
T: “The question is: what is the physical logic…. You try to explain what speed is and they tell you that it is the path divided by time, right? [Change in] position divided by time.”
G: “They said x divided by t…”
T: “He actually means, ah… because he remembers from his math lessons that the distance equals speed times time. This is what he learnt during his preparation for his Math exams.”
G: “More than that, he said before that…”
T: “[That] x is an unknown. Right, right. Ok, I say, we have here the position and we have here time and we would like to define for him what speed is.”
G: “Did you try to differentiate here between the mathematical definition and the physical one?”
T: “Not explicitly. But the explanation, the physical connotation was to try to explain, how ah… it [the definition] is related to the physics.”
G: “I would like to focus here on your teaching strategy. If you want to explain, and the student has a difficulty, which, like you said before, is related to his math studies, can you assist him by making the differentiation between math and physics explicit? The question is how to deal with students’ difficulties?”
T: “You can go over the definition few times, give them more and more examples until you realize that they have got the reason behind it…. I then gave them an exercise about constant speed to check their understanding and they answered it very well. They were able to explain that if the object covered a distance of 10 m within 5 s, its speed was 2 m/s because it advanced 2 m every second. So they really got the logic here.”
G: “So, do you think that they understood the difference between the mathematical definition and the physical one?”
T: “I said that it is the change in position over time. In mathematics they learn that S equals vt.”
G: “We can see [in the video] that you wrote on the board that x is position and t is time but the student, after 10 min of explanations, asked: ‘what is x?’. So, what was so difficult for him?”
T: “How can I explain more what position is? What is the problem here?”
…
G: “Everything was written correctly on the board. So, where does the difficulty come from?”

T: "Because he didn't feel it by his own hands?"
G: "When will it happen?"
T: "When he will make an experiment"

The above discussion can be viewed through the following components of our adopted PCK model (see the above list):

a. Help students develop the 'science process' skills
b. Represent a particular body of knowledge
c. Facilitate the development of scientific knowledge by confronting students with contexts to explain that challenge their naïve concepts

In the discussion the teacher is fully aware that there is a difference between how motion is addressed in mathematics and in physics. He realizes that for students who hold the mathematical conceptualization, time, speed and distance are merely three quantities related by an equation. However, he finds it difficult to *develop a teaching strategy* (see Fig. 1) to make this difference clear for his students, moving them away from their mathematical conceptualization into the physical one. Finally, the teacher begins, through the guidance, to consider the idea that what really makes physics different from mathematics are the former's empirical bases.

Example 2: "A Phys-Math surprise" This example describes a scene from a teacher's classroom representing a different strategy to address the same difficulty.

(T = Teacher; S = student(s)), excerpt from a classroom discussion:

T: "We want to describe motion. I have here few toys, each group will have one. I want you to describe the motion of the toy.
S: "What, the energy that it has?
T: "No. What kind of motion; Time and position.
S: "Position - classroom (S2: Desk) [is a position]. Time is t"
T: [Referring to a drawing on the board] "We have two drawings, each with a certain reading of my stopwatch. I have two things: I measure the time and the change in position. This change in position is called a displacement. … What concepts do we need to describe motion?"
S: "Time and distance."
S: "Speed, time and distance."
T: "OK. So we said that we can calculate the speed."
S: "No. we just said distance and time…"
T: "It is sufficient to measure the distance and time and from these we can calculate the speed and in fact to describe motion. You have studied it in Mathematics. In physics it is a little bit different. The concepts that we will use are position, relative position and displacement. Now, how can I in general measure motion?"
S: "To measure motion?
T: "Yes."
S: "Distance?"
S: "Speed"
S: "Speed, time and distance."

T: "[If] my car goes from Jerusalem to Tel Aviv…"
S: "[you can measure the speed by] a speedometer"
T: "The speedometer measures the speed."
S: "This is what you wanted. Right?"
S: "It [the speedometer] measures speed and minutes."
S: "It measures everything."
T: "What do you mean?"
S: "Speed and distance. It measures speed and time and derives the distance!"
T: "So, if we want to measure the motion of one of our toy-cars, how would we do that? Would we place a speedometer on it and measure its speed?"
Students: "Yes! Yes!"

Apparently, this line of thought was not what the teacher expected:

T: "OK. How else could we do it?"
S: "With an Equation. Speed multiplied by time equals distance."
T: "We can use a formula and calculate."
S: "This is what the speedometer does."

This teacher begins from a "hands-on" experience, develops the required vocabulary and tries to employ it in describing motion. However, she is not well aware of the *knowledge regarding student* (Fig. 1) with regard to two aspects:

a. The mathematical conceptualization that students bring to the class. For them all the three quantities that appear in the distance-time-speed equation are equivalent—you just have to know two of them in order to derive the third. They do not pay attention to what are the measured quantities and what is the derived one.[1]
b. The fact that their everyday experience ("speed is measured by a speedometer") does not conflict with their mathematical conceptualization.

Therefore, the teacher was apparently not expecting a situation in which students' mathematical knowledge not only hindered their physical understanding but enhanced their misinterpretation of the measured versus derived quantities.

4 Discussion

We provided here only two classroom examples (out of many) in order to depict how teachers' PCK with regard to the interplay between Physics and Mathematics may affect their teaching. As mentioned in the two examples, the findings fit rather well Magnusson's et al. PCK model adapted by us previously with regard to two

[1]It is possible to measure the speed directly via the Doppler Effect. However, this was not the strategy adopted here by the teacher.

components: knowledge regarding students and the need to develop a teaching strategy.

The examples provided above address components a–f (see the list above). They demonstrate that within the context of physics teaching, the juxtaposition of physics and mathematics carries its unique students' pre-knowledge and misinterpretations. Furthermore, the Phys-Math interplay requires teachers to develop specific assessments in order to reveal students' difficulties and their origins and specific teaching strategies to assist students in overcoming their learning difficulties.

With regard to the patterns we have recognized in the teachers' description of their instruction, we may identify two such patterns in our two teachers' practice. The first teacher follows the "application pattern" and employs the already known (to the students) mathematical representations of motion. This teacher exhibits difficulties in changing the students' previous mathematical knowledge and provide them with physical insights. Apparently, the teacher is not fully aware of the importance of making the Phys-Math interplay explicit to the students.

The second teacher seems to practice the "construction pattern". Similar to the first teacher, she begins from a physical situation and then constructs the mathematical tools to describe and analyse physical phenomena. Importantly, both teachers seem to follow the patterns intuitively and show little awareness with regard to possible difficulties that students may have with regard to the Phys-Math content.

It would thus be advisable to regard the Phys-Math interplay as a sub-content of its own and develop special teachers training programs in order to address the special challenges posed by it. Consequently, our next step in the coming year is to invite experienced teachers to develop Phys-Math teaching strategies and try them in their own classes.

References

Abell, S. K. (2007). Research on science teacher knowledge (Chapter 36). In S. K. Abell & N. G. Lederman (Eds.), *Research on science teacher education* (pp. 1105–1149). New York: Routledge.

Bagno, E., Eylon, B., & Berger, H. (2007). Meeting the challenge of students' understanding formulas in high-school physics—A learning tool. *Physics Education, 43*(1), 75–82.

Etkina, E. (2010). Pedagogical content knowledge and preparation of high school physics teachers. *Physical Review Special Topics: Physics Education Research, 6*, 2.

Eylon, B., Cohen, E., & Bagno, E. (2010). The interplay of physics and mathematics in a graduate quantum mechanics course for physics teachers, talk given in GIREP-conference 2010 in the Symposium. *Addressing the Role of Mathematics in Physics Education*.

Hull, M., Kuo, E., Gupta, A., & Elby, A. (2013). Problem-solving rubrics revisited: Attending to the blending of informal conceptual and formal mathematical reasoning. *Physical Review Special Topics: Physics Education Research, 9*, 1.

Karam, R. (2014). Framing the structural role of mathematics in physics lectures: A case study on electromagnetism. *Physical Review Special Topics: Physics Education Research, 10*(1).

Karam, R., & Krey, O. (2015, February). Quod erat demonstrandum: Understanding and explaining equations in physics teacher education. *Science & Education, 24*(5), 661–698.

Kuo, E., Hull, M. M., Gupta, A., & Elby, A. (2013). How students blend conceptual and formal mathematical reasoning in solving physics problems. *Science & Education, 97*, 32.

Lehavi, Y., Bagno, E., Eylon, B.-S., & Cohen, E. (2013). Can math for physics teachers impact their conceptual knowledge of physics? An oral presentation given in the 2013 Girep conference, Prague.

Lehavi, Y., Bagno, E., Eylon, B. S., Mualem, R., Pospiech, G., Böhm, U., et al. (2015). Towards a PCK of Physics and Mathematics interplay. In *The GIREP MPTL 2014 Conference Proceedings* (Accepted for publication).

Magnusson, S., Krajcik, J., & Borko, H. (1999) Nature, sources and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95–133). Dordrecht: Kluwer Academic Publishers.

Pospiech, G., & Matthias, S. (2011). Quantum physics in teacher education. In *Proceedings of the 12th International Symposium Frontiers of Fundamental Physics [FFP12], Udine*. http://www.fisica.uniud.it/~ffp12/proceedings.html

Pospiech, G., Eylon, B. S., Bagno, E., Lehavi, Y., & Geyer, M. A. (2015). The role of mathematics for physics teaching and understanding. In *The GIREP MPTL 2014 Conference Proceedings* (Accepted for publication).

Redish, E. F., & Smith, K. A. (2008). Looking beyond content: Skill development for engineers. *Journal of Engineering Education, 97*(3), 295–307.

Rozier, S., & Viennot, L. (1991). Students' reasoning in thermodynamics. *International Journal of Science Education., 13*, 159–170.