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COMPARISON OF STUDENT UNDERSTANDING OF LINE GRAPH SLOPE IN PHYSICS AND MATHEMATICS

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ABSTRACT. This study gives an insight into the differences between student understanding of line graph slope in the context of physics (kinematics) and mathematics. Two pairs of parallel physics and mathematics questions that involved estimation and interpretation of line graph slope were constructed and administered to 114 Croatian second year high school students (aged 15 to 16 years). Each pair of questions referred to the same skill in different contexts—one question in the context of mathematics and the other in the context of kinematics. A sample of Croatian physics teachers ($N = 90$) was asked to rank the questions according to their expected difficulty for second year high school students. The prevalent ranking order suggests that most physics teachers expected mathematics questions to be more difficult for students than the parallel physics questions. Contrary to the prevalent teachers' expectations, students succeeded better on mathematics than on physics questions. The analysis of student answers and explanations suggests that the lack of mathematical knowledge is not the main reason for student difficulties with graphs in kinematics. It appears that the interpretation of the meaning of line graph slope in a physics context presents the largest problem for students. However, students also showed problems with the understanding of the concept of slope in a mathematical context. Students exhibited slope/height confusion in both contexts, but much more frequently in the context of physics than in the context of mathematics.

KEY WORDS: kinematics, line graph, mathematics education, physics education, slope

Introduction

The fact that many students at high school, or even university level, lack the ability to understand and interpret graphs in physics is not new. It has been investigated in several physics education research studies (e.g. Arons, 1983; Beichner, 1990, 1994; McDermott, Rosenquist & Zee, 1987; Wavering, 1989; Woolnough, 2000; Brassel & Rowe, 1993). This topic has also been the subject of mathematics education research (Dreyfus & Eisenberg, 1990; Graham & Sharp, 1999; Habre & Abboud, 2006; Kerslake, 1981; Leinhardt, Zaslavsky & Stein, 1990; Swatton & Taylor, 1994), since

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line graphs were first introduced into mathematics and then only later, into physics. Both disciplines require students to be able to extract various pieces of information from line graphs, but, in addition, physics also requires an interpretation of the obtained information in the context of a given physical situation. One such type of information that students are very often asked to estimate and interpret is the slope of the line graph.

The concept of slope (gradient) is very important for physics since many physical quantities are defined as gradients (e.g. velocity, acceleration) and represented with line graphs. The concept of slope is also important for mathematics as a necessary prerequisite for the development of the concept of derivation. Students study line graph slope in both mathematics and physics, but, because of differences in contexts, they may not necessarily realize that they are studying the same concept. Student difficulties with the concept of slope were identified through both physics and mathematics education research, usually as a part of studies which investigated general student difficulties with graphs.

The study of McDermott et al. (1987) presented a good overview of student difficulties with graphs. Regarding slope, it was found that students have difficulties discriminating the slope and height of a graph and interpreting changes in height and changes in slope. Students often do not know whether to extract the desired information from the slope or the height of the graph, and instead of looking for changes in slope, many students focus on the more perceptually obvious changes in height.

On the basis of reports on student difficulties with graphs, Beichner (1994) constructed the Test of Understanding Graphs in Kinematics (TUG-K) and applied it to 895 high-school- and college-level students. He pointed out that the most common mistakes students make with kinematics graphs are thinking that a graph is a picture of the situation and confusing the meaning of the slope of the line with the height of a point on the line. This study also stressed that many students were unable to choose which feature of the graph represented the information that was needed to answer the question (for example, they calculated slope when they should have been calculating the area). Graphing skills of high school students were investigated in a study (Brassel & Rowe, 1993), which found that at least one fifth of the students did not have adequate graphing skills. Students had difficulties with linking the graph and the verbal descriptions of a given event, and they did not understand graphs as a means of representing relationships among variables. Students' facility with graphs was found to be generally superficial, being based on a few simplistic algorithms.

The use of microcomputer-based labs and real-time graphing was investigated and found to have an effect, especially on reducing “graph as picture” errors (e.g. Mokros & Tinker, 1987).

Most of the research on student understanding of graphs in physics was done in the context of kinematics, because of the very broad use of graphical representations in kinematics. As well, some of the research in mathematics education was also based on kinematics motion graphs (Graham & Sharp, 1999; Kerslake, 1981). General findings are similar to those in physics education research: Students tend to use graphs as actual pictures of the motion. They do not realize which feature of the graph to use in a particular situation, and they tend to use a position criterion instead of a gradient-based criterion when considering velocities. One of the studies (Graham & Sharp, 1999) focused only on able 13-year-old students with interest in mathematics and found that relatively few of them had a sound understanding of graphs that could be applied consistently.

The study of Woolnough (2000) was particularly interesting because it revealed the existence of student resistance to applying their mathematical knowledge to physics. The study suggested that senior secondary students operated in three distinct contexts: the real world, the physics world, and the mathematical world, each with different characteristics and belief systems. It was suggested that, most students, even those who do well in mathematics and physics, do not make substantial links between these contexts. Some students even thought that it was not appropriate to transfer concepts from mathematics to physics. For example, when calculating the slope of a line graph, some students thought that it was inappropriate to assign units to the slope because of their perception that slope is a mathematical concept. The same study also found that about half of the high school students entering Year 11 physics were not overly familiar with the concept of slope and could not determine the slope of a simple straight line. In Year 12, the situation greatly improved, and it was found that 90% of students were able to calculate a slope of a line through origin, but almost a third of those students were not able to provide a physical interpretation of the calculated slope.

Studies in a purely mathematical context (e.g. Dreyfus & Eisenberg, 1990; Habre & Abboud, 2006; Leinhardt et al. 1990; Swatton & Taylor, 1994) have shown that student understanding of mathematical concepts (such as functions) tends to be typically algebraic and not visual. Visual information is more difficult for students to learn and is considered less mathematical (Dreyfus & Eisenberg, 1990; Habre & Abboud, 2006). Leinhardt et al. (1990) classified student difficulties into three categories:

interval/point confusions, where students focus on a single point instead of on an interval; slope/height confusions where students mistake the height of the graph for the slope; and iconic confusions, where students incorrectly interpret graphs as pictures.

It appears that the results of investigations of student difficulties with graphs in physics and mathematics have a lot in common. The same difficulties are found in both contexts—the slope/height confusion and iconic confusion seeming to be the most prominent.

Theoretical Background

The first step in extracting any information from a graph is realizing that it is a symbolic representation of the relationship between variables. Processing of visual symbolic information, such as line graphs, requires the ability to perceive and remember a pattern of spatially arranged visual data as well as the ability to reason about spatial visual information. It is therefore not surprising that understanding of kinematics graphs, and in particular, slope calculation, was found to be related to logical thinking, spatial ability, and mathematics achievement (Bektasli, 2006).

How graphs are related to student cognitive development was investigated in a study (Wavering, 1989) which found a correspondence between categories in student graphing skills and Piagetian stages of cognitive development. This study and other similar ones (Berg & Phillips, 1994) suggest that constructing and interpreting graphs requires formal operational reasoning. Students who had low levels of logical thinking were not able to construct and interpret graphs. Berg & Phillips (1994) state that, without developed logical thinking, students are dependent upon their perceptions and low-level thinking, which will lead them to a "graph as picture" error, slope/height confusion, etc.

Students who have not yet reached the formal operational stage of cognitive development are likely to view graphs as something concrete rather than as indicators of abstract trends (Beichner, 1990). This view was criticized by Roth & McGinn (1997) who emphasized graphing as practice (as opposed to cognitive ability) and attributed student difficulties with graphing to students' lack of experience with the construction and use of graphs, as well as to a lack of opportunities for students to endow graphs with meaning.

Spatial ability can be defined as the intuition about shapes and the relationships among shapes, that is, as the ability to generate, retain, retrieve, and transform well-structured visual (mental) images (Lohman, 1996). Researchers in cognitive psychology have suggested that measures on spatial ability tests reflect both subjects' ability to maintain and transform spatial images, as well as the capacity of their visual–spatial working memory (Miyake, Friedman, Rettinger, Shah & Hegarty, 1991; Salthouse, Babcock, Mitchell, Palmon & Skovronek, 1990; Shah & Miyake, 1996). It was found that people who differ in spatial abilities also differ in their ability to solve physics problems that involve multiple spatial parameters (Kozhevnikov, Motes & Hegarty, 2007). Interpretation of kinematics graphs is an example of such problems. Vekiri (2002) found that students with low prior knowledge and low spatial ability have difficulties extracting information from graphs.

Kozhevnikov, Hegarty & Mayer (1999) investigated student graph interpretation in correlation with visual ability (the ability to construct mental images of vivid color and detail) and spatial ability. Visual ability refers to physical objects in the real world and is therefore more related to concrete operations, whereas spatial ability refers to three-dimensional transformations of these objects and is more related to formal operations.

The same study searched for a relationship between imagery and problem solving in kinematics. Two different types of imagery were defined, visual and spatial. Visual imagery refers to the external appearance of objects (e.g. color, shape). Spatial imagery refers to spatial relationships between parts of an object and the location of objects in space. It was found that spatial imagery contributes to student graph interpretation in physics, whereas visual imagery is an obstacle on the same task. The authors selected kinematics as the context for their study because this topic is related to visual and spatial abilities since it involves both graphical and concrete physical representations. The authors argue that focusing on concrete aspects of the situation prevents students from thinking formally. Students who are more visual (low spatial ability), who generated vivid images of the concrete situation (visual imagery), tended to interpret the graph as a literal picture of the situation and were unable to abstract any relevant information from the graph. Students who are more spatial, on the other hand, considered the graph to be an abstract spatial representation, and none of them referred to the graph as a concrete picture of the motion. Students of low spatial ability are therefore expected to have more problems with graph interpretation than high-spatial-ability students.

Another important factor in graph interpretation is students' conceptual knowledge in physics. It appears that a correlation between spatial ability and kinematics problem solving is no longer present after students receive physics instruction through the use of rich visualization technologies (Kozhevnikov & Thornton, 2006). Spatial ability and physics conceptual knowledge are probably interrelated since high spatial ability may enhance people's ability to gain a conceptual knowledge of physics (Kozhevnikov et al., 2007). It is suggested that a physics curriculum that provides external visualizations can help students who have difficulty generating such visualizations on their own (low spatial ability students), to compensate for such shortcomings. In the case of kinematics graphs, visual simulations should highlight segments of data and associated tick-mark ranges along the axes, rather than the overall shape of the graph, and should lead the learner to analyze and imagine event changes occurring within the subintervals (Kozhevnikov et al., 2007).

To summarize, most cognitive studies cited above point to the importance of logical thinking and spatial ability as prerequisites for understanding kinematic graphs, and some also suggest that studying kinematics graphs in physics and mathematics can help develop these abilities in students.

Physics Teachers' Attitudes

Physics teachers are often inclined to attribute the observed student difficulties with graphs to students' lack of mathematical knowledge. However, some researchers in physics education (McDermott et al., 1987) have suggested that the lack of mathematical skills may not be the main cause of students' difficulties with graphs in physics. Also, some researchers in mathematics education have pointed out that often students who solve graphing or function problems in mathematics seem to be unable to access their knowledge in science (Leinhardt et al., 1990). To further investigate this issue, we have undertaken a study to investigate Croatian students' understanding of line graph slope in mathematics and kinematics contexts, as the first step in the development of an instrument which could measure and compare student understanding of graphs in the broader contexts of physics and mathematics. We have also surveyed a group of Croatian physics teachers ($N = 90$) and asked them to rank parallel physics and mathematics questions which are concerned with the concept of slope, according to their expected difficulties for students. Teachers' predictions were compared with the empirically obtained difficulties of questions.

Research Questions

This study attempts to answer the following research question: How does student ability to estimate and interpret slopes of line graphs in mathematics relate to their ability to estimate and interpret slopes of line graphs in physics, e.g. kinematics? Although a number of studies investigated these abilities separately, we are not aware of the existence of a study which attempted to compare them directly on parallel physics and mathematics problems. Such an attempt could provide new insights into the relationship between student understanding of line graph slope in mathematics and physics and the frequency of typical student errors in each context. An additional purpose of the study is to document and compare Croatian student difficulties concerning line graphs, with the corresponding documented difficulties of students in other countries.

METHODOLOGY

Two pairs of parallel multiple choice mathematics and physics questions about graphs were developed by authors (Figures 1 and 2). Each pair of questions referred to the concept of line graph slope in different contexts —one question in a mathematics context and the other in a physics context. The first pair of questions referred to positive slope, and the next pair to negative slope. Physics questions (labeled P1, P2) were adjusted questions from Beichner's TUG-K (Beichner, 1994), and mathematics questions (labeled M1, M2) were constructed to be analogous to the

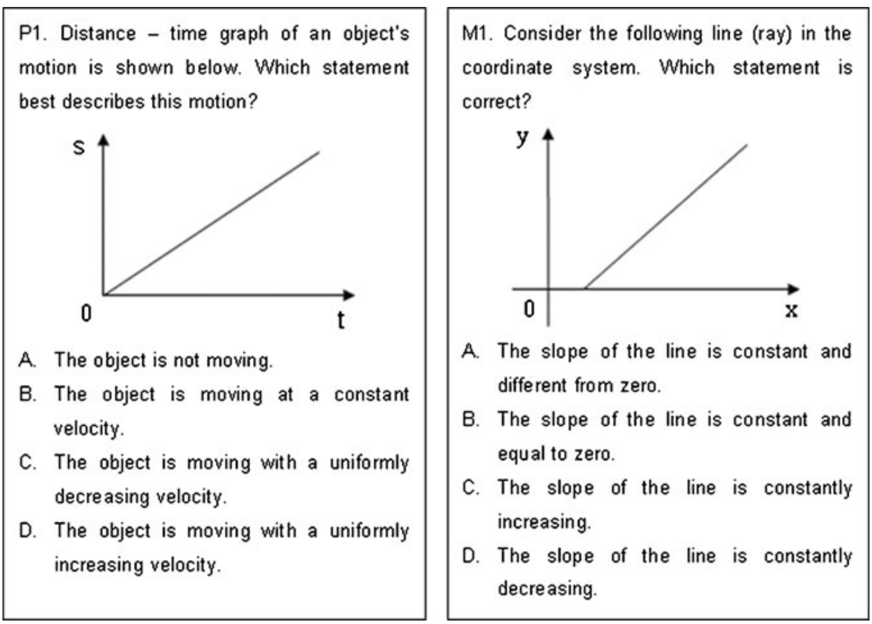

Figure 1. Questions P1 and M1

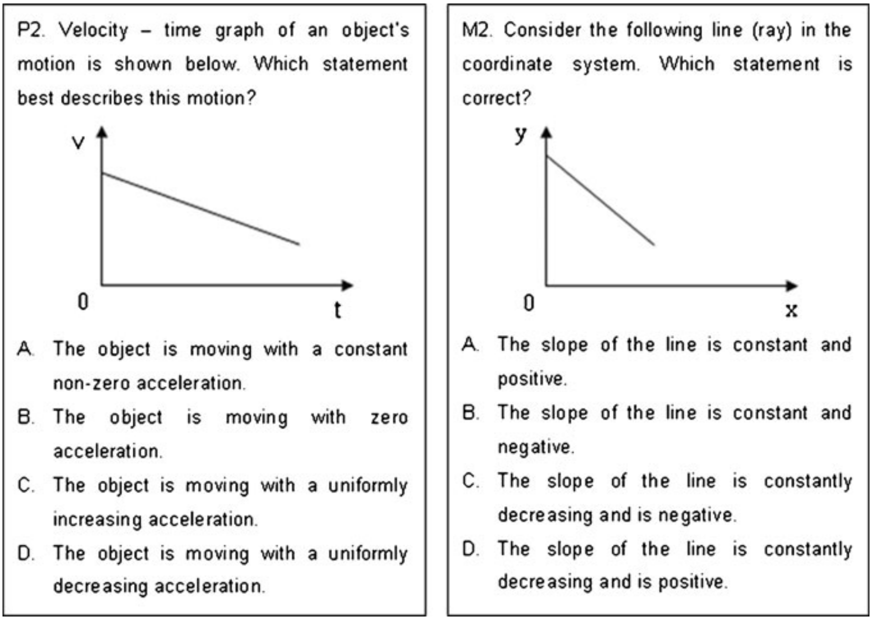

Figure 2. Questions P2 and M2

respective physics questions. The mathematics and physics questions are regarded in this study as parallel (M1 to P1 and M2 to P2) in the sense that the required mathematical reasoning in both corresponding questions is the same.

In question P1, the correct answer is given in option B and, in question M1, in option A. In both questions, students were expected to reason about the slope of the line graph in order to recognize the slope as constant and different from zero and in question P1, to also interpret the slope as the magnitude of the object's velocity. In question P2, the correct answer is given in option A and, in question M2, in option B. In these two questions, students were expected to recognize that the slope of the graph was constant and negative. In Question P2, they were also expected to interpret the slope of the graph as the magnitude of the object's acceleration.

All four questions were administered to 114 second year high school students (aged 15 to 16 years) from two Croatian cities, Zagreb and Slunj, as a part of a longer (16 questions) multiple choice test about graphs. Parallel questions analyzed in this study did not follow each other in the test but were separated with other questions. The test was administered in written form, and the allocated time for taking the whole test was 45 min. One of the researchers (H. K.) was always present at the time of the testing to answer students' questions and supervise the testing. There was no incentive such as grades offered for taking the test. Students were informed that the test was a part of the research on student understanding of graphs, and they were generally willing to participate. In addition to choosing the correct answer, students were asked to provide explanations for their answers so that an insight into the underlying student reasoning could be obtained. Unfortunately, not all students provided explanations for their answers. In cases where there was no explanation, the answer was taken at its face value.

There were 52% male students and 48% female students in the sample. Most of the sample was from the Croatian capital Zagreb (80 students), and 34 students were from the smaller town of Slunj. In the first year of high school, all students have studied, among other topics, linear functions and linear graphs in mathematics and motion graphs and kinematics in physics.

Physics is a compulsory school subject in most Croatian high schools which can be of different types. Sixty students were from high schools which specialize in natural sciences and mathematics (NSM); 33 students were from high schools which are of a general education type (GE), and there were 21 students from a vocational high school for informatics (VS). Students in NSM schools had three class periods of physics and four periods of mathematics per week, whereas students in GE schools had two periods of physics and four periods of mathematics per week. Students in VS schools had two periods of physics and three periods of mathematics per week in the first year of high school.

Regarding abilities in mathematics and physics, the students from NSM schools are from the top 20% of the Croatian high school students; GE students can be considered as being average and VS students slightly below average.

Physics teachers, who gathered in April 2011, at the biannual Croatian Symposium on Physics Teaching, were surveyed about their expectations of the relative difficulties of the four questions P1, P2, M1, and M2. In the written questionnaire, teachers were asked to rank these four questions according to their expected difficulty for second year high school students, starting with the question with the highest level of difficulty. They based question rankings on their experience and opinion. Teachers were also asked to provide reasons for the ranking order that they proposed. Ninety teachers filled out the questionnaire. Their answers were analyzed and compared with the actual level of difficulties established for the four questions. Frequencies of different ranking combinations were determined, and the explanations accompanying these combinations were grouped according to similarities in teachers' reasoning. In addition, the answers of teachers who teach both mathematics and physics were analyzed separately and compared with the answers of other teachers who teach physics, but not mathematics.

Results and Discussion

Table 1 gives an overview of student average success on each question for different types of schools. It is evident that students in all types of schools scored higher on mathematics questions than on the corresponding physics questions. This is not surprising because, in addition to mathematical skills, the physics questions also required students to interpret mathematical results in the context of the physics situation. The overall success on the four questions was relatively high in NSM and GE schools but much lower in VS. This is not only the consequence of a larger number of physics and mathematics lessons in NSM and GE schools, but also of higher motivation and higher general ability of students in those schools. In Table 2, the frequencies of different combinations of correct and incorrect answers for questions P1, P2, M1, and M2 are given.

In Figure 3, percentages of correct answers for pairs of parallel physics and mathematics questions are shown. It is immediately noticeable that parallel physics and mathematics questions differ in level of difficulty. Also, the results on questions P1 and P2 are quite close to the results of university students in Beichner's study (1994) on the same questions, where there was 63% of correct answers for question P1 and 18% of correct answers for question P2.

TABLE 1

Number (*N*) and average success of students from different types of schools (NSM, GE, VS) on physics (P1, P2) and mathematics (M1, M2) questions

Type of school	*N*	*P1*	*P2*	*M1*	*M2*
NSM	60	71.7%	33.3%	91.7%	80.0%
GE	33	57.6%	12.1%	60.6%	48.5%
VS	21	28.6%	14.3%	52.4%	19.0%
All schools	114	59.6%	23.7%	75.4%	59.6%

TABLE 2

Number (*N*) and percentage of students for different combinations of correct and incorrect answers on questions P1, P2, M1, and M2

	P1 and M1 correct	*P2 and M2 correct*	*P1 correct, M1 incorrect*	*P1 incorrect, M1 correct*	*P2 correct, M2 incorrect*	*P2 incorrect, M2 correct*
N	56	24	12	30	3	44
%	49, 1	21, 1	10, 5	26, 3	2, 6	38, 6

Positive Slope

Questions M1 and P1 both referred to the positive slope of a line graph. Both questions are shown in Figure 1. Mathematics question M1 was easier for students than the physics question P1. In question M1, the word ‘slope’ was already mentioned in the text of the question, whereas in question P1, students had to determine on their own that the slope of the graph was the decisive feature for answering the question about the motion of the body. They also had to know that the slope of *s* vs. *t* graph represents the magnitude of the body’s velocity. This obviously increased the difficulty of question P1 compared with question M1. In some explanations given by students with the correct answer to question P1,

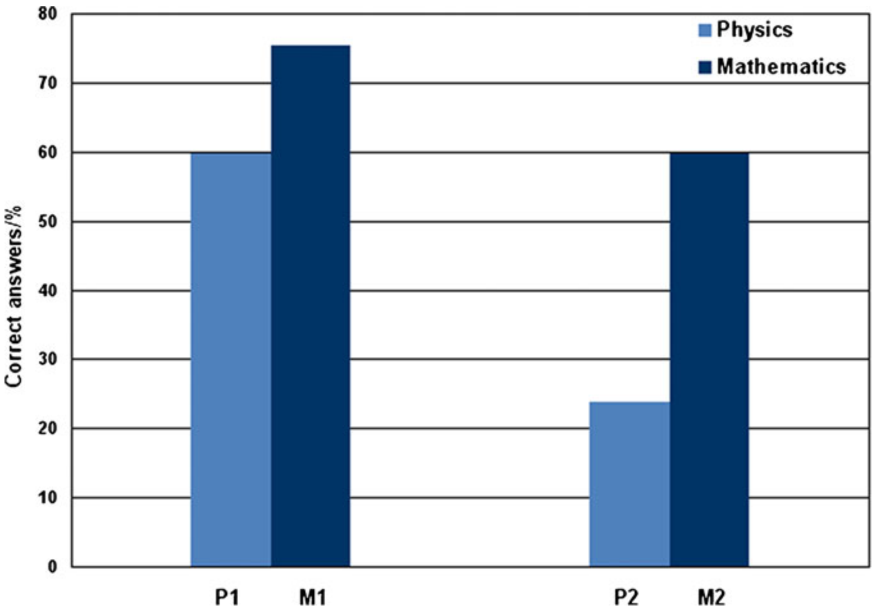

Figure 3. Percentages of correct answers for pairs of parallel physics and mathematics questions

neither the word slope nor the idea of the slope was present. Here are some examples:

The ratio of the covered distance and the time (the speed) is always the same.
Because the covered distance in each moment increases by the same amount.

In the second comment, the student does not distinguish a moment from a time interval, but they probably understand (at least vaguely) the idea of slope.

From some comments, it was hard to see what the student was thinking. An example of such a comment is the following one given with the correct answer to question P1:

$s = vt,\ v = const.$

Since the student had chosen the correct answer, he might have indicated with this equation his understanding of the velocity as the slope in s vs. t graph, but we cannot be completely sure of this.

Some students used the concept of slope in their reasoning, as can be deduced from the following comment:

The velocity is constant, since s vs. t graph is a straight line with the constant slope.

Some explanations that accompanied correct answers to question M1 revealed that these students' understanding of the concept of slope is at best only partially correct, or even completely wrong:

Because the slope is the angle between the straight line and the x axis.
The slope of the straight line is not changing and the line is all the time in the first quadrant.
The slope angle is not changing and it is greater than zero.
The straight line does not begin from zero, therefore it (the slope) is different from zero and the line is in the positive quadrant.

It can be inferred from the explanations above that some students tend to attribute zero value of slope to the straight line that passes through the origin of the coordinate system, while others associate slope value with the quadrant in which the line is drawn (not realizing that the straight line extends to infinity on both sides and has only one value of slope). Some identify slope with the angle between the straight line and the x axis.

The leading and largely prevailing wrong answer chosen by students in P1 (40%) was the answer D (the body moves with uniformly increasing velocity), and in M1, it was the answer C (23%) (the slope of the line constantly increases). The most frequent wrong choice on the question similar to P1 in Beichner's study (1994) was also the one that suggested that the body moves with uniformly increasing velocity. In both contexts, physics and mathematics, students most often made the same well-known mistake, which consisted in confusing the slope of the graph with the height of the graph (Beichner, 1994; McDermott et al., 1987; Leinhardt et al., 1990). However, the frequency of this mistake was much higher in the context of physics than in the context of mathematics.

One student showed in his answer to M1, an even more profound confusion:

> The correct answer is C (the slope constantly increases) because the body's motion is uniformly accelerated.

This student confused the mathematics graph with the *v* vs. *t* graph, which could be a sign of a known student tendency to memorize shapes of kinematics graphs without paying attention to what physical quantities (if any) are depicted in the graph (Beichner, 1994).

Most of the students who answered the physics question P1 correctly also succeeded on the parallel mathematics question (about half of the students got both questions correct). It is interesting, however, that about 26% of students (Table 2) correctly answered the mathematics question but failed on the parallel physics question. (The opposite was the case in about 10% of students. Some students could have solved P1 without understanding the slope concept if they, for example, memorized the shape of the *s* vs. *t* graph for motion at constant velocity). This indicates that mathematical knowledge is not a guarantee of success on a parallel physics problem. Problems with student understanding of the concept of line graph slope in mathematics were also observed. We see from explanations which accompanied answers to M1 that student mathematical understanding of the concept of slope was often only partially correct, even in cases when students chose the correct answer.

Negative Slope

Questions P2 and M2 are shown in Figure 2. They are both harder than P1 and M1, respectively (Figure 3, Tables 1 and 2). Negative slope is obviously a harder concept than positive slope, but this is even more pronounced for the concepts of acceleration and velocity.

Some students showed elements of correct understanding in their explanation for question P2, such as the student who gave the following explanation:

A is the correct answer, because in each moment velocity diminishes by the same amount.

Once again we notice that some students do not distinguish a moment of time from an interval of time, but the underlying reasoning about acceleration is essentially correct.

Most students did not answer question P2 correctly. The following explanations which accompanied answer D (acceleration decreases uniformly), represent incorrect reasoning based on poor understanding of uniformly decelerating motion.

In uniformly decelerated motion velocity decreases linearly, v=at.
The answer is D because the body decelerates uniformly.

Students notice correctly that the depicted motion is uniformly decelerated (velocity diminishes at a steady rate), but then incorrectly link this type of motion with uniformly decreasing acceleration. This indicates that they do not understand the concept of acceleration either as the rate of change of velocity, or as the slope of the line in the v vs. t graph.

There were more correct answers to question M2 than to question P2, but not many explanations were given for either of them. Here are some explanations for the correct answer to question M2:

The graph is a straight line and it has negative direction.
The angle between the straight line and the x axis is the same and the value of the function is decreasing.

The first explanation suggests that the student's reasoning is based on the appearance of the graph and probably also on the learned rule that the slope of the line is negative when the line has negative direction (negative direction meaning that y values become smaller as x values increase). It is not possible to judge whether this student really understands the concept of slope. In the second explanation, we see an indication of the correct reasoning based on the concept of slope, but we can also notice once again the possible identification of slope with the angle between the straight line and the x axis.

For both questions P2 and M2, the leading wrong answer was the same and indicated the presence of the well-known slope/height confusion: The acceleration of the body (in P2) or the slope of the graph (in M2) constantly decreases. The same was the case in Beichner's study (1994) for question P2, where 72% of students chose a similar answer. However, the frequency of the same mistake in our study was again very different in the contexts of physics and mathematics. In the physics context, 68% of students displayed slope/height confusion (even though some of them correctly stated in their explanations that the body uniformly decelerated) whereas, in mathematics context, only 33% of the students were similarly confused. Almost 40% of students correctly answered the mathematics question but failed on the parallel physics question, whereas the opposite was true for less than 3% of students (Table 2). This suggests that the interpretation of mathematical quantities in the context of physics is an important source of student difficulties with graphs in physics. Many students who are able to estimate whether the slope of a straight line is constant (as is visible from their answers to question M2) fail to conclude that the acceleration is constant in P2. This is obviously not the consequence of their lack of mathematical knowledge but rather of the missing link between mathematics and physics and of the lack of relevant conceptual physics knowledge. Students are probably not aware that the problem in P2 is mathematically the same as the problem in M2, because they do not see acceleration as the slope of the v vs. t graph. Also, student understanding of the concept of acceleration is obviously problematic—some associating finite change in velocity with an instant of time instead with a finite interval of time (interval/point confusion) and many thinking that uniform deceleration implies a decreasing acceleration.

The difference in student performance on parallel mathematics and physics questions may also be explained in terms of the different cognitive demands that these problems impose on students. The cognitive literature suggests that generating visual imagery when interpreting graphs is an obstacle to problem solving in physics. Mathematics problems are less likely to induce visual imagery than physics problems, which contain more "real-life" associations (object, motion, speed, etc.). The context of physics problems could have led some students to start thinking about what the object in question or its trajectory might look like and then to connect that trajectory erroneously with the shape of the graph, therefore failing to perceive the graph as a symbolic representation of a relationship between variables. Physics questions are also less direct. Mathematics questions directly called for the activation of knowledge related to slope, and students correctly answered questions M1 and M2 if they possessed such knowledge—even if that knowledge was in some cases only partially correct. However, on physics questions P1 and P2, many students did not even activate their mathematical knowledge about slope, because they lacked relevant conceptual knowledge about velocity and acceleration that should have led them to use this mathematical knowledge in a physics context. Instead, they activated naive thinking patterns under the influence of a strong visual cue (increasing/decreasing height of the graph). Different frequencies of naive answers in the two contexts suggest that students used different strategies for the same type of problems in mathematics and in physics. The added context in physics problems made these problems more complex than the parallel mathematics ones. In consequence, the results suggest that the physics problems required more information processing from students than the respective mathematics problems and were cognitively more demanding for them.

Survey of Physics Teachers

The results of the survey of physics teachers are shown in Figures 4, 5, and 6. The observed leading rank order (from the most difficult to the least difficult question) is M2–M1–P2–P1 (Figure 6), by which teachers expressed their opinion that mathematics is more difficult for students than physics. If we add up all combinations that have the both mathematics questions before the both physics questions, we find that 33 out of 90 teachers (37%) shared this opinion. It is interesting that the second most frequent ranking combination (P2–P1–M2–M1) is very close to the order of difficulty found in our study (P2–P1=M2–M1), in which P2 was the most difficult question, P1 and M2 were equally difficult, but easier than P2, and M1 was the least difficult question. If combination 3412 (P2-M2-P1-M1), which also cannot be distinguished from the empirically found ranking order, were added to the combination 3142 (P2–P1–M2–M1), we could say that 18 out of 90 teachers (20%) correctly estimated the order of difficulty for the four questions. The same number of teachers ranked physics questions as more difficult than mathematics questions. The prevailing attitude among teachers, expressed through the ranking of questions, was that each mathematics question would be

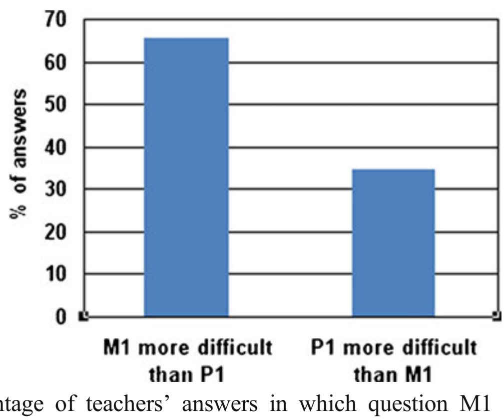

Figure 4. Percentage of teachers' answers in which question M1 is ranked as more difficult than question P1 and those in which P1 is ranked as more difficult than M1

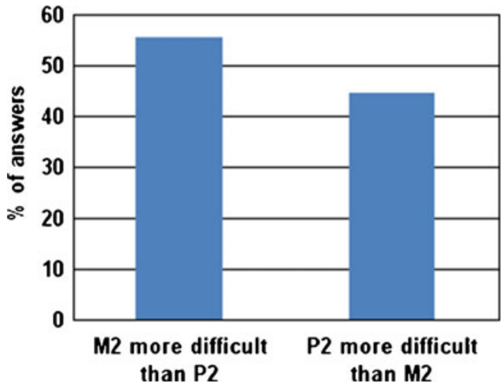

Figure 5. Percentage of teachers' answers in which question M2 is ranked as more difficult than question P2 and those in which P2 is ranked as more difficult than M2

harder for students than the respective parallel physics question (Figures 4 and 5). In the explanations for the ranking order that they provided, many teachers expressed the idea that the physics context is more familiar to students than the mathematics context and that it is therefore easier for students to understand graphs in physics than in mathematics. In other words, many teachers consider mathematics more abstract and therefore more difficult than physics. Some of the teachers also mentioned that one of the leading causes of student difficulties with physics is students' lack of mathematical knowledge and skills. Twenty of the surveyed teachers were both physics and mathematics teachers. They ranked P1 as more difficult than M1 in 11 cases (55%) and P2 as more difficult than M2 in

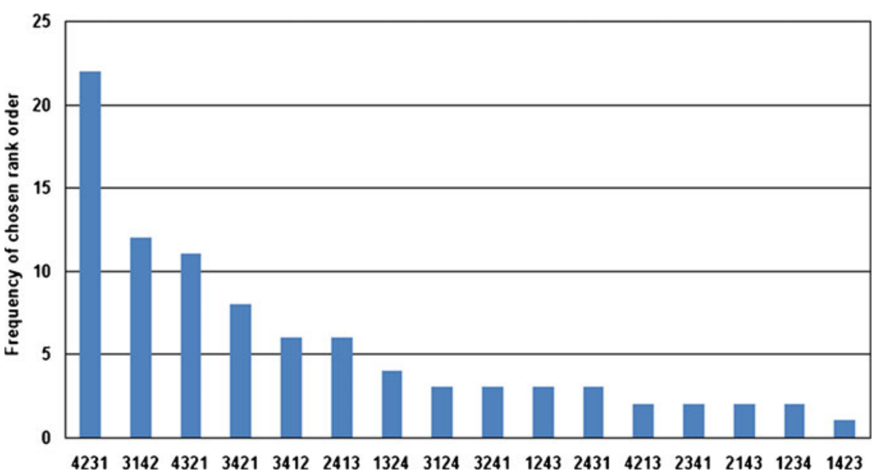

Figure 6. Frequency of different ranking combinations given by physics teachers ('*1*' stands for question P1, '*2*' for M1, '*3*' for P2, and '*4*' for M2)

ten cases (50%), which is higher than in the whole sample. This could indicate that teachers who teach both subjects have a somewhat better insight into student difficulties, but, because of the small number of these teachers, we cannot be sure that the observed differences are significant.

The study also provided some information on Croatian high school students. The same most common difficulties with the concept of line graph slope that were found in students from other countries (e.g. Araujo, Veit & Moreira, 2008; Cataloglu, 2007) are also found in this sample of Croatian students. Comparison of student success on questions P1 and P2 with the success of American students in Beichner's study (1994) shows very similar results. This might suggest that typical student difficulties with line graphs such as slope/height confusion reflect intuitive reasoning patterns that are common for all students.

The results of Croatian students suggest that the intensity of student difficulties varies with the type of school, but a larger sample would be required to investigate the differences between schools in detail. However, for students from all types of schools, the physics part of the test was more difficult than the mathematics part, and even the most able students (from NSM schools) had difficulties with interpretation of slopes in a physics context.

Conclusion

Student understanding of line graph slope is important for both physics and mathematics education. In this study, we have tried to compare student ability to estimate and interpret the slope of a line graph in both contexts by comparing student answers on two pairs of parallel mathematics and physics (kinematics) questions. Many physics teachers still believe that students' lack of mathematical knowledge is the main reason that students have difficulties with physics, as was confirmed also by the results of our survey of physics teachers. The results of our study seem to support the suggestion of McDermott et al., (1987) that the lack of mathematical skills is not the main cause of student difficulties with graphs in physics. Quite a number of students in our study were able to solve a mathematical problem concerning slope, but failed on a parallel physics problem. It is important to emphasize that some students really do lack mathematical skills, but even if they do not, mathematical knowledge is not a guarantee for their success on similar physics problems. Often, students did not even recognize that the problems were similar—they used different strategies for analyzing graphs in mathematics and physics.

The component of interpretation of mathematical quantities in the physics context (e.g. recognizing the slope of a velocity—time graph as acceleration) is often missing, and even if students possess the needed mathematical knowledge, they will not use it in a physics context if they cannot transfer understandings between a physics and mathematics situation.

The transfer of knowledge from mathematics to physics seems to be relatively weak, but that might be caused by the fact that students do not see the similarity between certain problems in mathematics and physics. Theories of transfer of knowledge are based upon the idea that knowledge can be transferred from one situation to another and linked with a new situation (Potgieter, Harding & Engelbrecht, 2008). Some researchers disagree and argue that learners' mental processes are structured by the context and the implemented activities and tools (Lave, 1988). Potgieter et al. (2008) suggest that teachers often expect students to rise above context, but that it is not easy for students to apply the mathematics in other contexts. Recognizing mathematics in a different context requires good understanding of the context (which is often missing), along with mathematical knowledge. This was also the case with students in this study.

It seems that student knowledge is very compartmentalized and that stronger links should be established between mathematics and physics during teaching, but students also need stronger conceptual knowledge in physics. However, we have also noticed from students' explanations that some of them already have significant problems with the concept of slope in a purely mathematical context. Many identify slope with the angle between the straight line and the x axis, or evaluate the sign of the slope according to the quadrant in which the line is drawn. Several cases of interval/point confusion were also found.

For learning and transfer of knowledge to succeed, both physics and mathematics should focus more on the meaning (interpretation) of graphs. It is very obvious from the analysis of student difficulties with graphs that finding the meaning of graphs and of their characteristics (such as slope) seems to be one of the most problematic aspects of graph analysis for students. Students need to make sense of graphs during instruction; discover and discuss their meaning through collaboration with their peers; and if possible, connect graphs with real examples of motion (for example through the use of motion detectors).

The findings of our study suggest that—contrary to the prevailing expectations of physics teachers—physics problems concerning the slope of a line graph are more difficult for students than parallel mathematics problems. Many physics teachers in our study thought that mathematics items would be more difficult because they are more abstract and lack context, whereas physics items, which are closer to real life and appear less abstract, should be easier for students. The results suggest quite the opposite. Even though mathematics questions appear more abstract, they are more direct and require less processing of information and less conceptual understanding than parallel physics questions. This study suggests that it is the added physics context that makes parallel physics problems more difficult, "masks" the mathematical essence of the problem, and increases the cognitive demand on students. Teachers should realize that it is very important to work on student conceptual understanding and interpretation of physical and mathematical quantities, as well as on building stronger links between the two subjects.

Another interesting result of this study is that the same common student difficulty known from other studies as slope/height confusion is found to be dominant in both contexts but is much more frequent in the context of physics than in the context of mathematics. It also occurs more frequently in questions concerning negative slope than in questions concerning positive slope. A graph as a visual cue seems to provoke the same naïve thinking in both contexts but not with equal frequency. In mathematics, this kind of reasoning will be characteristic for students who lack knowledge about slope. In physics, mathematical knowledge about slope will often not be activated at all (even if it is present), because of a lack of students conceptual knowledge. More students will therefore tend to revert to their intuitive reasoning patterns on physics problems than on similar mathematics problems.

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