STRONG, LIGHT STRUT (when stability under compression is the limit)

F: strut (puntone)

C: L, $\boxed{F_{CR} > F^*}$ $F_{CR} = \frac{n^2\pi^2 E I}{L^2}$

O: min mass → $m = A L \rho$

F.V.: A, material

$$\frac{n^2\pi^2 E A^2}{12 L^2} > F^* \qquad A > \left(\frac{12 F^* L^2}{n^2\pi^2 E}\right)^{\frac{1}{2}}$$

$$m > \left(\frac{12 F^* L^2}{n^2\pi^2}\right) L \frac{\rho}{E^{\frac{1}{2}}} \qquad \boxed{M = \frac{\rho}{E^{\frac{1}{2}}}}$$

CHEAP, STIFF TIE

F: tie

C: L, $S > S^*$ $S = \frac{EA}{L} > S^*$ cost per unit mass

O: min cost $C = A L \rho C_m$

F.V.: A, material

$$C > \frac{L S^*}{E} L \rho C_m \Rightarrow \boxed{M_{IC} = \frac{\rho C_m}{E}}$$

Slope of a line for a strong, light beam

$$M = \frac{\sigma^{\frac{2}{3}}}{\rho} \qquad \log M = \frac{2}{3}\log\sigma - \log\rho$$

$$\underbrace{\log\sigma}_{y} = \underbrace{\frac{3}{2}}_{m}\underbrace{\log\rho}_{x} + \overbrace{\frac{3}{2}\log M}^{q}$$

CHANGING FREE VARIABLES

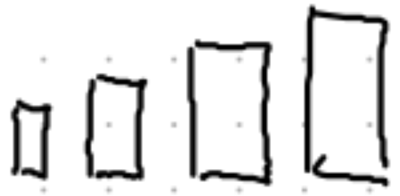
Example: light stiff beam with different free variables

O: min mass $m = hbL\rho$

C: L, $S > S^*$ $\quad S = \frac{F}{\delta} = \frac{CEI}{L^3} = \boxed{\frac{CE}{L^3}\frac{bh^3}{12}} > S^*$

F V ?

1) FV: A cross section area (aspect ratio is fixed $\Rightarrow b = \alpha h$) $\Rightarrow \boxed{M = \frac{\rho}{E^{\frac{1}{2}}}}$



2) FV: b $\quad A = bh$

$$b > \frac{12 S^* L^3}{CEh^3} \Rightarrow m > \frac{12 S^* L^3}{Ch^3} hL \frac{\rho}{E} \qquad \boxed{M = \frac{\rho}{E}}$$

3) FV: h

$$h > \left(\frac{12 S^* L^3}{CEb}\right)^{\frac{1}{3}} \Rightarrow m > \left(\frac{12 S^* L^3}{Cb}\right)^{\frac{1}{3}} \cdot bL \frac{\rho}{E^{\frac{1}{3}}} \Rightarrow \boxed{M = \frac{\rho}{E^{\frac{1}{3}}}}$$

The choice of the free variable strongly influences materials selection

NOTE: this is the same as the one for the board in bending