Minimizing Volumes - examples

Small, stiff, tie

F: tie

C: $s > s^*$ $\quad s = \frac{EA}{L}$ $\quad \frac{EA}{L} > s^*$ $\quad A > \frac{s^* L}{E}$

O: min V $\rightarrow$ $V = L \cdot A$

$\Rightarrow \quad V > \frac{s^* L^2}{E} \quad \Rightarrow \quad \boxed{M = \frac{1}{E}}$

Small, stiff beam

F: beam

C: $s > s^*$ $\quad s = \frac{CEI}{L^3} = \frac{CEA^2}{12L^3}$ $\quad A > \left(\frac{12L^3 s^*}{CE}\right)^{\frac{1}{2}}$

O: min V $\rightarrow$ $V = L \cdot A$

$V > \left(\frac{12L^3 s^*}{C}\right)^{\frac{1}{2}} \cdot L \left(\frac{1}{E}\right)^{\frac{1}{2}}$ $\quad \boxed{M = \frac{1}{E}}$

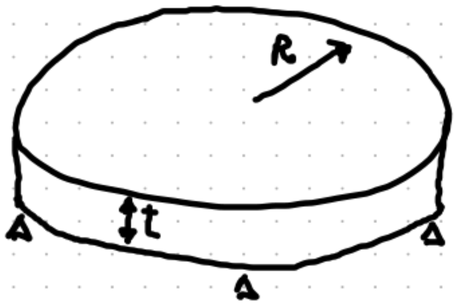
LARGE TELESCOPE

F: plate under bending

C: R, δ_{max}, α low, surf.

F: t

O: min mass (cost)



$$\delta = \frac{3}{4}(1-\nu^2)\frac{\Delta p R^4}{E t^3} \simeq \frac{3}{4}\frac{mg}{\pi R^2}\frac{R^4}{E t^3} = \frac{3}{4}\frac{\pi R^2 \rho g t}{\pi R^2}\frac{R^4}{E t^3}$$

(Δp)

$\delta < \delta_{max} \Rightarrow \quad \frac{3}{4}\frac{\rho g}{E}\frac{R^4}{t^2} < \delta_{max} \quad t > \left(\frac{3}{4}\frac{g R^4}{\delta_{max}}\frac{\rho}{E}\right)^{1/2}$

$m = \pi R^2 t \rho \Rightarrow \quad m = \ldots \frac{\rho^{\frac{3}{2}}}{C}$