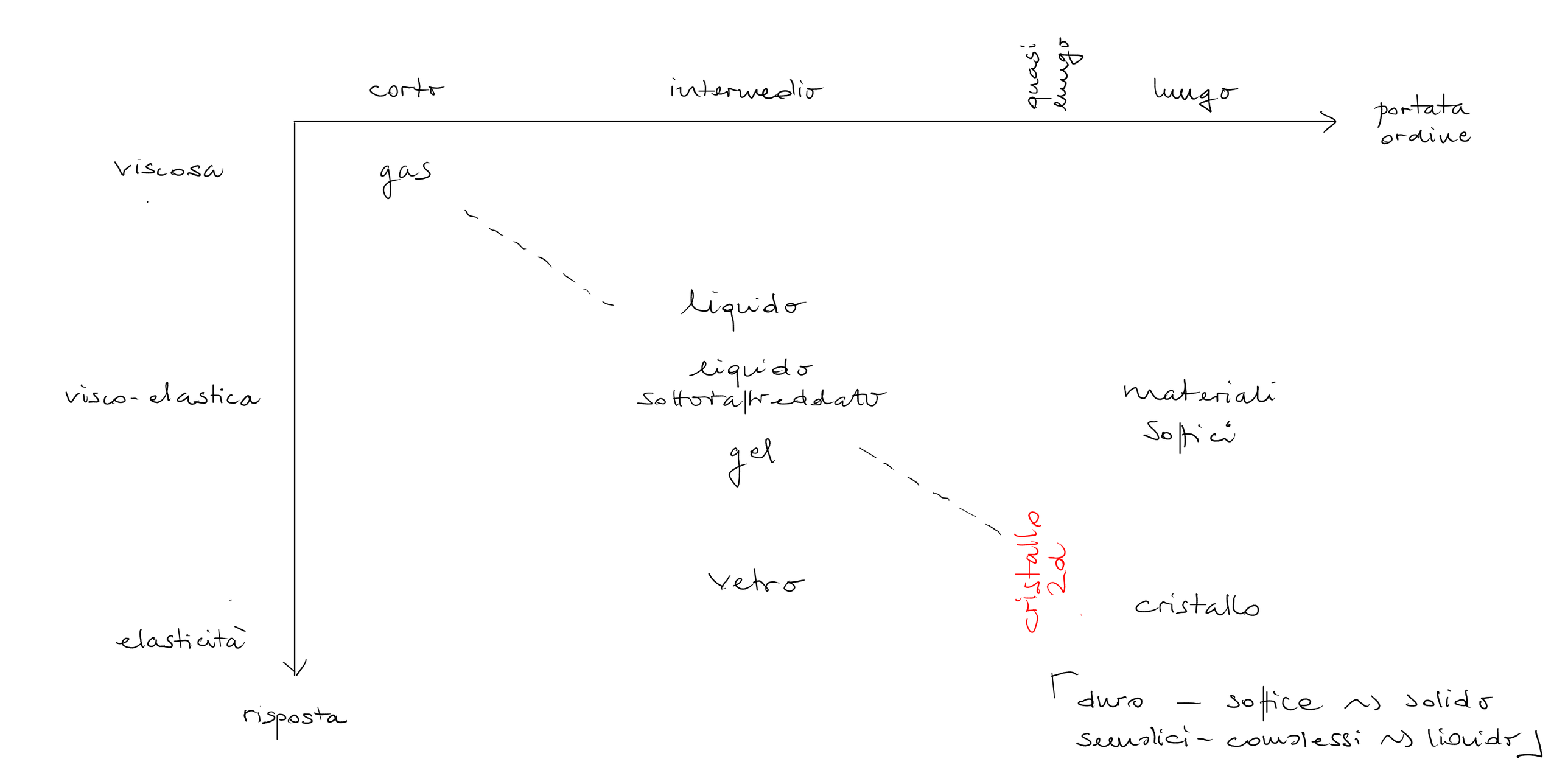
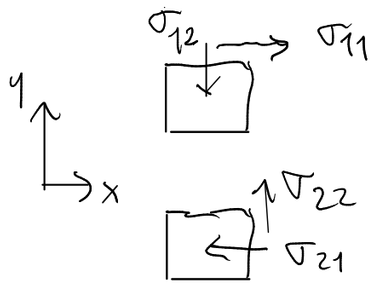

corto
intermedio
quasi lungo
lungo
portata ordine
viscosa
gas
liquido
liquido sottoraffreddato
visco-elastica
materiali soffici
gel
cristallo 2d
vetro
cristallo
elasticità
risposta
duro — soffice ⇝ solido
semplici - complessi ⇝ liquido

ELASTICITÀ

sforzo = modulo × deformazione

$\frac{F}{A}$ → Pa ; adim.

$[\ F = K \cdot \Delta x\ ]$ **solido Hookiano**



tensore degli sforzi : $\sigma_{\alpha\beta}$

$$\sigma_{\alpha\beta} = \sum_{\delta\sigma} C_{\alpha\beta\delta\sigma}\, \gamma_{\delta\sigma}$$

↑ tensore elastico
modulo elastico

1) meccanica
2) reologia

$\vec{R} \rightarrow \vec{r}$

tensore di deformazione :

$$\gamma_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial r_\alpha}{\partial R_\beta} + \frac{\partial r_\beta}{\partial R_\alpha}\right)$$

→ 81 componenti !

↓

36

↙ ↘

cristallo cubico

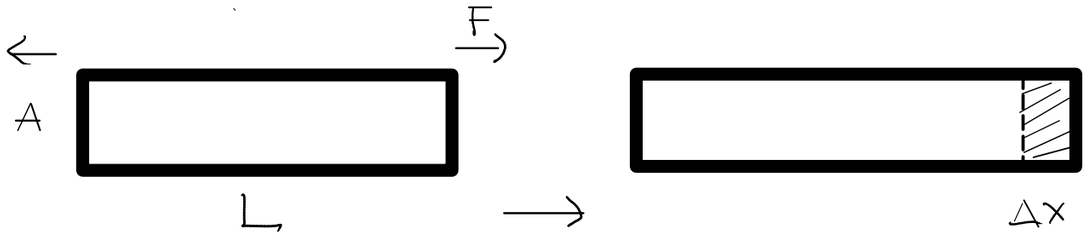
3

omogeneità, isotropia

2

modulo di Young Y
modulo di taglio G

1) Sforzo di trazione



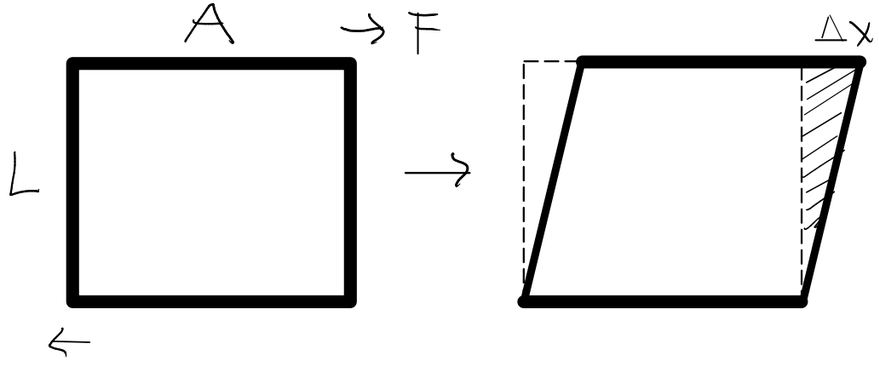
$$\sigma = \frac{F}{A} \qquad \gamma = \frac{\Delta x}{L} \rightarrow \boxed{\sigma = Y\,\gamma}$$

Y ↑ modulo di Young

diamante : $Y \sim 10^3$ GPa

gomma : $Y \sim 0.1 - 0.01$ GPa @ Tamb

2) Sforzo di taglio



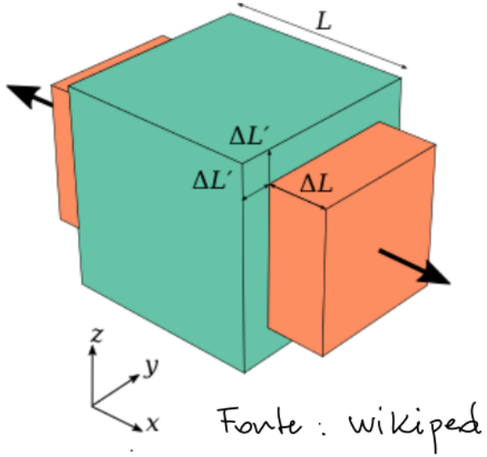
diamante : 500 GPa

gomma : 0.0006 GPa @ Tamb

$$\sigma = \frac{F}{A} \qquad \gamma = \frac{\Delta x}{L}$$

$$\boxed{\sigma = G\,\gamma}$$

G ← modulo di taglio



Fonte: wikipedia "Poisson's ratio"

$\Delta L'$ trasversale

ΔL assiale

Coeff. Poisson : $\nu = -\frac{\Delta L'}{\Delta L}$

$0 \lesssim \nu \lesssim 0.5$

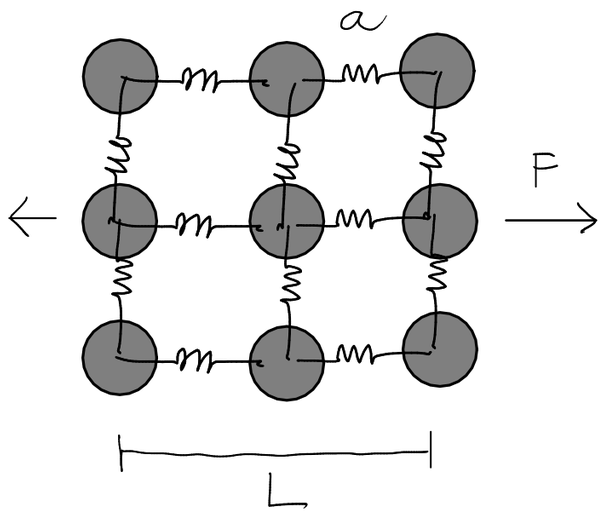
↑ sughero ↑ gomma

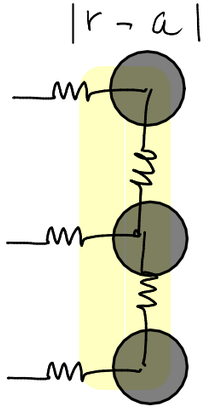
$$\boxed{Y = 2G(1+\nu)}$$

Modello micro

goal: materia soffice / dura

ipotesi: Solido armonico, sforzo di trazione



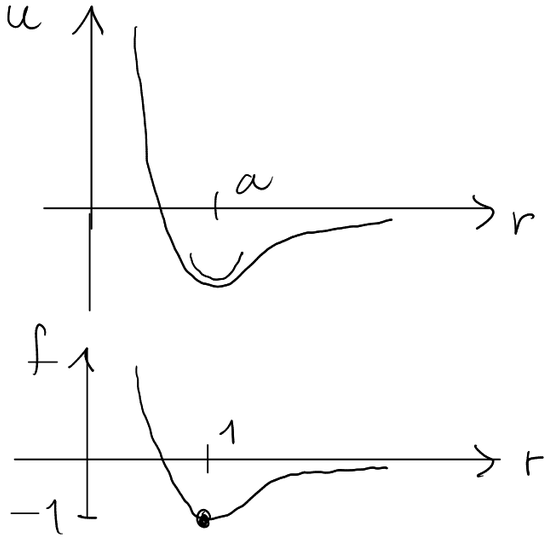


$$\sigma = \frac{F}{A} = \frac{k\,|r-a|}{a^2}$$

$$\gamma = \frac{\Delta x}{L} = \frac{|r-a|}{a}$$

$$\sigma = Y\,\gamma \qquad \frac{k\,|r-a|}{a^2} \simeq Y\,\frac{|r-a|}{a}$$

$$Y = \frac{K}{a} \sim \frac{\epsilon}{a^3} \sim \text{densità di energia legame}$$



$$u(r) \approx u(a) + \frac{1}{2}\,u''(a)\,(r-a)^2$$

$$u(r) = \epsilon\, f(r/a)$$

↑ scala di energia

$$u'' \simeq \frac{\epsilon}{a^2}\, f''\left(\frac{r}{a}\right)$$

prefattore ↓

$$u(r) \approx u(a) + \frac{1}{2}\underbrace{\frac{\epsilon}{a^2}}\, f''(1)\,(r-a)^2$$

dura

$\epsilon \sim 100\, K_B T a$

$a \sim 10^{-10}$ m

soffice

$\epsilon \sim 1 - 10\, K_B T a$

$a \sim 10^{-6}$ m

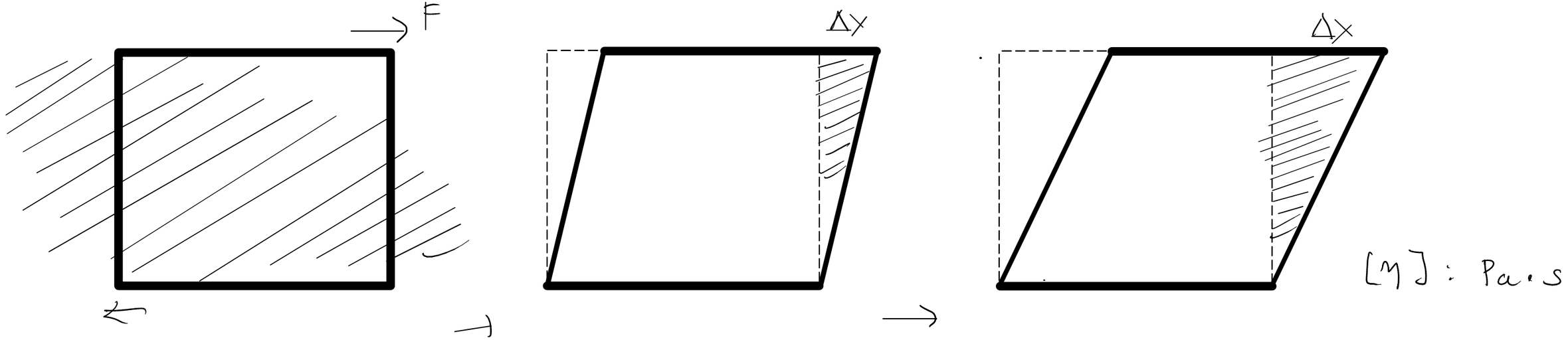
$$\frac{Y_{dura}}{Y_{soffice}} \sim 10 \cdot \left(\frac{10^{-6}}{10^{-10}}\right)^3 \sim \underline{\underline{10^{13}}}$$

$$[\sigma] = [Y] = \frac{[F]}{[A]} = \frac{[E]}{[V]} \sim \frac{\epsilon}{\sigma^3}$$

VISCOSITÀ

Solido: sforzo ⇒ deformazione
stato di equilibrio

fluido: sforzo ⇒ tasso di deformazione
regime stazionario



$[\eta]$: Pa·s

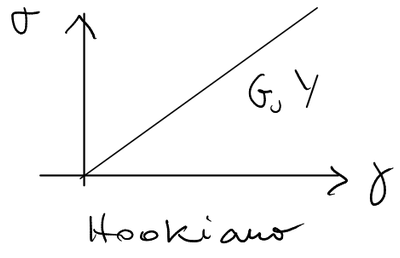
$$\gamma = \frac{\Delta x}{L} = \frac{v \cdot t}{L} \quad ; \quad \sigma = \frac{F}{A} \quad \Rightarrow \quad \sigma = \eta \dot{\gamma}$$

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{v}{L}$$

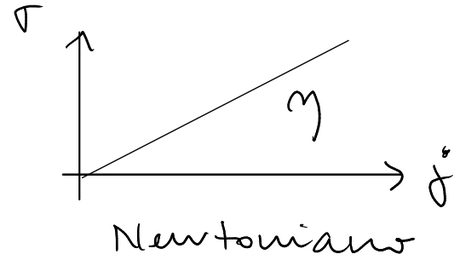
↑ tasso di deformazione

Pa — Viscosità

fluido Newtoniano



Hookiano



Newtoniano

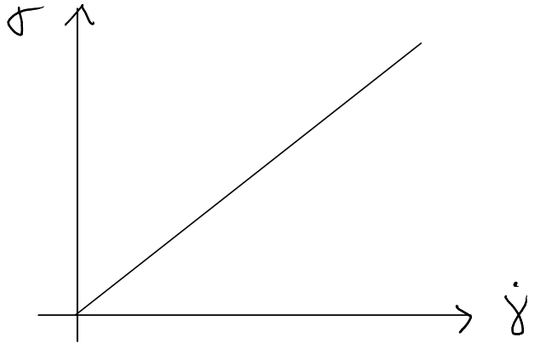
Es.: H_2O @Tamb
$\eta \sim 10^{-3}$ Pa·s
$T \simeq 90°C$
$\eta \sim 10^{-4}$ Pa·s
Miele $\eta \sim 1$ Pa·s

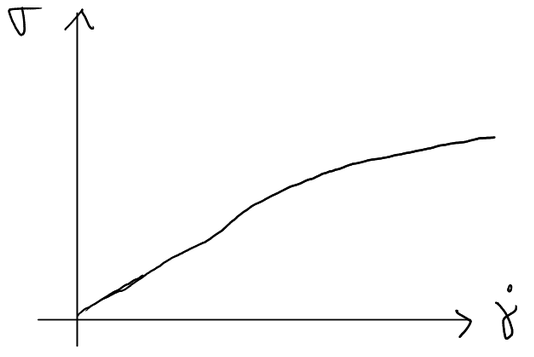
Comportamento non - Newtoniano

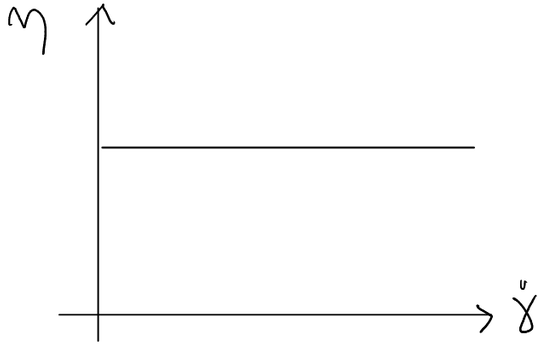
Newtoniano

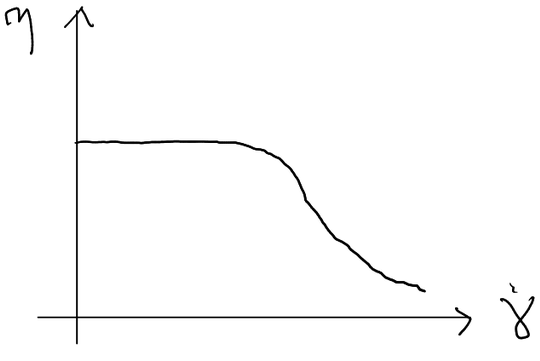
non - Newtoniano ⟶

$$\sigma = \eta(\dot{\gamma})\,\dot{\gamma}$$



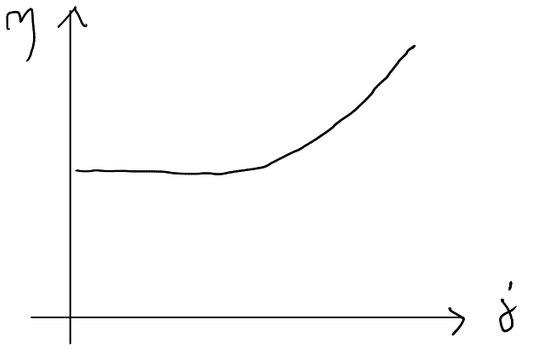






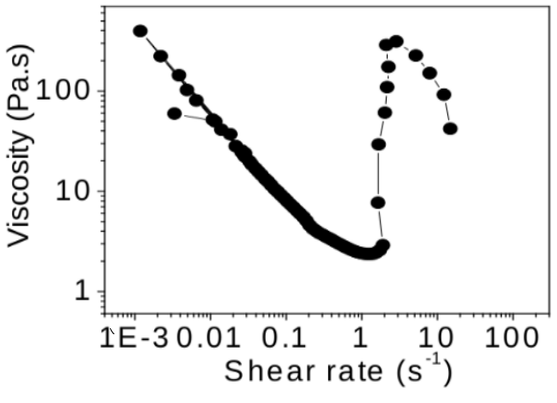
assoltigliamento al taglio (shear thinning)

Es: Ketchup, pittura



ispessimento al taglio (shear thickening)

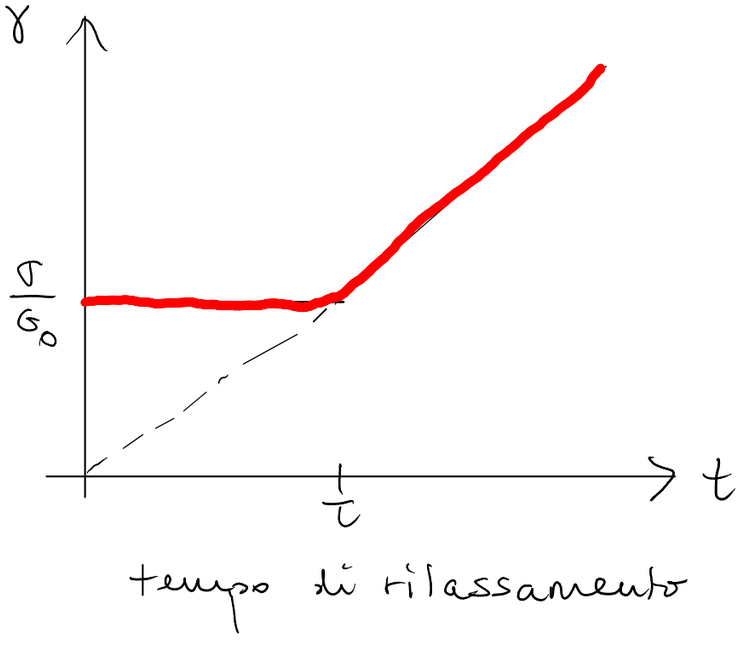
Es: amido di mais + H_2O



Fall et al. J. of Rheology '12
"Shear thickening of cornstarch suspensions"

VISCO - ELASTICITA'

Sistema si comporta in modo elastico o viscosa a seconda della scala di tempo



tempo di rilassamento

- solido Hookiano: $\sigma = G_0 \gamma \Rightarrow \gamma = \frac{\sigma}{G_0}$

  G_0 ↑ modulo istantaneo

- fluido Newtoniano: $\sigma = \eta \dot{\gamma}$

$$\begin{cases} \sigma = \eta \dot{\gamma} \\ \gamma = \dot{\gamma} \cdot t \end{cases}$$

modello di Maxwell

$$\gamma = \frac{\sigma}{G_0} = \dot{\gamma} \tau = \frac{\sigma}{\eta} \tau \Rightarrow \eta = G_0 \tau$$

Es: $\tau \sim 10^{-12}$ s ; $G_0 \sim 10^9$ Pa $\Rightarrow \eta \sim 10^{-3}$ Pas