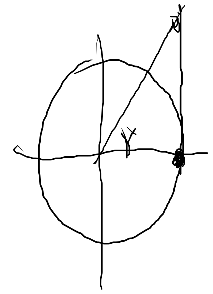
$$tg\,x = \frac{sen\,x}{\cos x}$$

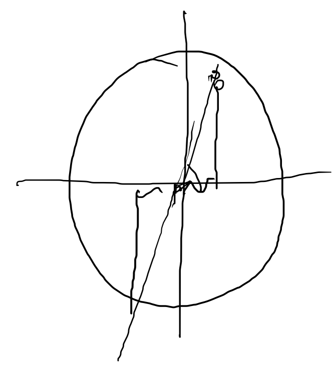


$$tg(-x) = -tg(x)$$

$$f(-x)\cdot g(-x) = f(x)\left(-g(x)\right) = -f(x)\cdot g(x)$$

pari   dispari

$$tg(x+\pi) = \frac{sen(x+\pi)}{\cos(x+\pi)} = \frac{-sen(x)}{-\cos(x)} = tg(x)$$



$\forall x$

su $[0,\frac{\pi}{2}[$ $f(x) = tg(x)$ $f(x) > 0$ $sen(x)$ è crescente e >0 su $[0,\frac{\pi}{2}[$

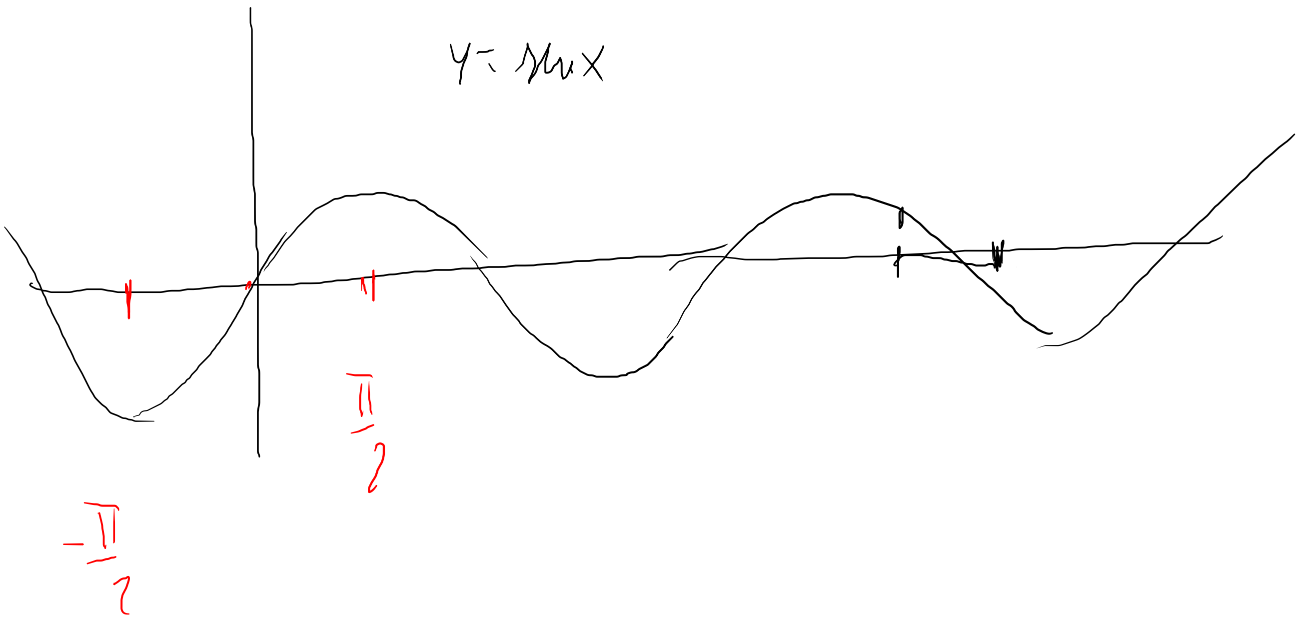
$\cos(x)$ è decrescente e >0

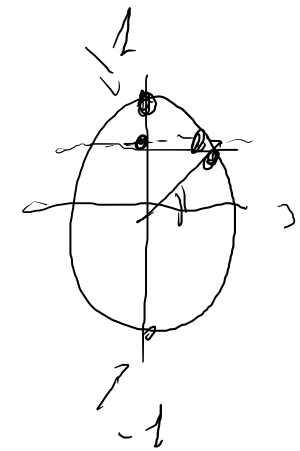
$g(x) > 0$ decrescente $\Rightarrow$ $\frac{1}{g(x)} > 0$ e crescente $x_1 < x_2 \Rightarrow g(x_1) > g(x_2)$

$\Rightarrow \frac{1}{g(x_2)} > \frac{1}{g(x_1)}$

$\frac{1}{\cos(x)}$ è crescente su $[0,\frac{\pi}{2}[$

⚠ Una funzione periodica NON è MAI invertibile!





$\max \operatorname{sen} x = 1$ ($\cos x$: 1) $\sup \operatorname{tg} x = +\infty$

$\min \operatorname{sen} x = -1$ ($\cos x$: -1) $\inf \operatorname{tg} x = -\infty$

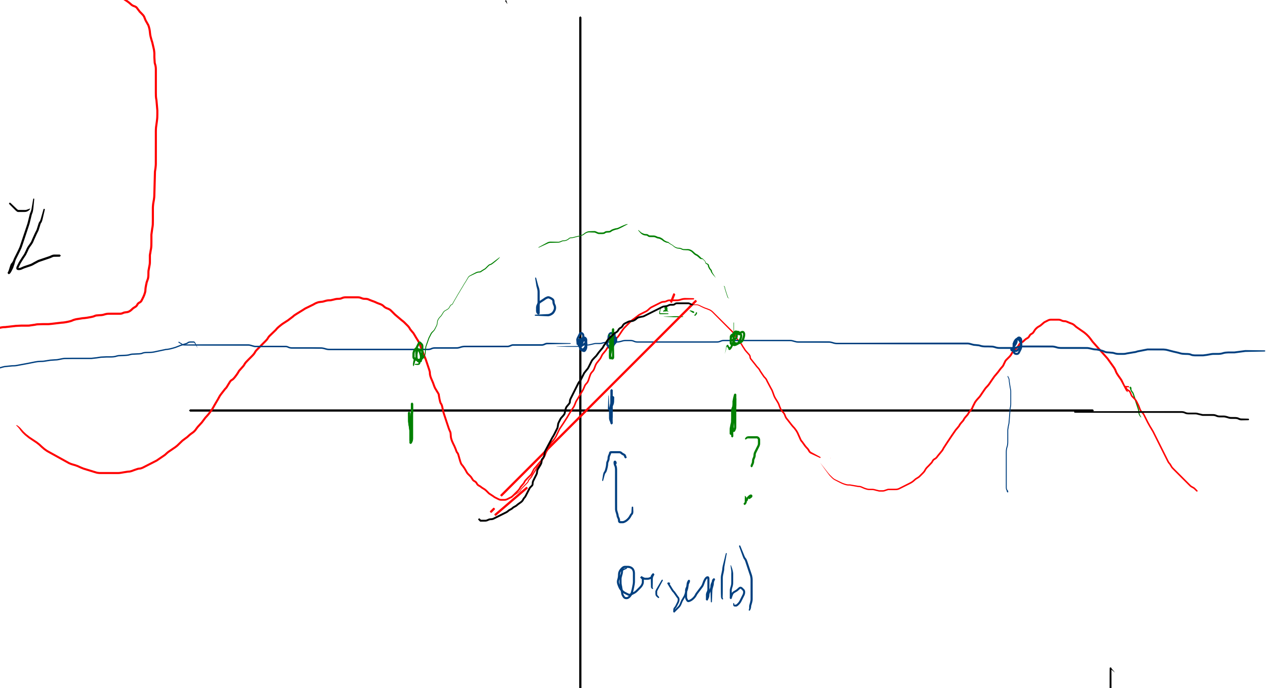
$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ $f(x) = \operatorname{sen} x$ è biiettiva; la sua inversa $f^{-1}(y)$

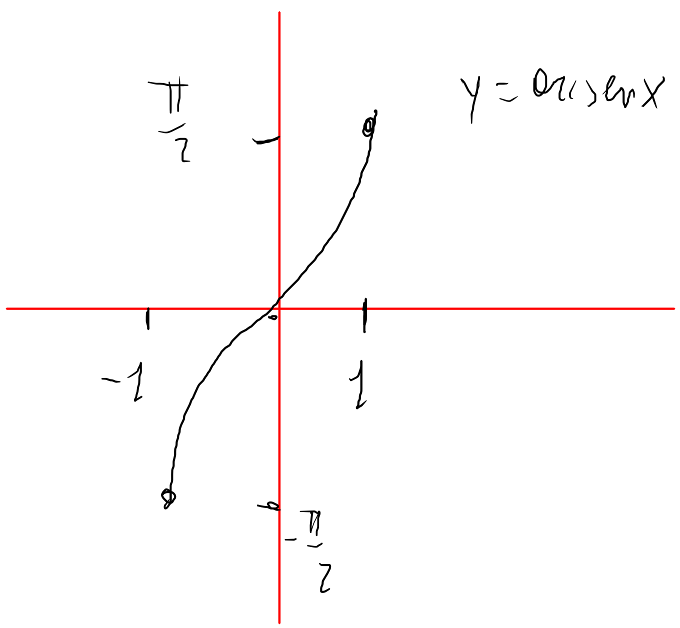
si dice arcoseno $\operatorname{arcsen}(y) = x$ se e solo se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ e $\operatorname{sen}(x) = y$

$[\forall y \in [-1, 1]]$

NB: l'equazione $\operatorname{sen} x = b$ ha soluzioni se e solo se $|b| \leq 1$;
in questo caso sono infinite:

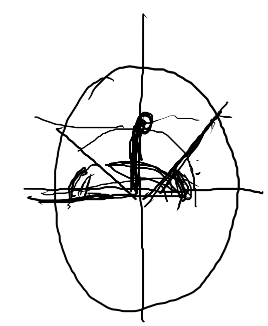
$$\boxed{\begin{aligned} x &= \operatorname{arcsen}(b) + 2k\pi \quad k \in \mathbb{Z} \\ &= \pi - \operatorname{arcsen}(b) + 2k\pi \quad k \in \mathbb{Z} \end{aligned}}$$

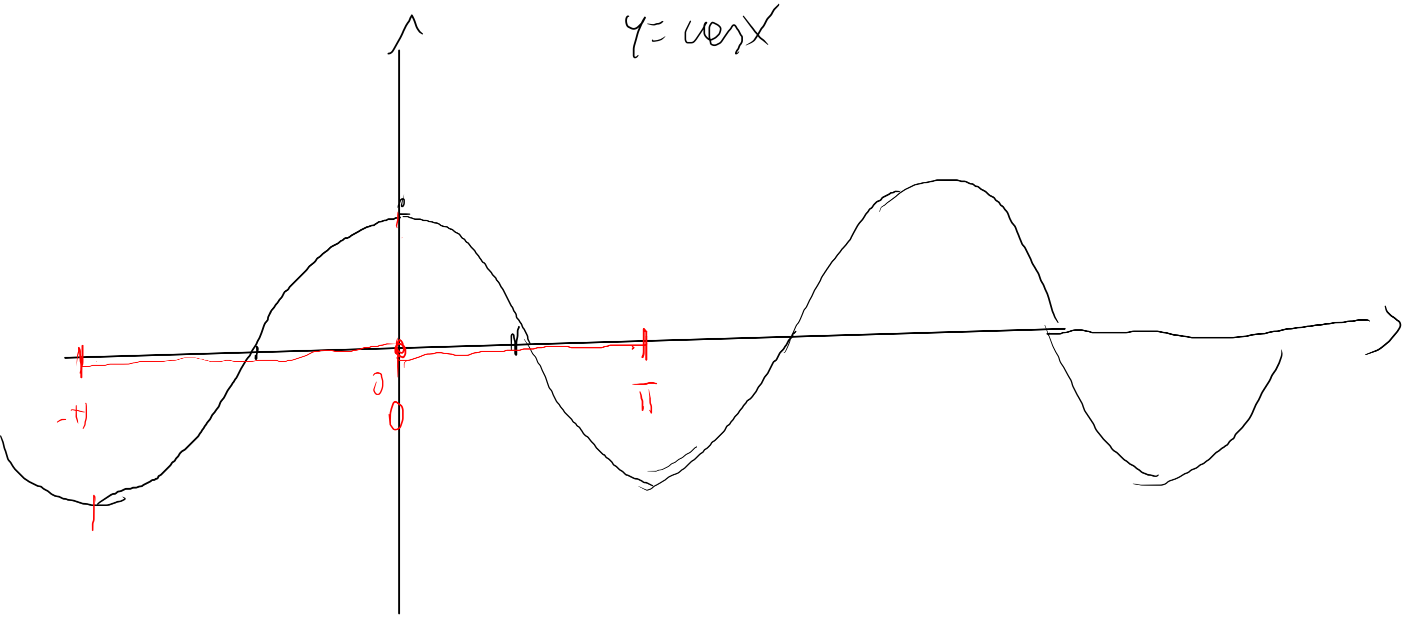




funzione dispari

$$\operatorname{arcsen}(-x) = -\operatorname{arcsen}(x)$$





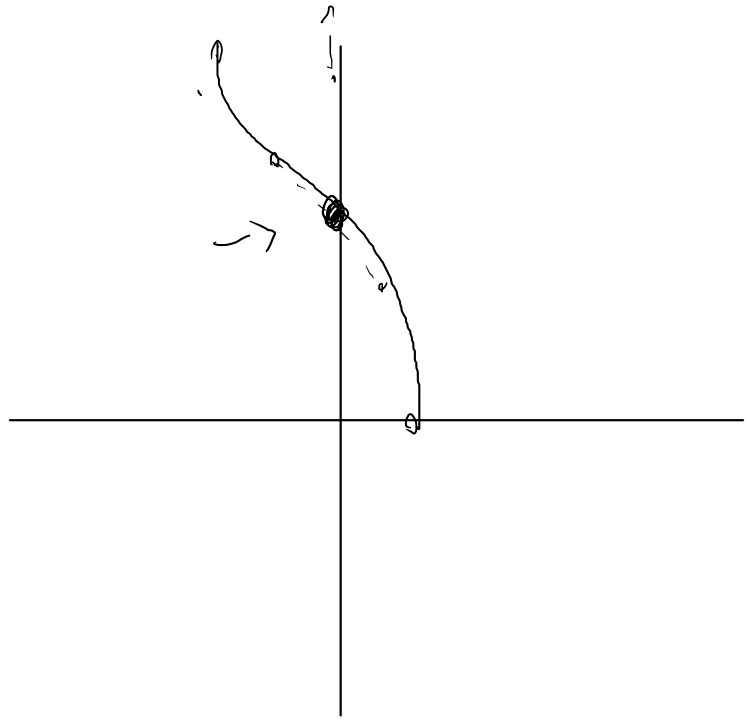
$f:[0,\pi]\to[-1,1]$ $\quad f(x)=\cos x$ $\quad$ f è decrescente

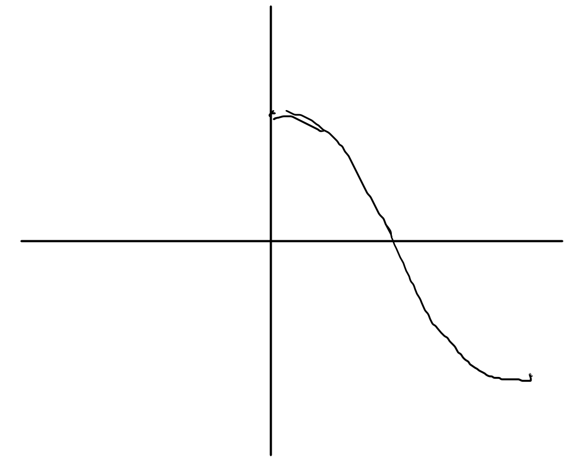
$f^{-1}:[-1,1]\to[0,\pi]$ è decrescente $\quad f^{-1}(y)=\arccos(y)$ $\quad$ arcocoseno

$f^{-1}(y)=x$ se e solo se $x\in[0,\pi]$ e $\cos(x)=y$ $\quad [\forall y\in[-1,1]]$

$\min \arccos(x)=0$ $\quad \max \arccos(x)=\pi$ $\quad f^{-1}$ pari? NO!

è decrescente!

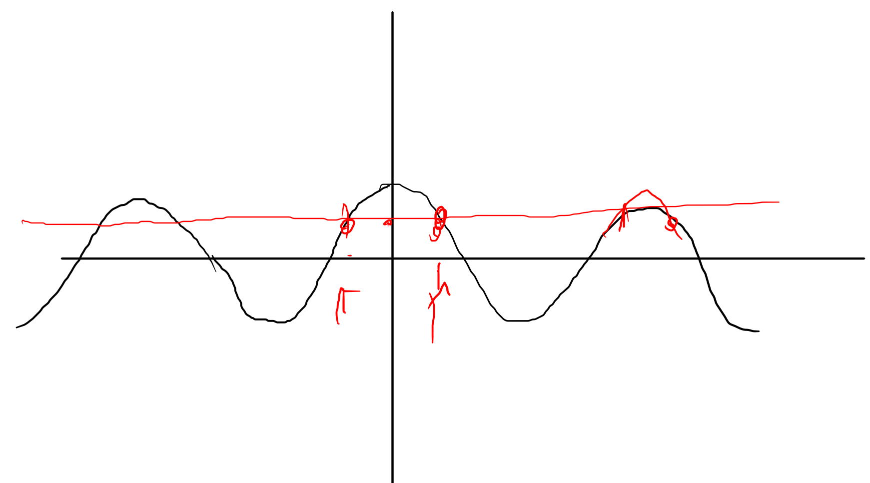




arccos x è una funzione simmetrica rispetto al punto $\left(0, \frac{\pi}{2}\right)$

$\arccos x - \frac{\pi}{2}$ è una funzione dispari!

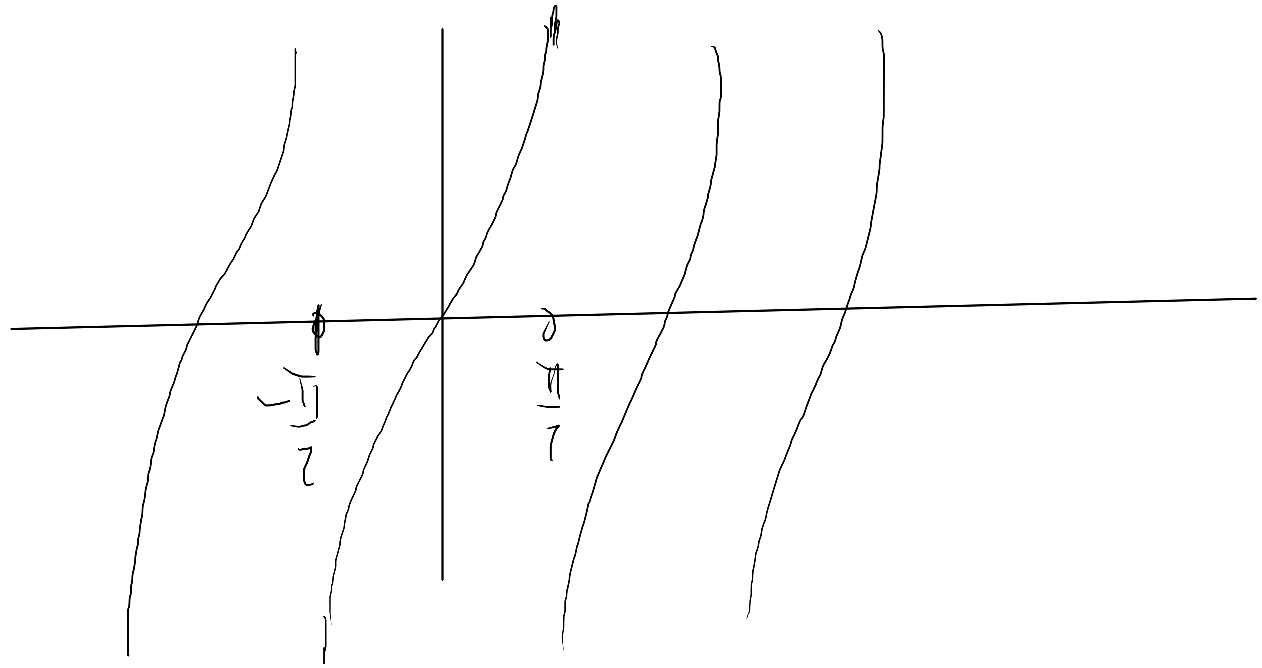
$\cos x = b \quad |b| \le 1$ ha soluzioni:



$$x = \arccos b + 2k\pi \qquad k \in \mathbb{Z}$$
$$\quad - \arccos b + 2k\pi$$

$f(x) = \operatorname{tg} x$

$f : \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\to \mathbb{R}$

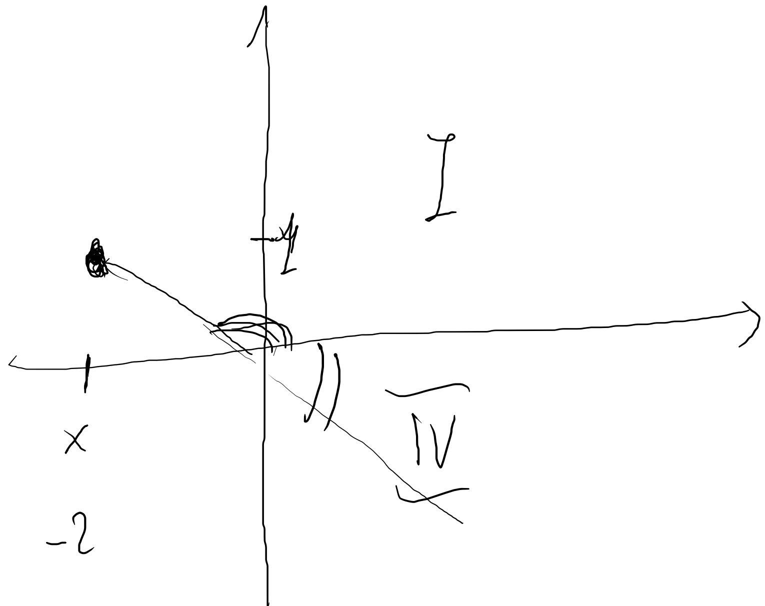


$f^{-1} : \mathbb{R} \to \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ $\qquad f^{-1}(y) = \operatorname{arctg} y$ $\quad$ [arctan , $\tan^{-1}$ tg^{-1} ...]

$\operatorname{arctg} y = x$ se e solo se $x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ e $\operatorname{tg} x = y$.

l'equazione $\underline{\operatorname{tg} x = b}$ ha infinite soluzioni $\forall b \in \mathbb{R}$

$x = \operatorname{arctg} b + k\pi$



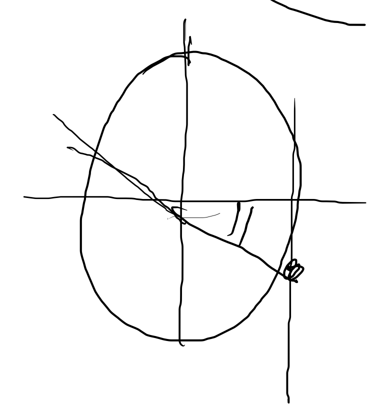
$x+iy=z$

$x=|z|\cos\vartheta = \rho\cos(\vartheta)$

$y=|z|\sin\vartheta = \rho\sin\vartheta$

$\rho=|z|=\sqrt{x^2+y^2}$

$\frac{y}{x}=\frac{\rho\sin\vartheta}{\rho\cos\vartheta}=\operatorname{tg}\vartheta \Rightarrow \vartheta=\operatorname{arctg}\frac{y}{x}$ NO!



$-2+i$

$\operatorname{arctg}\left(\frac{1}{-2}\right)\neq\vartheta$

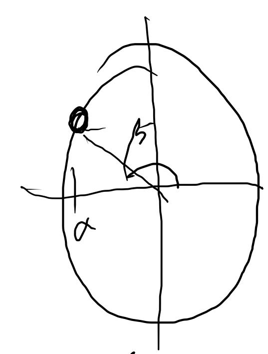
$\boxed{\vartheta=\operatorname{arctg}\left(-\frac{1}{2}\right)+\pi}$

$$f(x) = 3\cos(x) - 2\,\mathrm{sen}(x)$$

Combinazioni lineari in seno e coseno

$$f(x) = a\cos(x) + b\,\mathrm{sen}(x) = \sqrt{a^2+b^2}\left[\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\alpha}\cos(x) + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\beta}\,\mathrm{sen}(x)\right] = (*)$$

$$\alpha^2+\beta^2 = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = 1$$



esiste $\varphi \in [0, 2\pi]$ tale che $\alpha = \mathrm{sen}\,\varphi$ e $\beta = \cos\varphi$

$$(*) = \sqrt{a^2+b^2}\left(\mathrm{sen}(\varphi)\cdot\cos(x) + \cos(\varphi)\cdot\mathrm{sen}(x)\right) = \sqrt{a^2+b^2}\cdot\mathrm{sen}\left(x+\varphi\right)$$

In generale si considerano nel contesto delle funzioni periodiche funzioni del tipo

$$f(x) = A\,\mathrm{sen}(\omega x + \varphi)$$

periodo $\dfrac{2\pi}{\omega}$

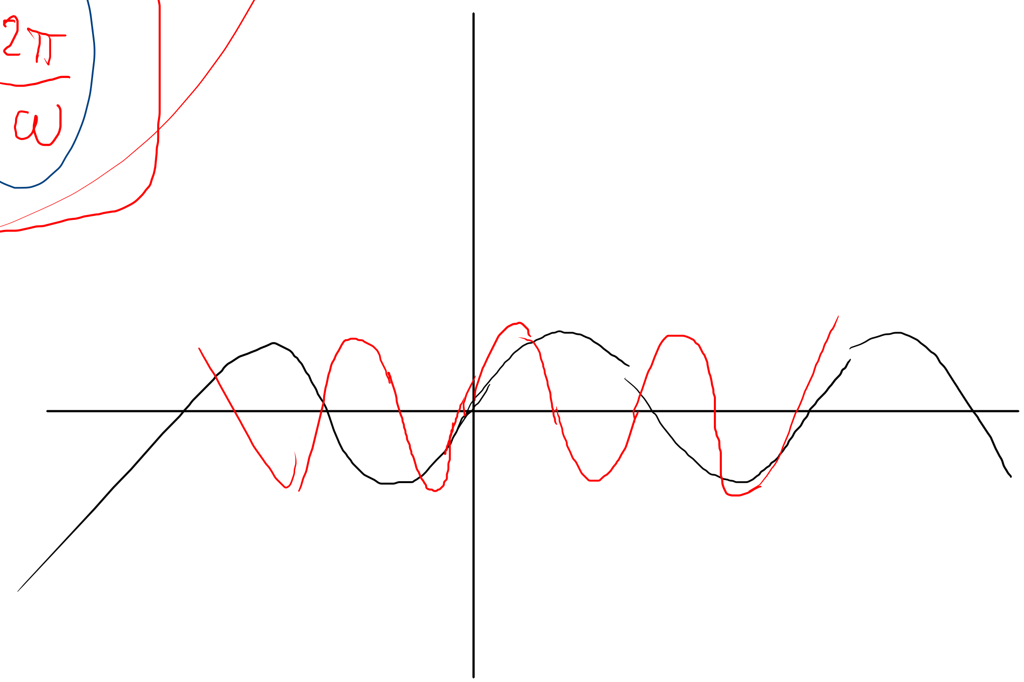
$$\mathrm{sen}(\omega x)$$

$$\mathrm{sen}(2x)$$

ω pulsazione

$\dfrac{\omega}{2\pi}$ frequenza

$$\mathrm{sen}\left(\omega\left(x + \frac{2\pi}{\omega}\right)\right) = \mathrm{sen}(\omega x + 2\pi) = \mathrm{sen}(\omega x)$$

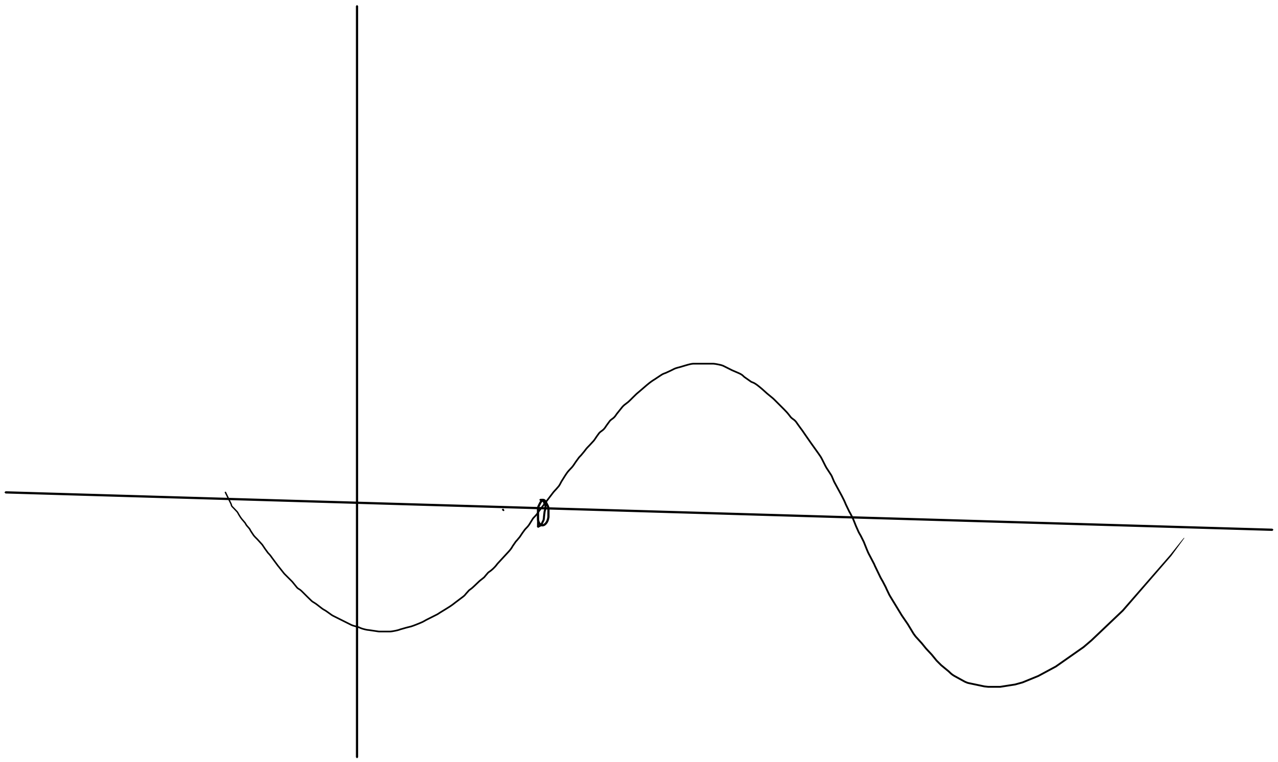


$$\mathrm{sen}(2(x+\pi)) = \mathrm{sen}(2x)$$

$$\| $$

$$\mathrm{sen}(2x + 2\pi)$$

$$T = \pi$$



$A \, \text{sen}(\omega x + \varphi)$

A: semiampiezza

$\min f = -A$

$\max f = A$

$\text{sen}(\omega x + \varphi)$

φ: sfasamento

Studiare una funzione significa descrivere le proprietà della funzione

Convenzione sul dominio

$f : E \subseteq \mathbb{R} \to \mathbb{R}$

$f(x) = \underline{\log(x-1)}$ "si cerchi l'insieme di $\mathbb{R}$ più grande dove ha significato l'espressione data"

⚠ $h(x) = x^x$ $\qquad f(x)^{g(x)}$ si intende $f(x) > 0$

$h(x) = [\sin(x)]^{\cos(x)} = e^{\log((\sin(x))^{\cos x})} =$ $\qquad \left[e^{\log z} = z \quad \forall z > 0 \right]$

$= e^{\boxed{\cos(x) \cdot \log(\sin(x))}}$

$\log x$, $\ln x$

segno $f(x) > 0$

$= 0$

< 0

limitato $\sup f$ $\inf f$ $\max f$ $\min f$

simmetrie pari/ dispari/ altro

monotonia (eventualmente su opportuni intervalli)

insieme immagine

$f(x) = \operatorname{arctg} x$ $\qquad f : \mathbb{R} \to \underbrace{\left] -\frac{\pi}{2}, \frac{\pi}{2} \right[}$

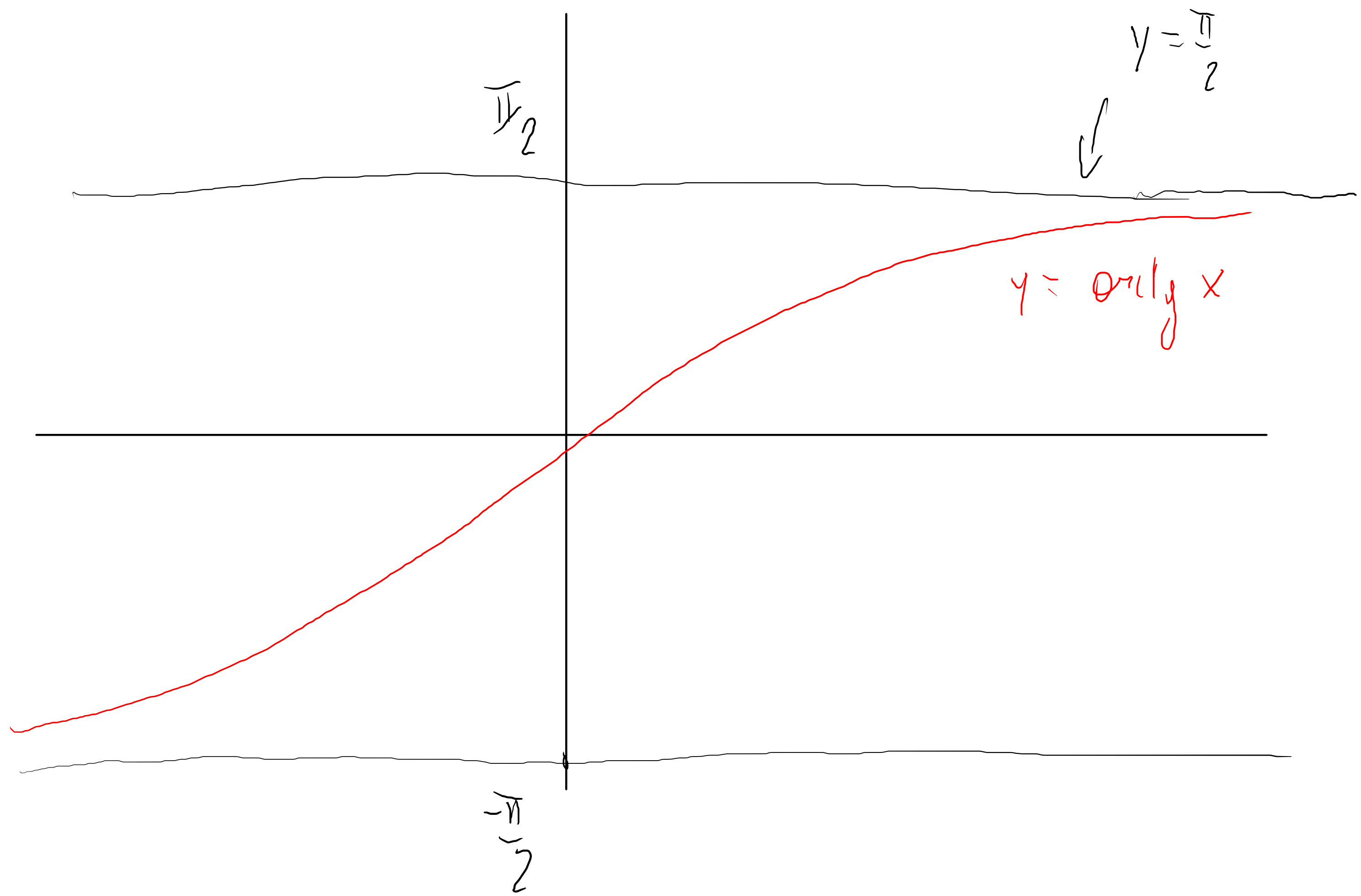
f crescente strettamente

f limitata

$\sup f = \frac{\pi}{2}$

$\inf f = -\frac{\pi}{2}$

f dispari



$sen(\omega x)$

$f(x)$ $\omega \in \mathbb{R}^+$ $g(x) = f(\omega x)$?

Funzioni polinomiali

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ $a_i \in \mathbb{R}$ $a_n \neq 0$

n grado

$f: \mathbb{R} \to \mathbb{R}$

Divisione tra polinomi / Teorema di Ruffini

Polinomi riducibili / irriducibili

Zero di molteplicità k $f(x) = (x-\alpha)^k q(x)$ $q(\alpha) \neq 0$

α

$f(x) = \sin x \cdot \frac{1}{\cos x}$ è crescente strettamente

$tg(x)$ è dispari $\Rightarrow$ $tg(x)$ è crescente su $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$