$0 < \frac{p}{q}$ $\quad q \neq 0$

$p, q \in \mathbb{Z}$

$\Updownarrow$

$p \cdot q > 0$ $\quad p, q$ "hanno lo stesso segno"

sono concordi

$\frac{p}{q} < 0$

$\Updownarrow$

$p \cdot q < 0$ $\quad p, q$ sono discordi

Consideriamo due frazioni

$\frac{p}{q}$ $\frac{n}{m}$ q, m diversi da 0

① $\frac{p}{q} < 0$ $0 < \frac{n}{m}$ $\Rightarrow$ $\frac{p}{q} < \frac{n}{m}$

② $0 < \frac{p}{q}$ $0 < \frac{n}{m}$

Dico che

$\frac{p}{q} < \frac{n}{m} \Longleftrightarrow p \cdot m < q \cdot n$

$\frac{p}{q} = \frac{p \cdot m}{q \cdot m}$ $\frac{n}{m} = \frac{nq}{q \cdot m}$

③

$\frac{p}{q} < 0 \qquad \frac{n}{m} < 0$

l'opposto di $\cancel{p/q} < 0$ quindi positivo

$$\frac{p}{q} < \frac{n}{m} \iff -\frac{p}{q} > -\frac{n}{m} > 0$$

l'opposto di $\frac{n}{m} < 0$

$n \in \mathbb{N}$ $\left(\frac{p}{q}\right)^n = \underbrace{\left(\frac{p}{q}\right) \cdot \ldots \cdot \left(\frac{p}{q}\right)}_{n \text{ volte}} = \frac{p^n}{q^n}$

$\left(\frac{p}{q}\right)^0 = 1$

$m \in \mathbb{Z}$ $\left(\frac{p}{q}\right)^m = \left(\frac{q}{p}\right)^{-m} =$ >0

$m < 0$

$p \neq 0$

$$0 < \frac{p}{q} < \frac{n}{m}$$

$$\begin{array}{ccc} \left(\dfrac{p}{q}\right)^k & < & \left(\dfrac{n}{m}\right)^k \\ \| & & \| \\ \dfrac{p^k}{q^k} & & \dfrac{n^k}{m^k} \end{array}$$

$k \in \mathbb{N}$

$k \neq 0$

Stabiliamo il segno di $\left(\frac{p}{q}\right)^{k}$ $\quad k \in \mathbb{N} \quad k \neq 0$

sapendo il segno di $\frac{p}{q}$

$$\left(\frac{p}{q}\right)^{k} = \frac{p^k}{q^k}$$

>0 se k è pari sia che $\frac{p}{q}>0$ $\frac{p}{q}<0$

<0 se k dispari e $\frac{p}{q}<0$

Tornando all'osservazione

$$0 < \frac{p}{q} < \frac{n}{m} \Longrightarrow 0 < \left(\frac{p}{q}\right)^k < \left(\frac{n}{m}\right)^k \qquad k \in \mathbb{N} \quad k \neq 0$$

pertanto la funzione

$$f_k : \mathbb{Q}^+ = \left\{ \frac{p}{q} \ \frac{p}{q} > 0 \right\} \longrightarrow \mathbb{Q} \qquad k \in \mathbb{N} \quad k \neq 0$$

$$0 < \left(\frac{p}{q}\right) \longmapsto \left(\frac{p}{q}\right)^k$$ è crescente

cioè

$$f_k\left(\frac{p}{q}\right) < f_k\left(\frac{n}{m}\right) \quad \text{se} \quad \frac{p}{q} < \frac{n}{m}.$$

$$\left(\frac{p}{q}\right)^k \qquad \left(\frac{n}{m}\right)^k$$

$$\mathbb{Q}^+ = \left\{ \frac{p}{q} \quad \frac{p}{q} > 0 \right\}$$

Essendo crescente, f_k è iniettiva. $\left(\forall k \in \mathbb{N}, \ k \neq 0\right)$

Quindi f_k è bigettiva sull'immagine $f_k(\mathbb{Q}^+)$

La funzione INVERSA di f_k (potenza alla k $k \in \mathbb{N}$ $k \neq 0$)
è detta RADICE $k$esima

Notazione $\sqrt[k]{\ }$ ← Radice k-esima

convenendo che quando $k = 2$ $\sqrt[2]{\ } = \sqrt{\ }$

$$\left(\frac{p}{q}\right)^k = \frac{p^k}{q^k} \qquad k \in \mathbb{N}$$

$$\boxed{\left(n^k\right)^{k'} = n^{k \cdot k'}}$$

$$k' = 1/k$$

$$\left[\left(\frac{p}{q}\right)^k\right]^{k'} = \left(\frac{p}{q}\right)^{k \cdot k'} = \left(\frac{p}{q}\right)^1 = \frac{p}{q}$$

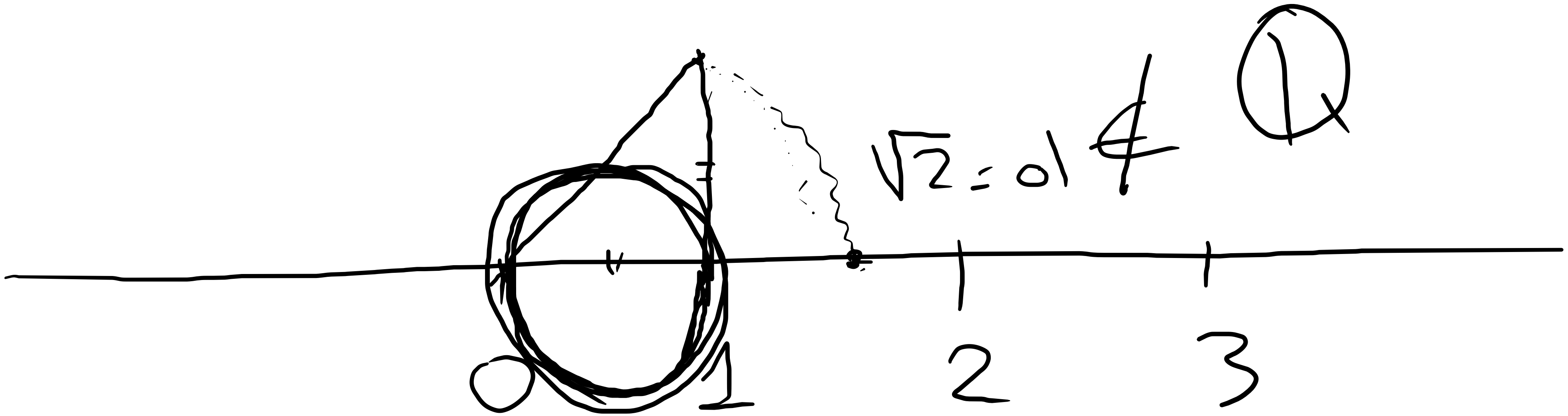
$$\left[\left(\frac{p}{q}\right)^{k}\right]^{\frac{1}{k}} = \left[\left(\frac{p}{q}\right)^{\frac{1}{k}}\right]^{k} = \left(\frac{p}{q}\right)^{1} = \frac{p}{q}$$

$\underline{\underline{k > 0}}$

$$\sqrt[k]{\frac{p}{q}} = \left(\frac{p}{q}\right)^{\frac{1}{k}}$$

$$\left(\frac{p}{q}\right)^{\frac{n}{m}} =: \left[\left(\frac{p}{q}\right)^{n}\right]^{\frac{1}{m}} = \sqrt[m]{\left(\frac{p}{q}\right)^{n}}$$

$q \neq 0 \quad m \neq 0 \quad \underline{\underline{\left(\frac{p}{q}\right) > 0}}$


√2 = 01 ∉ Q
0
1
2
3

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$\sqrt{2} \notin \mathbb{Q}$

l'insieme dei numeri reali

Assioma di completezza dei numeri reali

$A, B \subset \mathbb{R}$ $\quad A, B$ non vuoti

① $\forall a \in A \quad \forall b \in B \quad a < b$

② : $\varepsilon = \frac{1}{k} \; k \in$

$\forall \varepsilon > 0 \; \exists \, a_\varepsilon \in A$

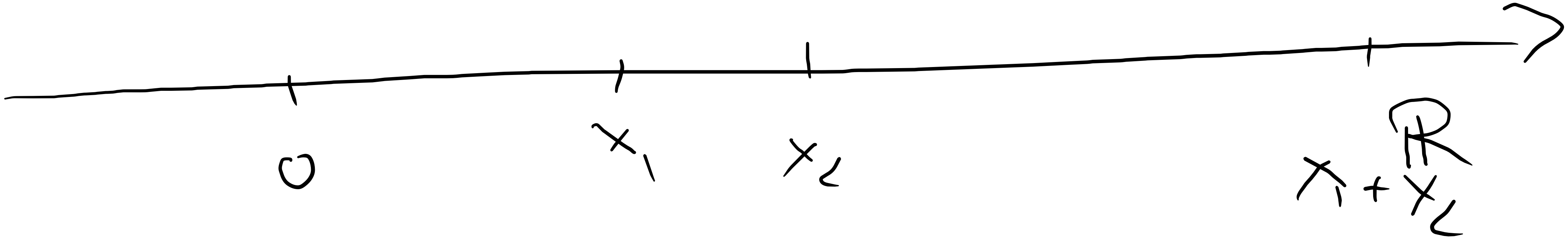
$b_\varepsilon \in B$ tale che

$0 < b_\varepsilon - a_\varepsilon < \varepsilon$

$\Rightarrow \exists! \; c \in \mathbb{R}$

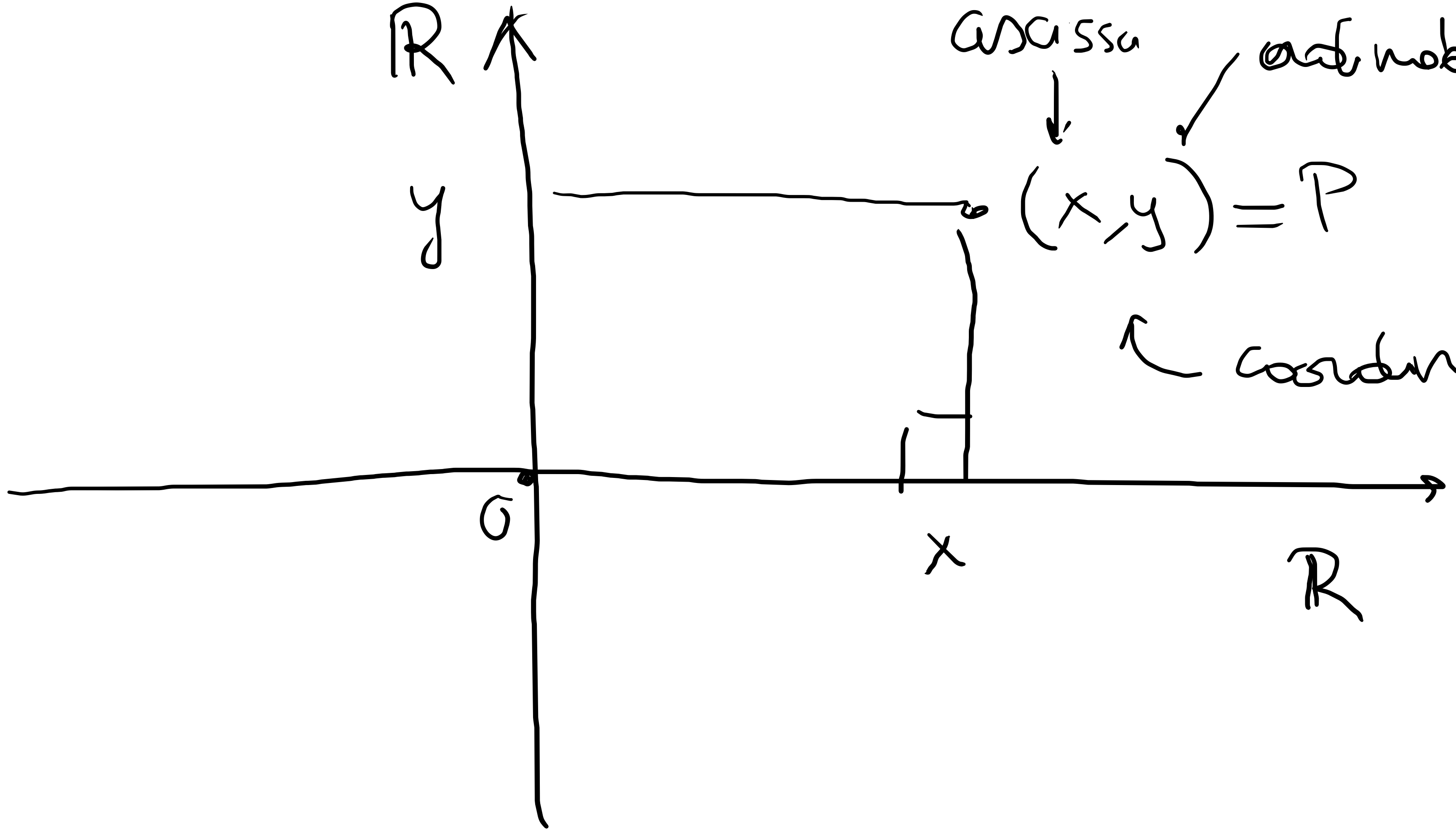
$a \leq c \leq b$

$\forall a \in A \quad \forall b \in B$



$x_1 \in \mathbb{R}$ $x_2 \in \mathbb{R}$ $x_1 + x_2$

$$\mathbb{R}\times\mathbb{R} = \mathbb{R}^2 = \{(x,y) \quad x\in\mathbb{R} \quad y\in\mathbb{R}\}$$



ascissa

ordinata

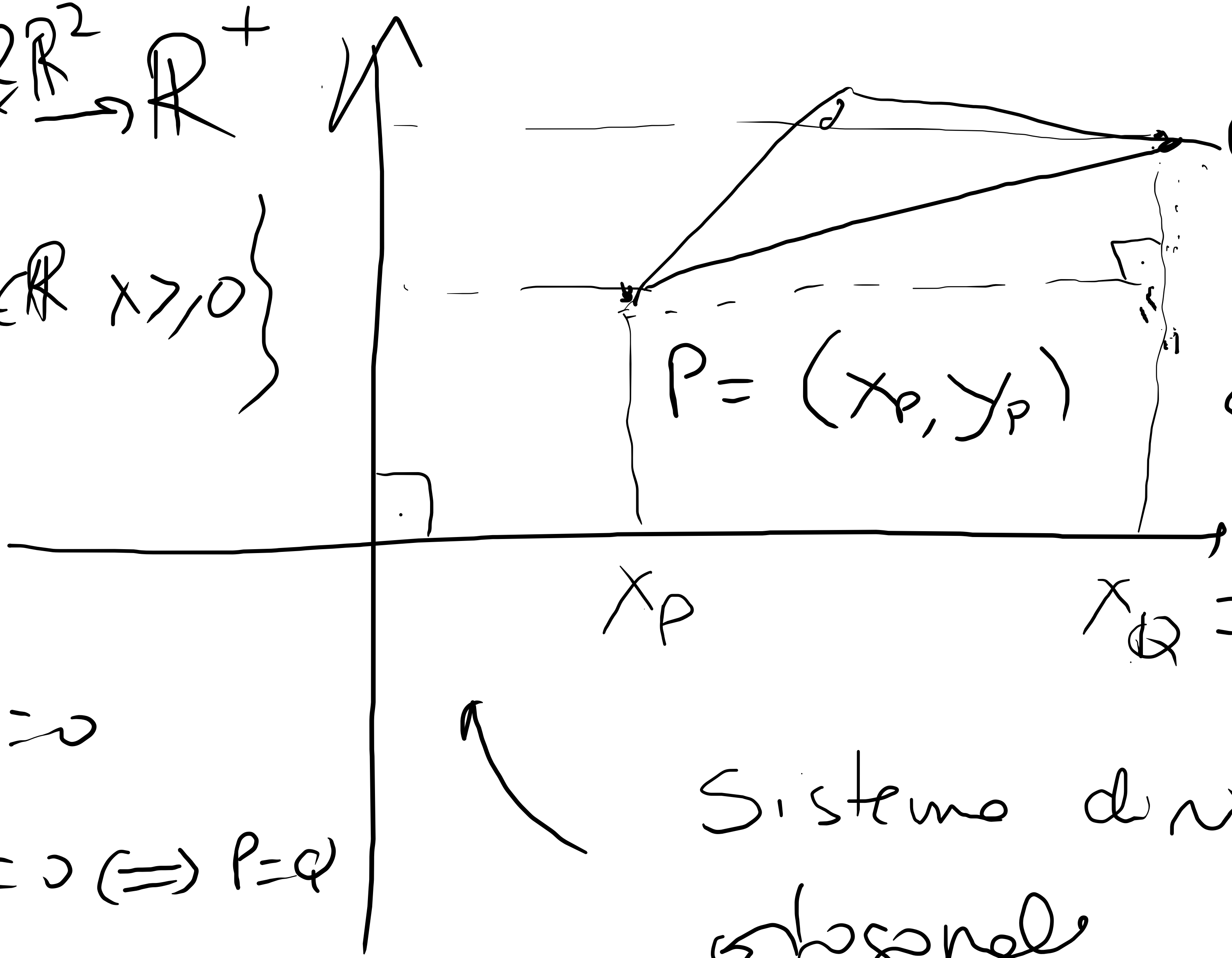
$(x,y) = P$

Piano CARTESIANO

coordinate cartesiane

$d: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^+$

$= \{x \in \mathbb{R} \;\; x \geq 0\}$



$Q = (x_Q, y_Q)$

$d(P,Q) = d(Q,P)$

$= \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

Sistema di riferimento cartesiano ortogonale

$d(P,P) = 0$

$d(P,Q) = 0 \iff P = Q$

$d(P,Q) = d(Q,P)$

$d(P,Q) \leq d(P,R) + d(R,Q)$