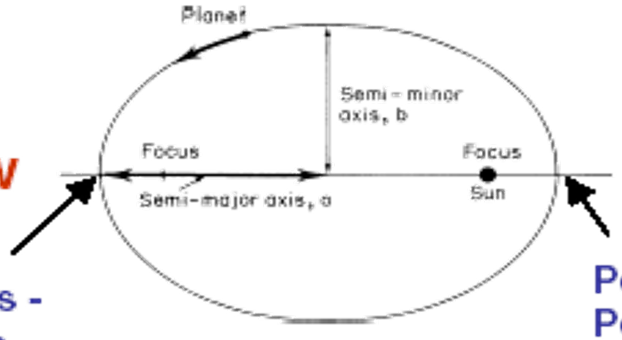
Le orbite di un satellite

- Leggi di Keplero
- Leggi di Newton
- Equazioni del moto
- Anomalie
- Elementi Orbitali
- Perturbazioni
- Orbite varie

Leggi di Keplero

1st Law



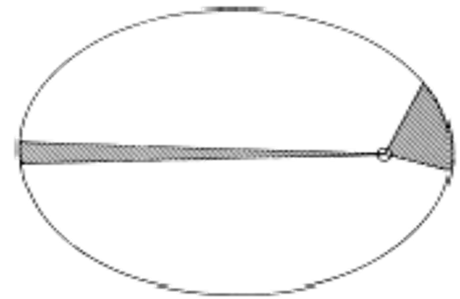
Apoapsis - Aphelion

Periapsis - Perihelion

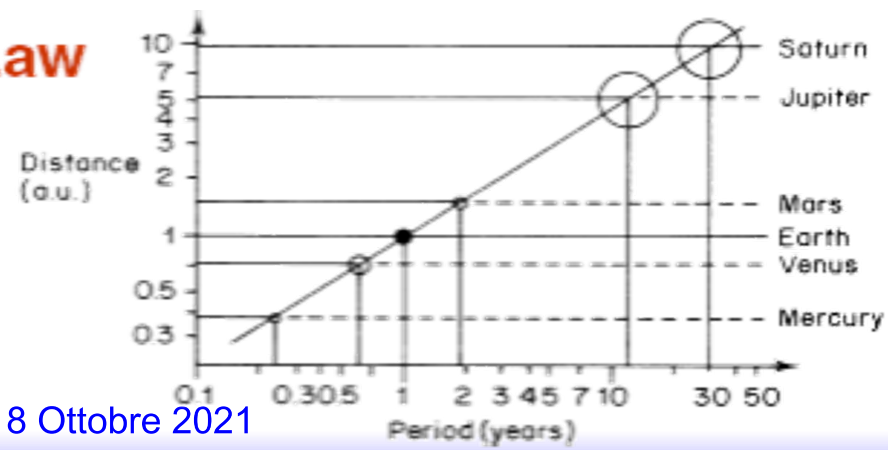
L'orbita di un pianeta è un'ellisse con il Sole in uno dei fuochi

Una linea congiungente il Sole ad un pianeta spazza aree uguali in intervalli di tempo uguali

2nd Law



3rd Law



Il quadrato del periodo dell'orbita di un pianeta è proporzionale al cubo della sua distanza media dal Sole

Leggi di Newton 1/2

- 1ª Legge: La legge di inerzia
- 2ª Legge: Forza = massa × accelerazione
- 3ª Legge: Azione e reazione

[illegible]
opposto

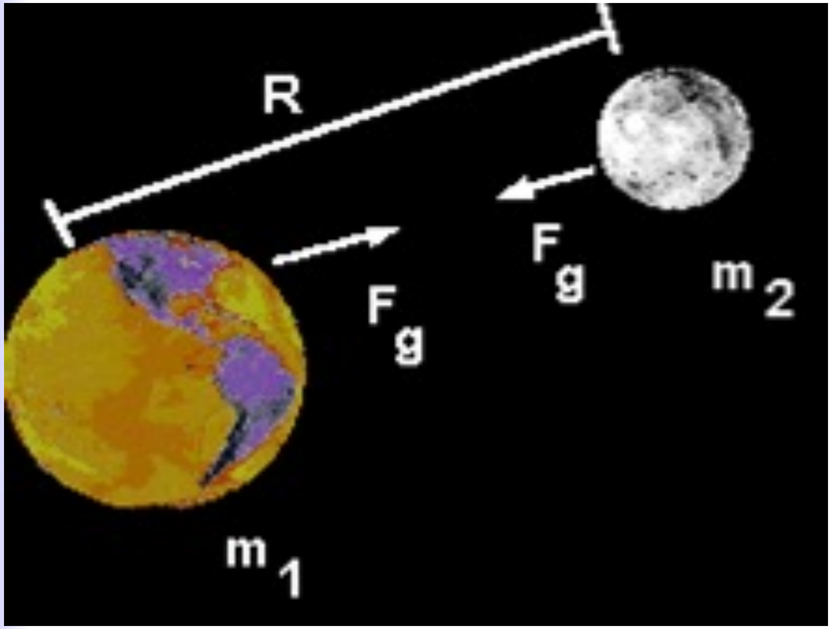
LA LEGGE DI GRAVITA': $F = \dfrac{GMm}{r^2}$

F	Forza gravitazionale tra due corpi
G	Costante di gravitazione universale: $G = 6.670 \times 10^{-11}\ N.m^2.kg^{-2}$
M	Massa di un corpo (Terra o Sole)
m	Massa di un altro corpo (il satellite)
r	Separazione tra i corpi

$$GM_{Earth} = \mu = 3.986004418 \times 10^{14}\ m^3.s^{-2}$$

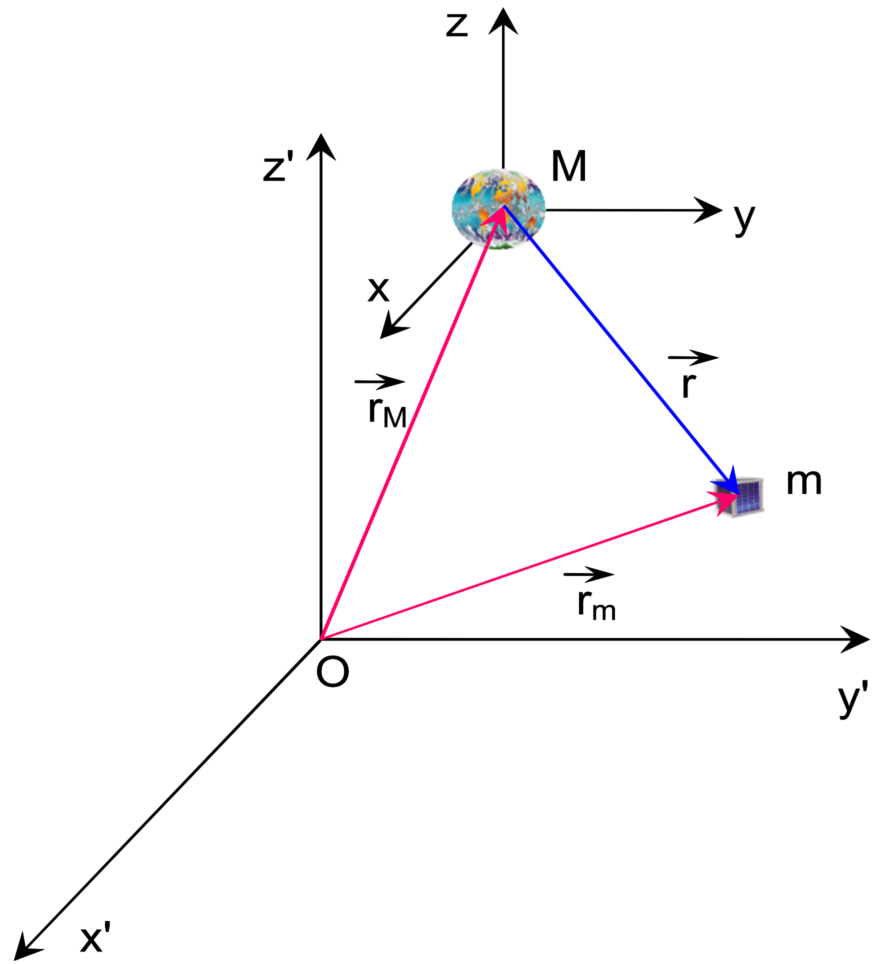
$$\sim 400\dot{}000\ km^3.s^{-2}$$

Leggi di Newton 2/2



Sistema a due corpi
di massa M e m

(M >> m)



Equazioni del moto 1/2

$$\ddot{\vec{r}} = -\mu \, \vec{r} / r^3$$

$$\Downarrow$$

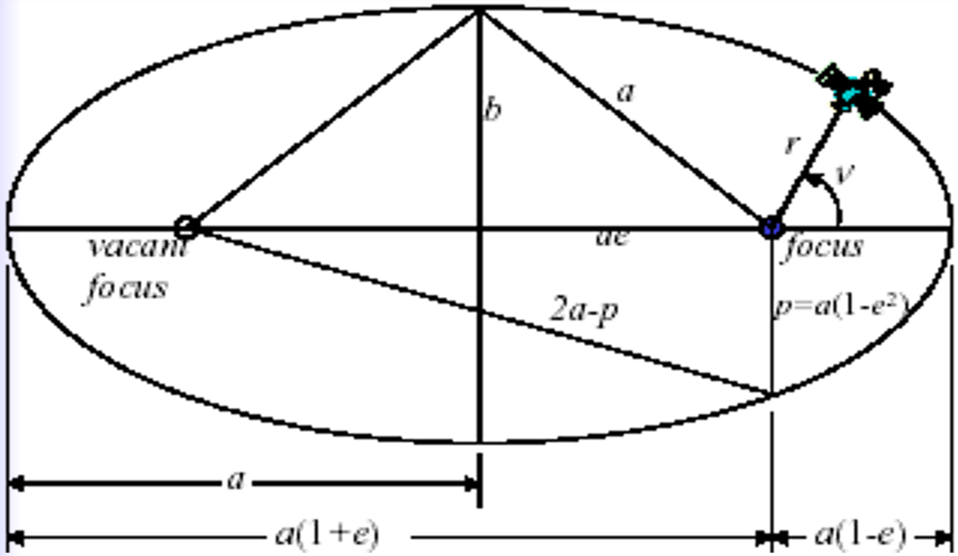
$$a_r = -\mu / r^2$$

$$a_\theta = 0$$

MOTO CENTRALE

$v_r = dr/dt$	velocità radiale
$v_\theta = r \, d\theta/dt$	velocità tangenziale
$a_r = d^2r/dt^2 - r \, (d\theta/dt)^2$	accelerazione radiale
$a_\theta = r \, d^2\theta/dt^2 + 2 \, dr/dt \, d\theta/dt$	accelerazione tangenziale
$= 1/r \, d(r^2 d\theta/dt)/dt$	

Equazioni del moto 2/2



S = satellite

T = terra (fuoco)

$\vec{r}$ = vettore posizione S rispetto al centro di T

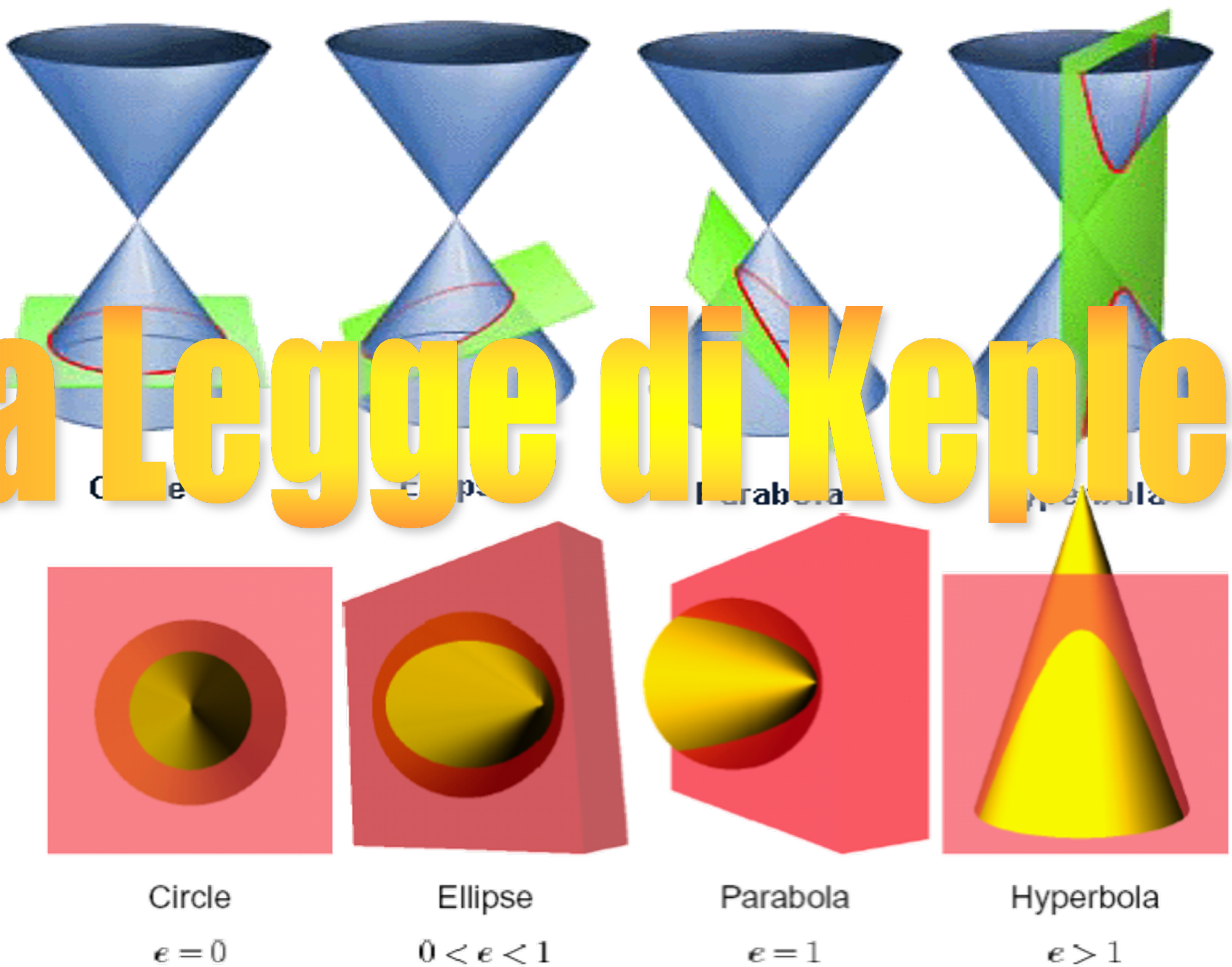
$\vec{v}$ = vettore velocità S rispetto a T

a	semiasse maggiore
b	semiasse minore: $b = a\sqrt{1-e^2}$
c	semidistanza fra i fuochi
e	eccentricità: $e = c/a = \sqrt{a^2-b^2}/a$
p	semi-latus rectus: $p = a(1-e^2)$
r_p	raggio del perigeo
r_a	raggio dell'apogeo
θ	angolo polare (anomalia vera)
φ	anomalia eccentrica

Coniche 1/3

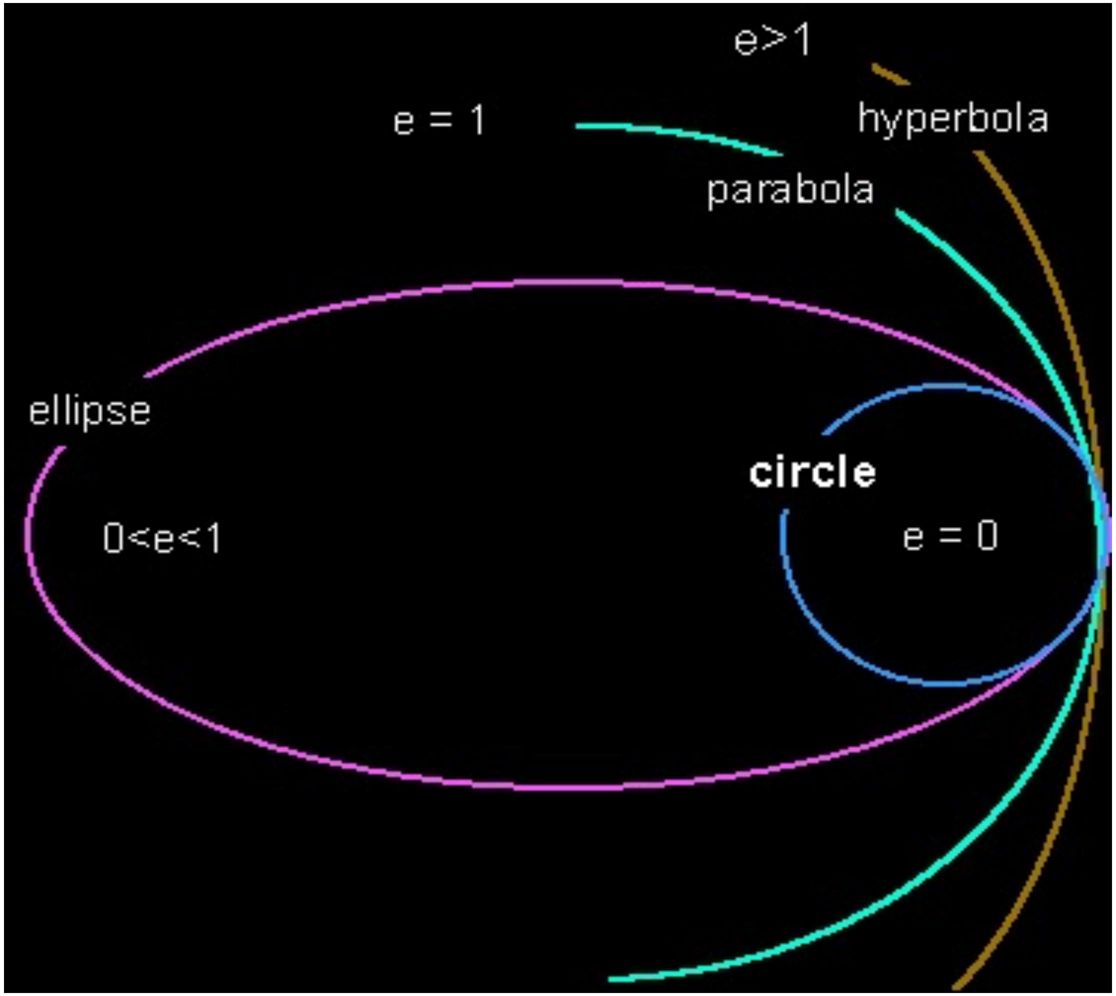
$$r = p / (1 + e \cos \theta)$$

1a Legge di Keplero



Coniche 2/3

$$r = p / (1 + e \cos \theta)$$

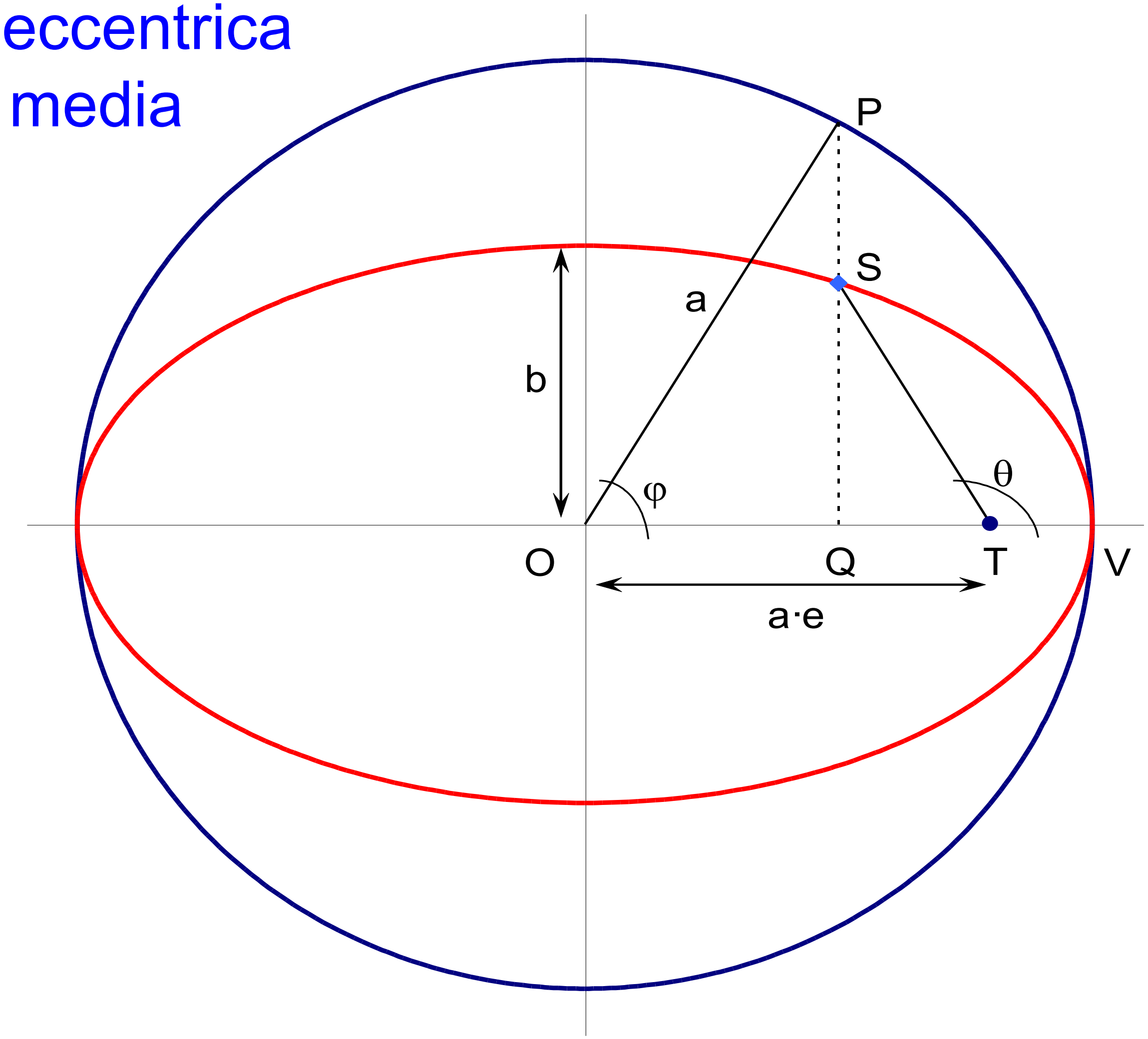


Coniche 3/3

θ = anomalia vera

φ = anomalia eccentrica

M = anomalia media



Anomalie

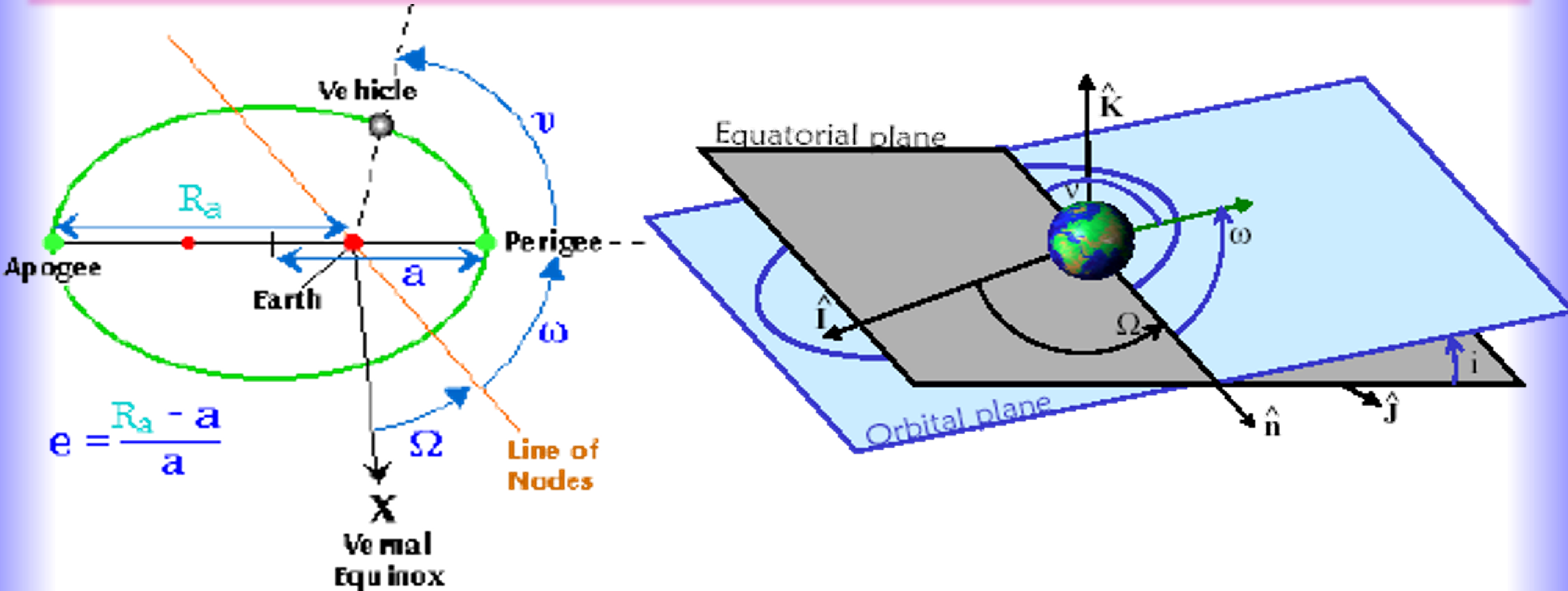
anomalia vera: $$r = \frac{a\,(1-e^2)}{1+e\cdot\cos\theta}$$

anomalia eccentrica: $$r = a\,(1-e\cdot\cos\varphi)$$

anomalia media: $$M = \varphi - e\cdot\sin\varphi = \sqrt{\frac{\mu}{a^3}}(t - t_p)$$

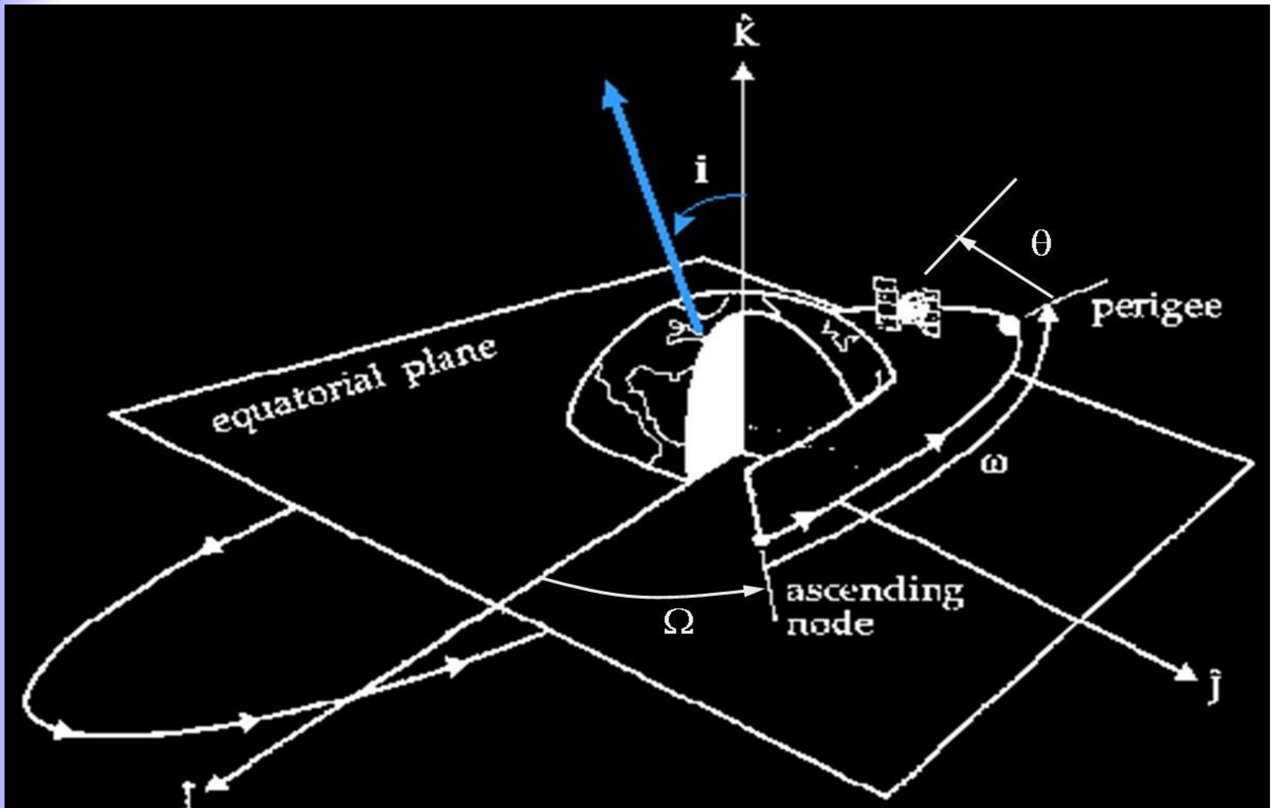
$$\cos\theta = \frac{e - \cos\varphi}{e\cdot\cos\varphi - 1}$$

Elementi Orbitali 1/2



Simbolo	Nome	Significato
a	semiasse maggiore	dimensione
e	eccentricità	forma
i	inclinazione	inclinazione del piano orbitale (nello spazio)
Ω (RAAN)	ascensione retta del nodo ascendente	orientazione del piano orbitale (nello spazio)
ω	argomento del perigeo	orientazione del perigeo (nel piano orbitale)
θ (ν)	anomalia vera	posizione dello S/C sull'orbita
T	istante di passaggio al periapside	posizione dello S/C sull'orbita

Elementi Orbitali 2/2



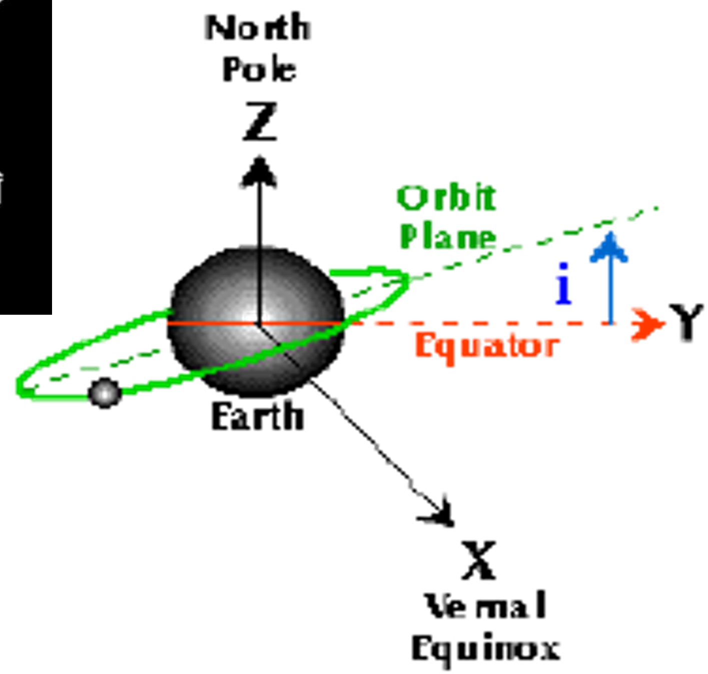
esempi:

orbita circolare, i=0, Ω,ω?

orbita circolare, i=10^o, Ω,ω?

Ω=0, ω=0, 90° , 180°

Ω=90° , ω=0, 90° , 180°



Perturbazioni 1/3

$$d^2\mathbf{r}/dt^2 + \mu/r^3\,\mathbf{r} = \mathbf{f}$$

Energia:

$da/dt \quad = 2a^2 / \sqrt{\mu p}\,(f_r\, e \sin\theta + f_\theta\, (1+e \cos\theta))$

Momento Angolare Specifico:

$d\Omega/dt \sin i \quad = 1/\sqrt{\mu p}\; r\, f_n \sin\upsilon \qquad (\upsilon = \omega+\theta)$

$di/dt \quad = 1/\sqrt{\mu p}\; r\, f_n \cos\upsilon$

$de/dt \quad = \sqrt{p/\mu}\,(f_r \sin\theta + f_\theta\, (\cos\theta + \cos\varphi))$

$d\omega/dt + d\Omega/dt \cos i \;= 1/e\, \sqrt{p/\mu}\,(-f_r \cos\theta + f_\theta\, (1+r/p) \sin\theta)$

Perturbazioni 2/3

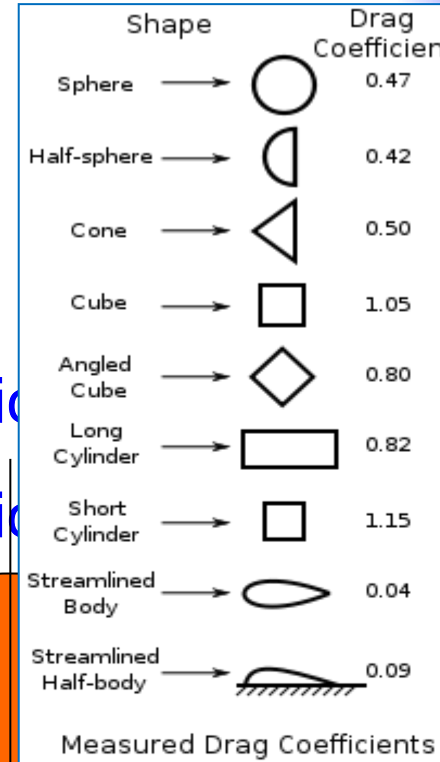
Triassialità della Terra

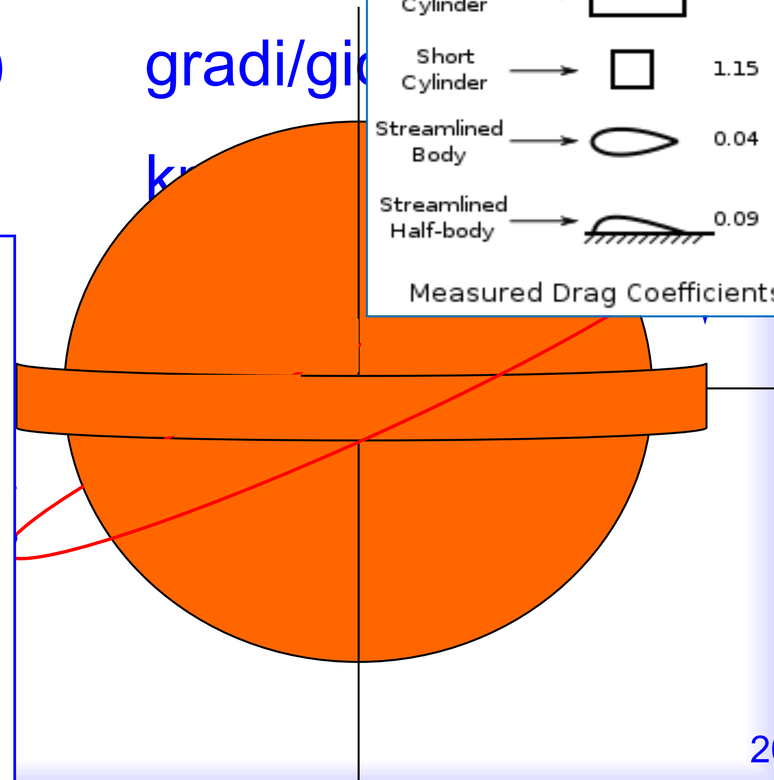
$$\Phi = \frac{\mu}{r}\left(1 - \Sigma_{n=2}^{\infty} J_n \left(\frac{R_{eq}}{r}\right)^n P_n(\sin(\lambda))\right) \qquad (f = -\nabla\Phi^*)$$

$d\Omega/dt \sim -9.97\,(R_{eq}/a)^{3.5}\,(1-e^2)^{-2}\cos(i)$ gradi/gi

$d\omega/dt \sim 4.98\,(R_{eq}/a)^{3.5}\,(1-e^2)^{-2}\,(5\cdot\cos^2(i)-1)$ gradi/gi

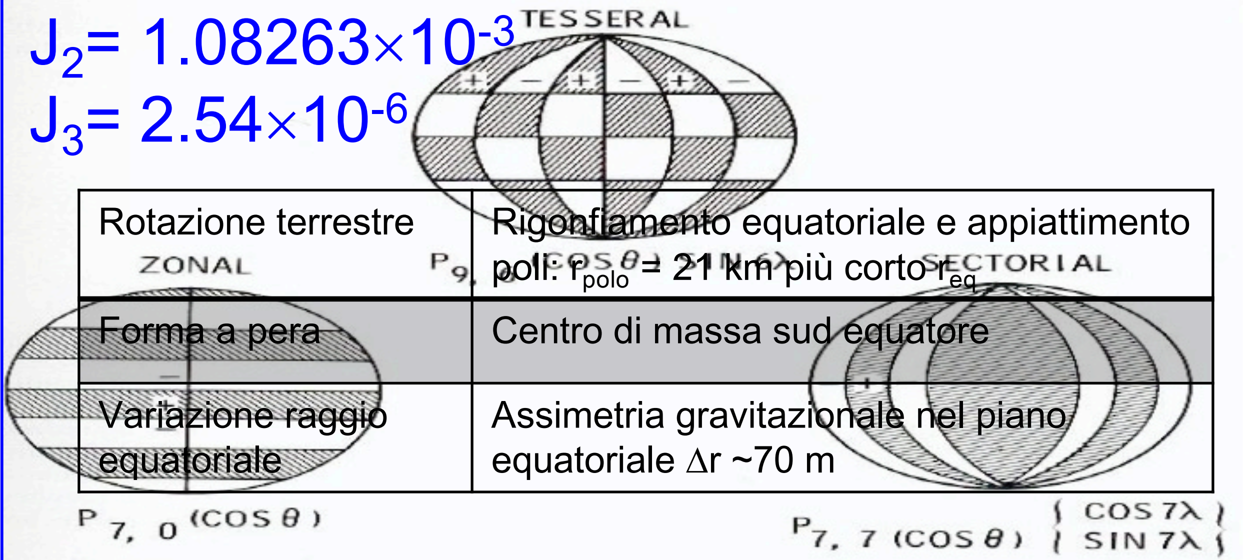
$r_p - r_{pe} \sim -6.8\sin(i)\sin(\omega)$ k





$J_2 = 1.08263\times10^{-3}$

$J_3 = 2.54\times10^{-6}$



Rotazione terrestre	Rigonfiamento equatoriale e appiattimento poli: r_{polo} = 21 km più corto r_{eq}
Forma a pera	Centro di massa sud equatore
Variazione raggio equatoriale	Assimetria gravitazionale nel piano equatoriale $\Delta r \sim 70$ m

Perturbazioni 2/3

	a (km)	e	i (deg)	$\Delta\Omega$ (deg/day)	$\Delta\omega$ (deg/day)
Shuttle	6700	0	28	-7.35	12.05
GPS	26600	0	60	0.033	0.008
GEO	42160	0	0	-0.013	0.025
Molniya	26600	0.75	63.4	-0.30	0.00

Attrito Atmosferico

$$F = ma = -1/2\ \rho\ v^2\ A\ C_D \quad \Rightarrow \quad \Delta a_{rev} = -\ 2\pi\ (C_D A\ /m)\ \rho\ a^2$$

$$\Delta v_{rev} = \pi\ (C_D A\ /m)\ \rho\ a\ v$$

$$\tau_{life} = -\ H\ /\ \Delta a_{rev}$$

Perturbazioni 3/3

Forze Gravitazionali del Sole e della Luna

$d\Omega/dt_L = -3.38 \cdot 10^{-3} \cos(i) / n$ gradi/giorno

$d\Omega/dt_S = -1.54 \cdot 10^{-3} \cos(i) / n$ gradi/giorno

$d\omega/dt_L = 1.69 \cdot 10^{-3} (5 \cdot \cos^2(i) - 1) / n$ gradi/giorno

$d\omega/dt_S = 0.77 \cdot 10^{-3} (5 \cdot \cos^2(i) - 1) / n$ gradi/giorno

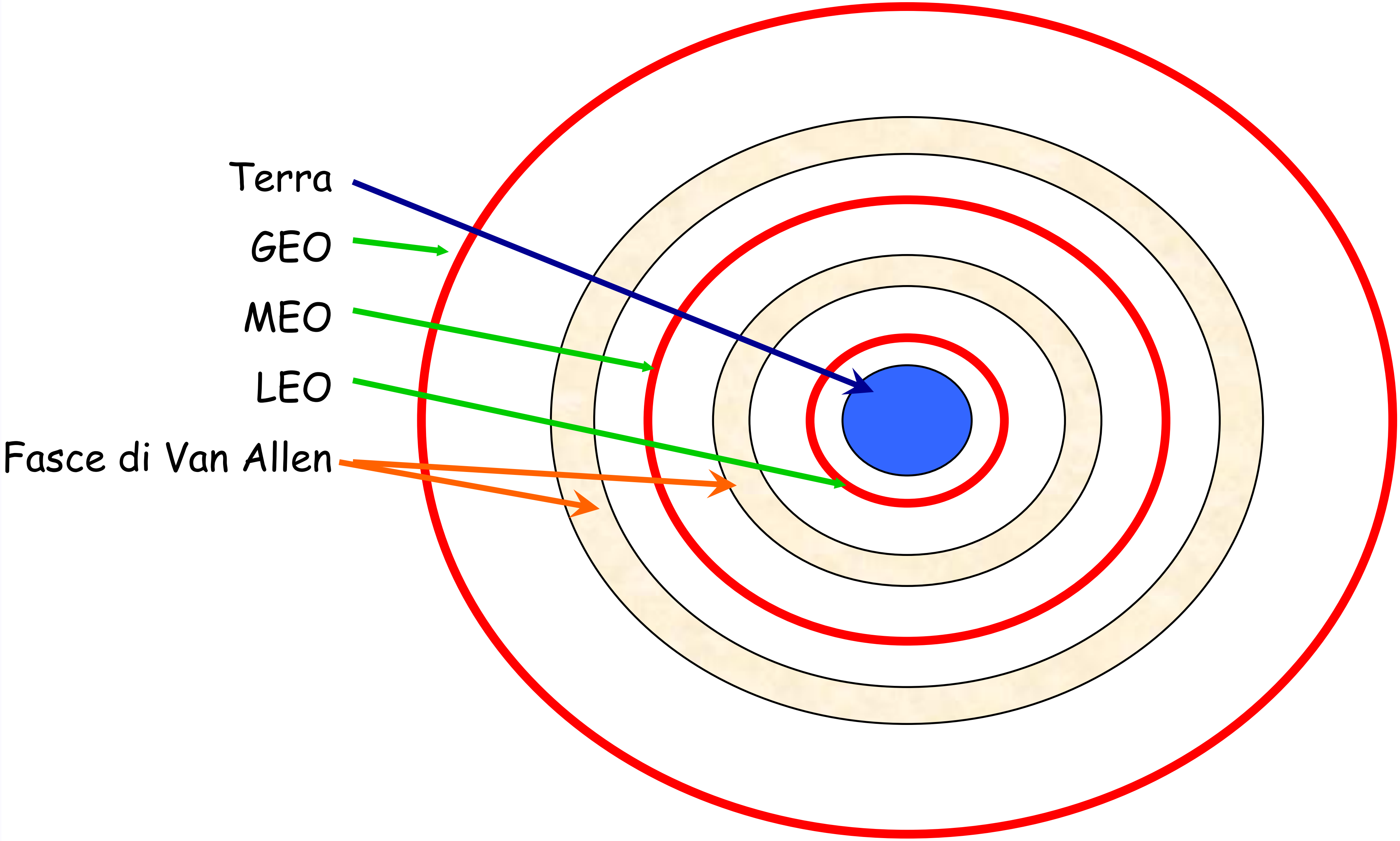
n = numero di rivoluzioni / giorno

Pressione di Radiazione

$f = - 4.5 \cdot 10^{-6} (1+r) A/m$ m/s^2

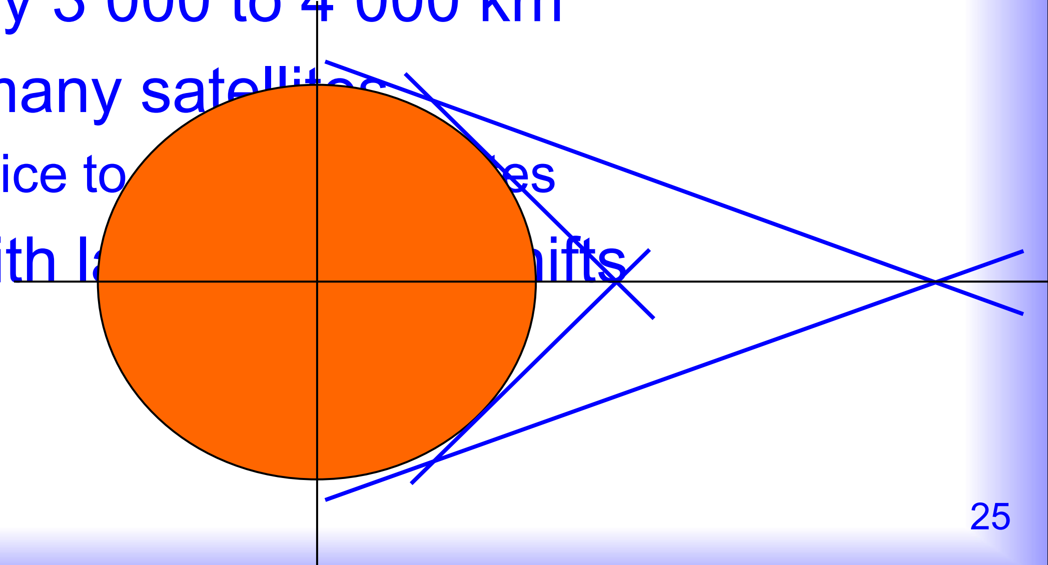
r = coefficiente riflessione

ORBITE



Low Earth Orbit (LEO)

- LEOs are either circular (or elliptical) orbits less than 2 000 km above the surface of the earth
  - Satellites generally some 700 to 1400 km up
  - Orbit periods between 90 to 120 minutes
    - Maximum time during which a satellite is above the horizon for an observer on the earth is 20 minutes.
  - Footprint radius is generally 3 000 to 4 000 km
  - A global system requires many satell
    - Needs to hand over the service to ... es
  - Need to be able to cope with l ... ifts



Medium Earth Orbits (MEO)

- MEOs are circular orbits at an altitude of around 10000 km, with an orbit period of around 6 hours

  - The time during which a MEO satellite is in view for an observer on the earth is in the order of a few hours
  - A global communications system using this type of orbit, requires a modest number of satellites (around 10 to 20) in 2 to 3 orbital planes to achieve global coverage
  - Compared to a LEO system, hand-over is less frequent, and propagation delay and free space loss are greater

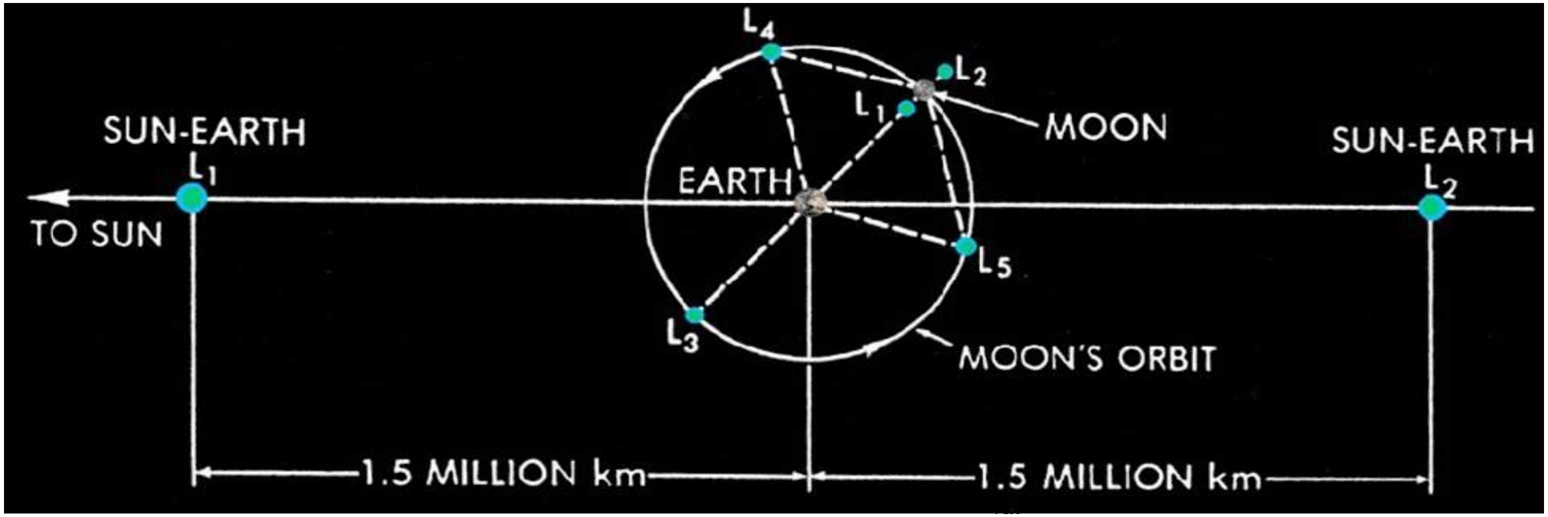
Geosynchronous Orbit (GEO)

- A circular orbit with an orbital period equal to that of the Earth
  - When in the equatorial plane (geostationary: inclination = 0^o), it appears fixed from an observer on Earth
    - This is achieved with an orbital height of 35 786 km (or an orbital radius of 6.6107 Equatorial Earth Radii)
  - A GEO orbit has small non-zero values for inclination and eccentricity
    - causing the satellite to trace out a small figure of eight in the sky
  - The round-trip delay is approximately 250 ms

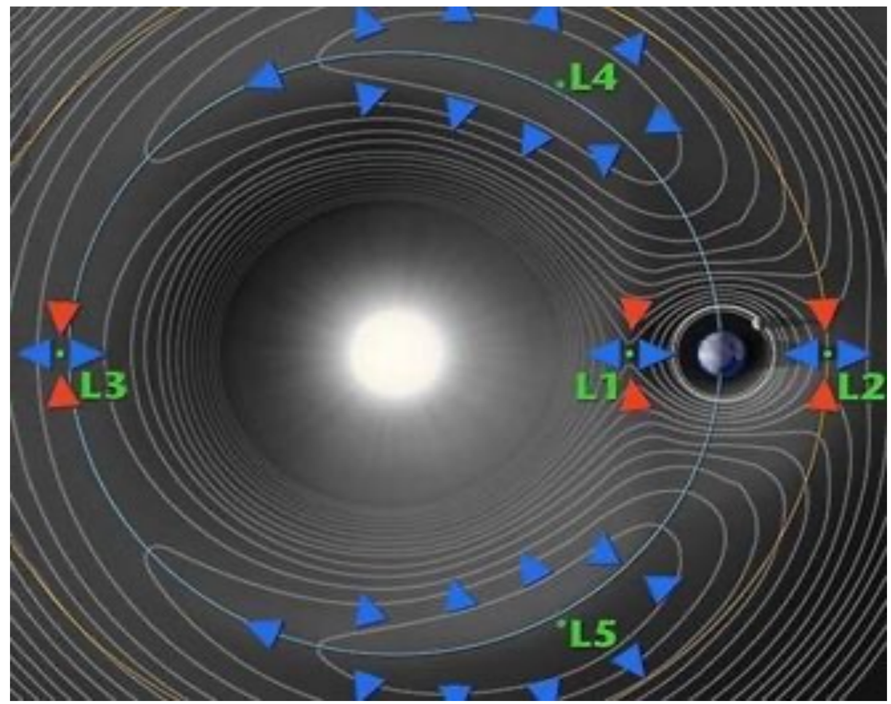
Orbite particolari

- ☺ Sun-Synchronous:
  - ➢ $d\Omega/dt_{J_2}$ = vel.ang. Terra
- ☺ Molniya:
  - ➢ τ = 12 hr, $d\omega/dt_{J_2} = 0$, e=0.75, i=63.5°
- ☺ Geo-Synchronous:
  - ➢ τ = vel.rot. Terra
- ☺ Geo-Stazionarie:
  - ➢ τ = vel.rot. Terra, i=0°
- ☺ Lagrangiane:
  - ➢ equilibrio Luna/Terra/sat

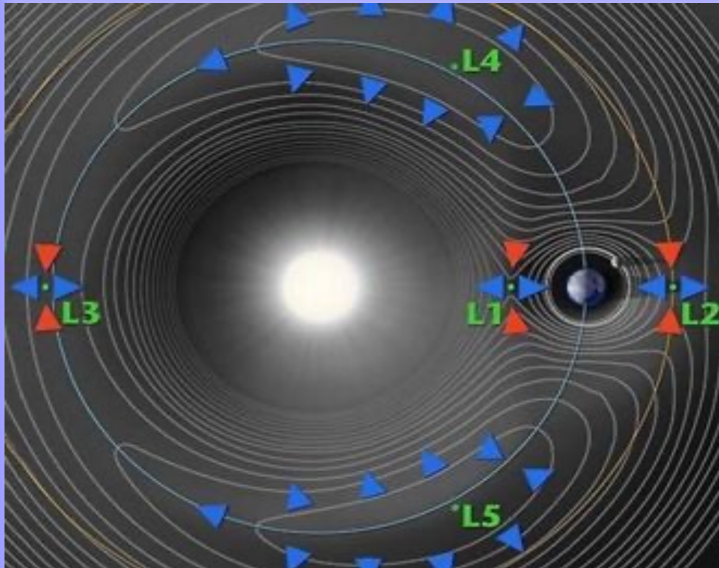
Lagrangiane



TL_1	=	322127 km
TL_2	=	442060 km
TL_3	=	386322 km
$TL_{4,5}$	=	384400 km
TS_2	=	1.5E+06 km
TS	=	1.5E+08 km

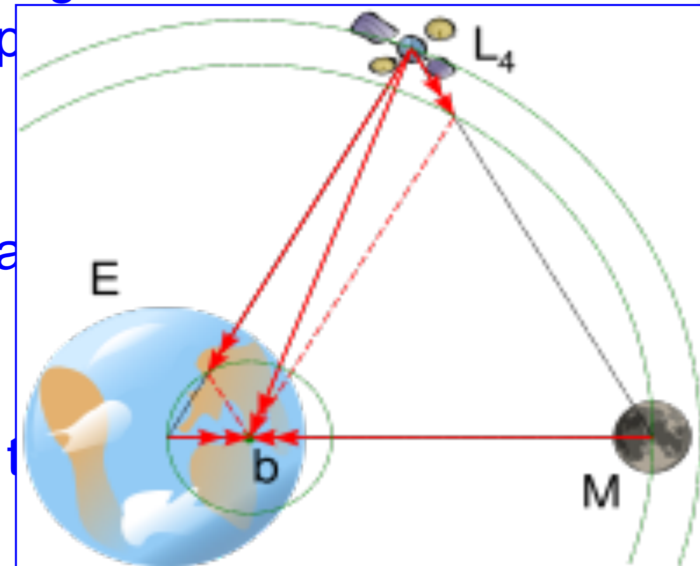


Lagrangian points

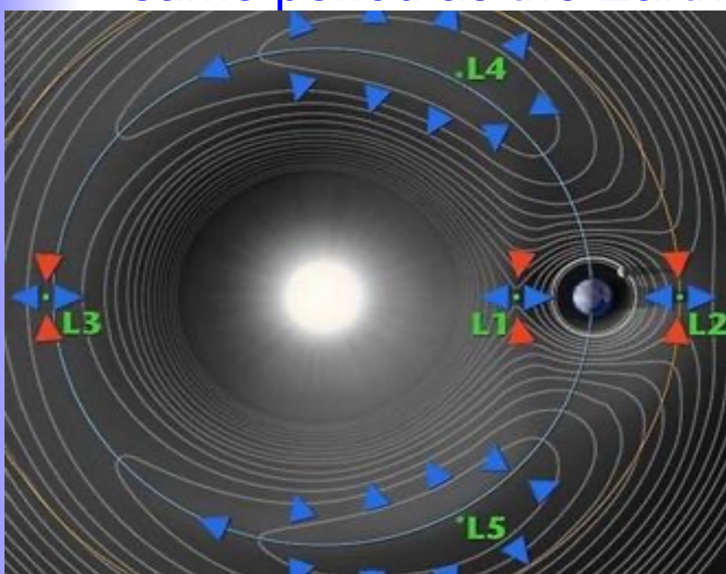


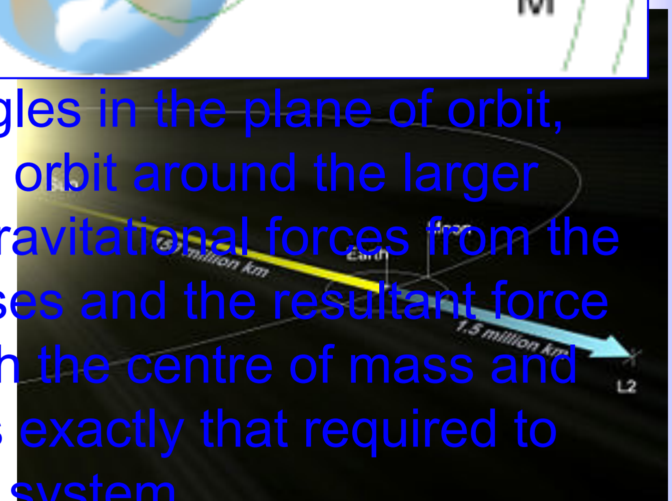
d by and between the 2 large masses M1 and M2. It is
stood of the Lagrangian p
s effectively cancel each

d by the 2 large masses,
f the 2 large masses bala



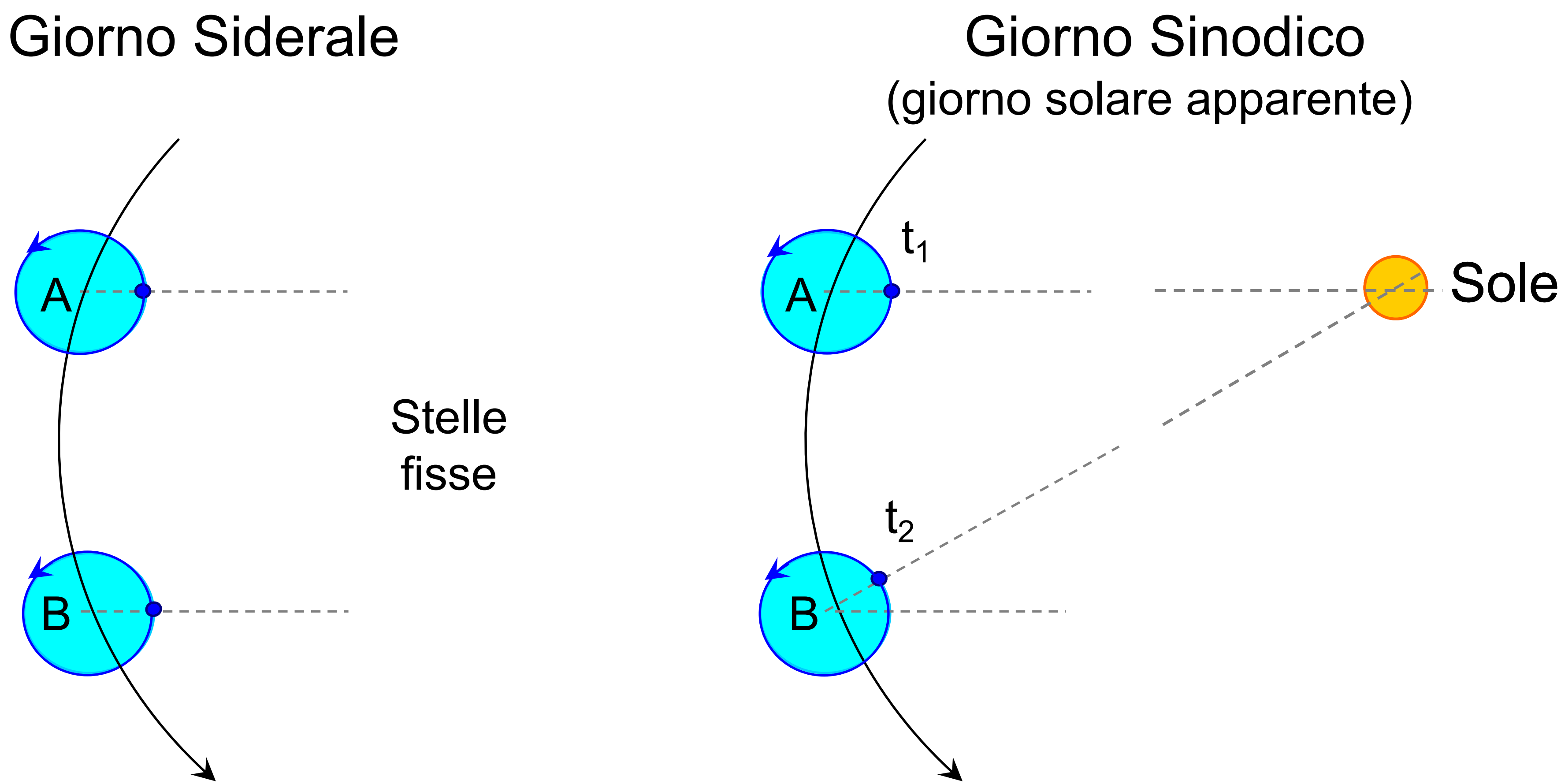
- L3: lies on the line defined by the 2 large masses,
  the combined pull of Earth and Sun again causes t
  same period as the Earth





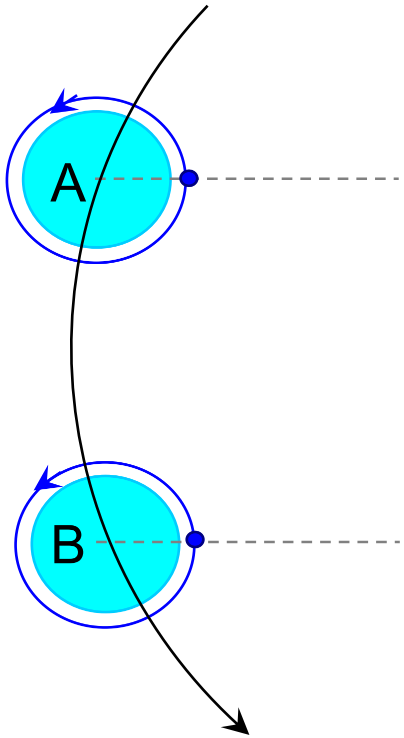
rs of the 2 equilateral triangles in the plane of orbit,
L4) the smaller mass wrt its orbit around the larger
e 2 masses are equal ⇒ gravitational forces from the
e same ratio as their masses and the resultant force
re of the system: being both the centre of mass and
ystem, the resultant force is exactly that required to
ilibrium with the rest of the system

Giorno siderale e sinodico



Periodo siderale e sinodico

Periodo Siderale τ_S

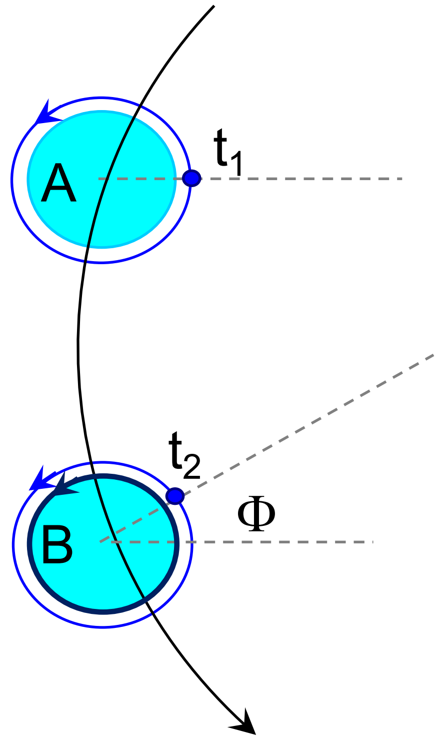


Stelle fisse

$$2\pi + \Phi = 2\pi/\tau_S\,(t_2 - t_1)$$

$$\Phi = 2\pi/\tau_M\,(t_2 - t_1)$$

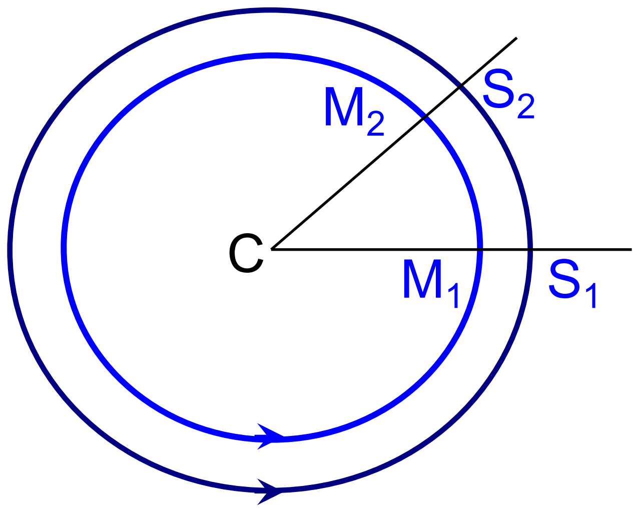
Periodo Sinodico τ_{SS}



LEO

$$\Rightarrow 1/(t_2 - t_1) = 1/\tau_S - 1/\tau_M = 1/\tau_{SS}$$

Terra e pianeti esterni e Luna



$1/\tau_M - 1/\tau_S = 1/\tau_{SS}$

Congiunzione S, T, P (C,M,S): periodo sinodico

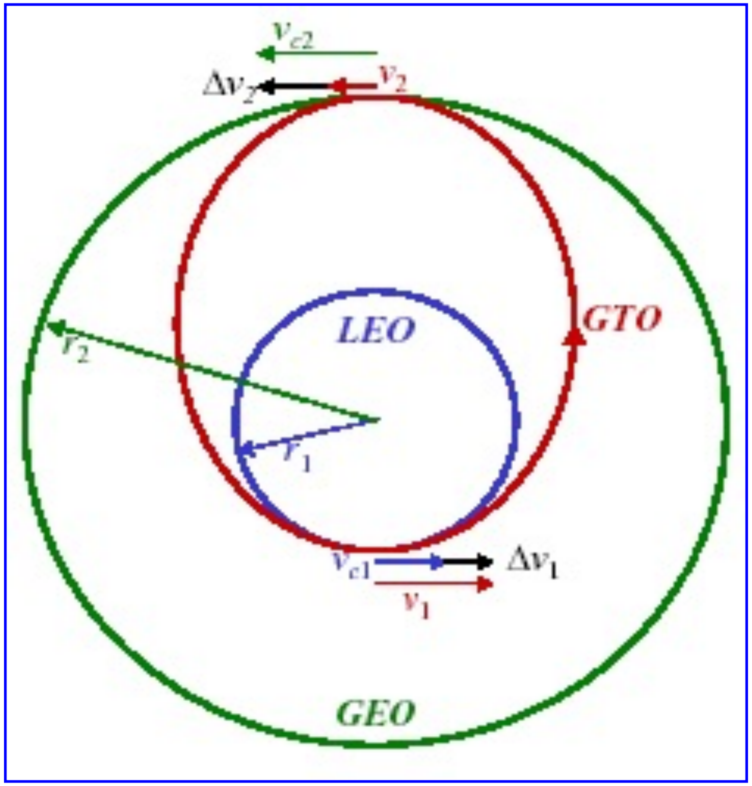
$\Rightarrow 1 - 1/\tau_S = 1/\tau_{SS}$

$\Rightarrow 1/\tau_P = 1 - 1/T_P$

Congiunzione T, L, S (C,M,S): “Luna Nuova”, periodo sinodico

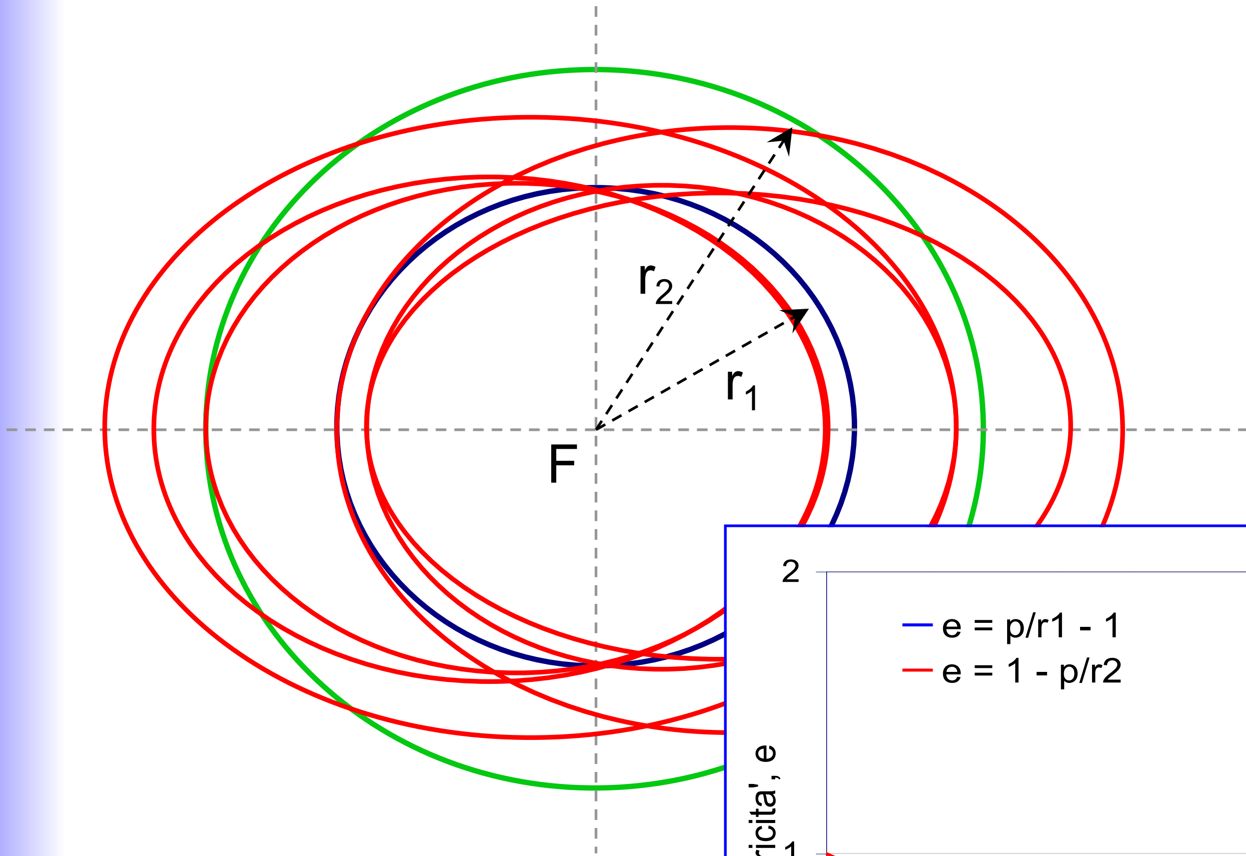
τ_S = 365 giorni, τ_L= 27.3 giorni, $\Rightarrow$ τ_{SS}= 29.5 giorni

Trasferimento LEO-GEO: GTO 1/3



Ellissi di Hohmann

Trasferimenti possibili

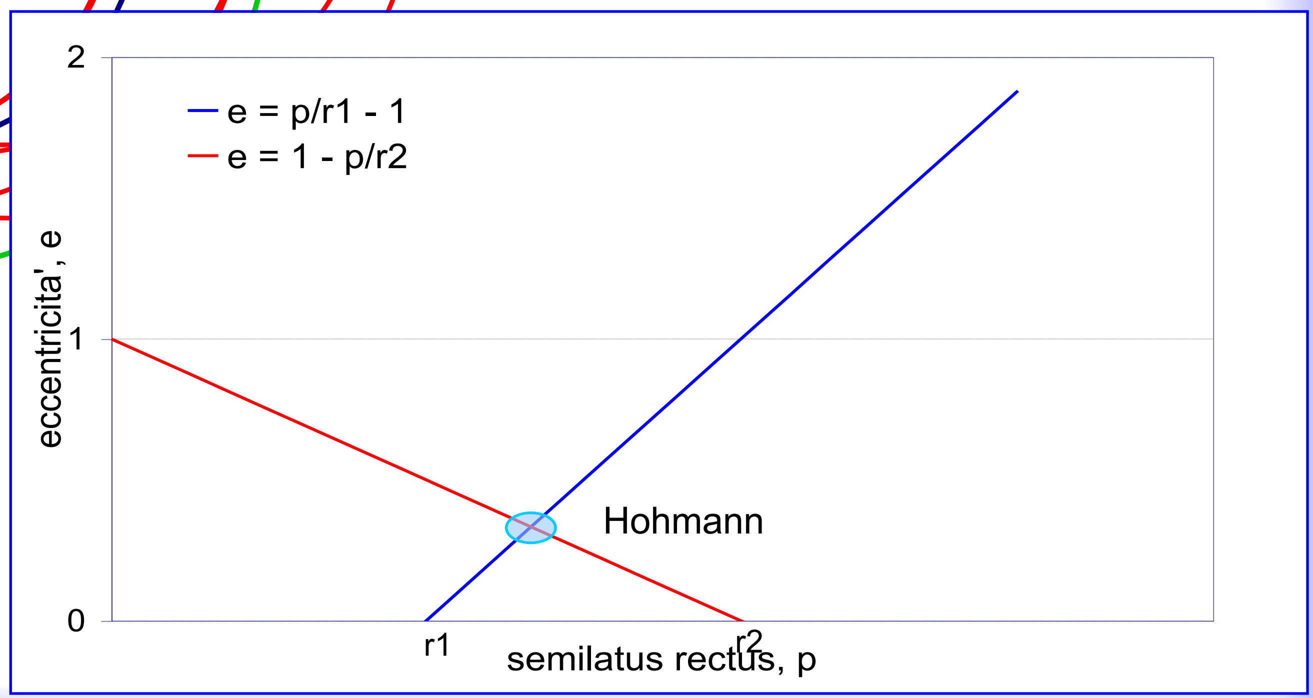


$\Rightarrow r_{peri} = p / (1+e) \leq r_1$

$\Rightarrow r_{apo} = p / (1-e) \geq r_2$

$\Rightarrow e \geq p/r_1 - 1$

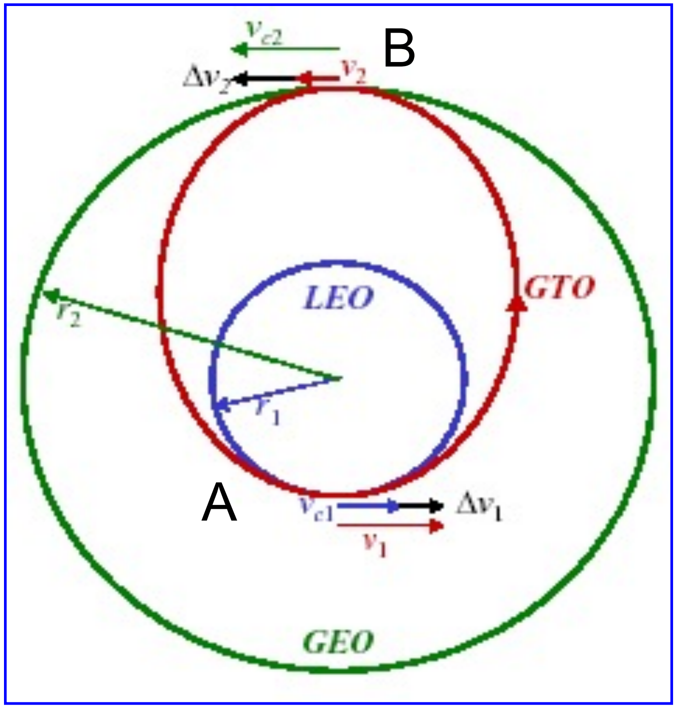
$\Rightarrow e \geq 1 - p/r_2$



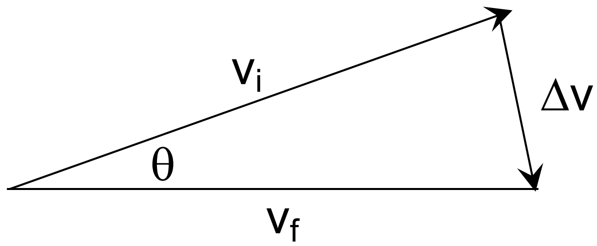
Trasferimento LEO-GEO: GTO 2/3

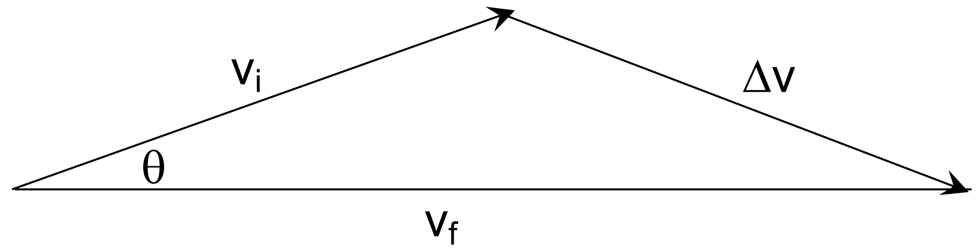
LEO		GEO	
h	400 km	h	35781 km
radius	6778 km	radius	42160 km
v	7.669 km/s	v	3.075 km/s

ELLISSE	
a	24469 km
r_A	6778 km
r_B	42160 km
v'_A	10.066 km/s
v'_B	1.618 km/s
τ	19046 s
	317.4 min
Δv_A	2.397 km/s
Δv_B	1.456 km/s
Δv_{TOT}	3.854 km/s



Trasferimento di piano





$\Delta v_{plane}/2 = v_i \sin \theta/2$

$\Delta v_{plane,if}^2 = v_i^2 + v_f^2 - 2\, v_i\, v_f \cos\theta$

Trasferimento LEO-GEO: GTO 2/3

LEO		GEO	
h	400 km	h	35781 km
radius	6778 km	radius	42160 km
v	7.669 km/s	v	3.075 km/s

ELLISSE	
a	24469 km
r_A	6778 km
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τ	19046 s
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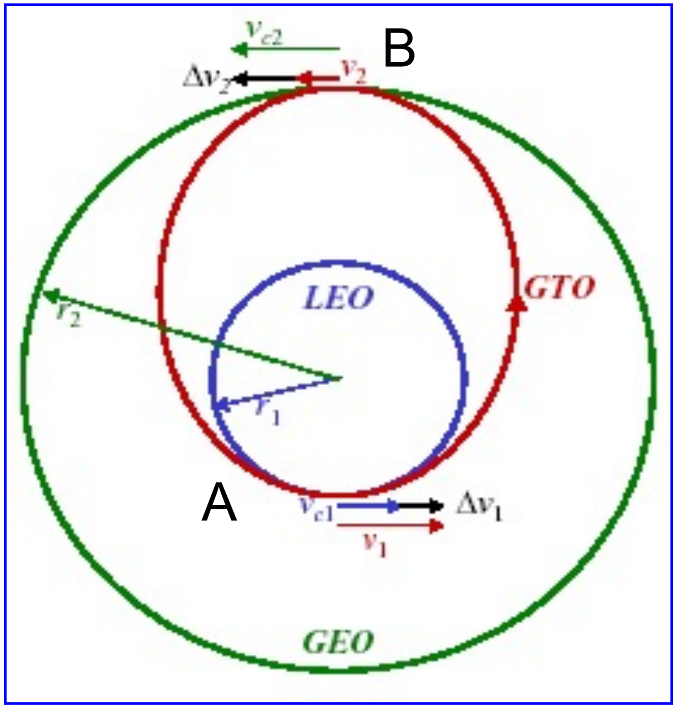
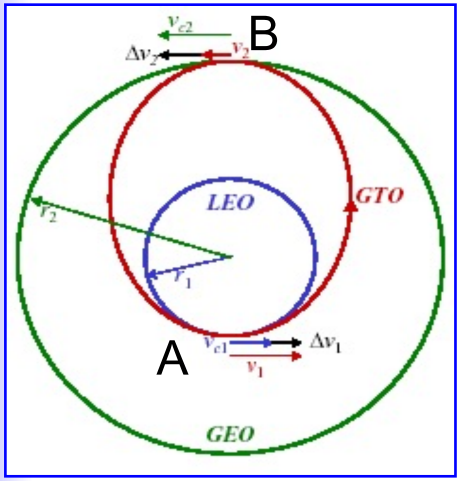


Trasferimento LEO-GEO: GTO 3/3

LEO			GEO		
h	400	km	h	35781	km
radius	6778	km	radius	42160	km
v	7.669	km/s	v	3.075	km/s
i	28	deg	i	0	deg
	0.4887	rad		0	rad

ELLISSE		
a	24469	km
r_A	6778	km
r_B	42160	km
v'_A	10.066	km/s
v'_B	1.618	km/s
τ	19046	s
	317.4	min
Δv_A	2.397	km/s
Δv_B	1.456	km/s
Δv_{TOT}	3.854	km/s
Δv_{plane}	1.4877	km/s
Δv_{TOT}	5.342	km/s
$\Delta v_{plane,B}$	1.8128	km/s
Δv_{TOT}	4.210	km/s

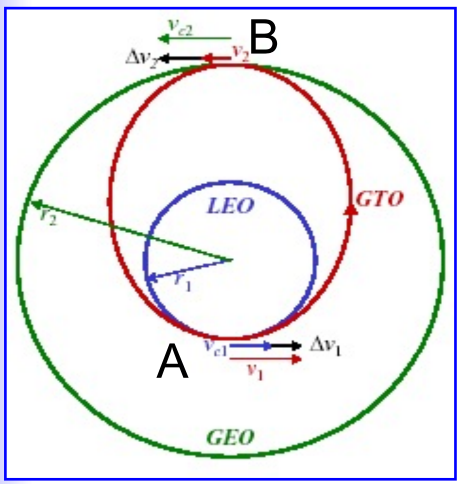


Tutto trasferimento effettuato in B

Trasferimento LEO-GEO: GTO 3/3

LEO		GEO	
h	400 km	h	35781 km
radius	6778 km	radius	42160 km
v	7.669 km/s	v	3.075 km/s
i	28 deg	i	0 deg
	0.4887 rad		0 rad

ELLISSE	
a	24469 km
r_A	6778 km
r_B	42160 km
v'_A	10.066 km/s
v'_B	1.618 km/s
τ	19046 s
	317.4 min
Δv_A	2.397 km/s
Δv_B	1.456 km/s
Δv_{TOT}	3.854 km/s
Δv_{plane}	3.710 km/s
Δv_{TOT}	7.564 km/s
$\Delta v_{plane,A}$	4.880 km/s
Δv_{TOT}	6.337 km/s



Tutto trasferimento effettuato in A

Aggiustamenti di orbita

LEO

Attrito Atmosferico

$$\Delta v_{rev} = \pi \, (C_D A / m) \, \rho \, a \, v$$

GEO, i=0°

Triassialità della Terra

$$\Delta v_{anno} = 1.715 \sin(2|long - long_s|) \text{ m/s} \qquad (long_s = 75^o/225^o \text{ E})$$

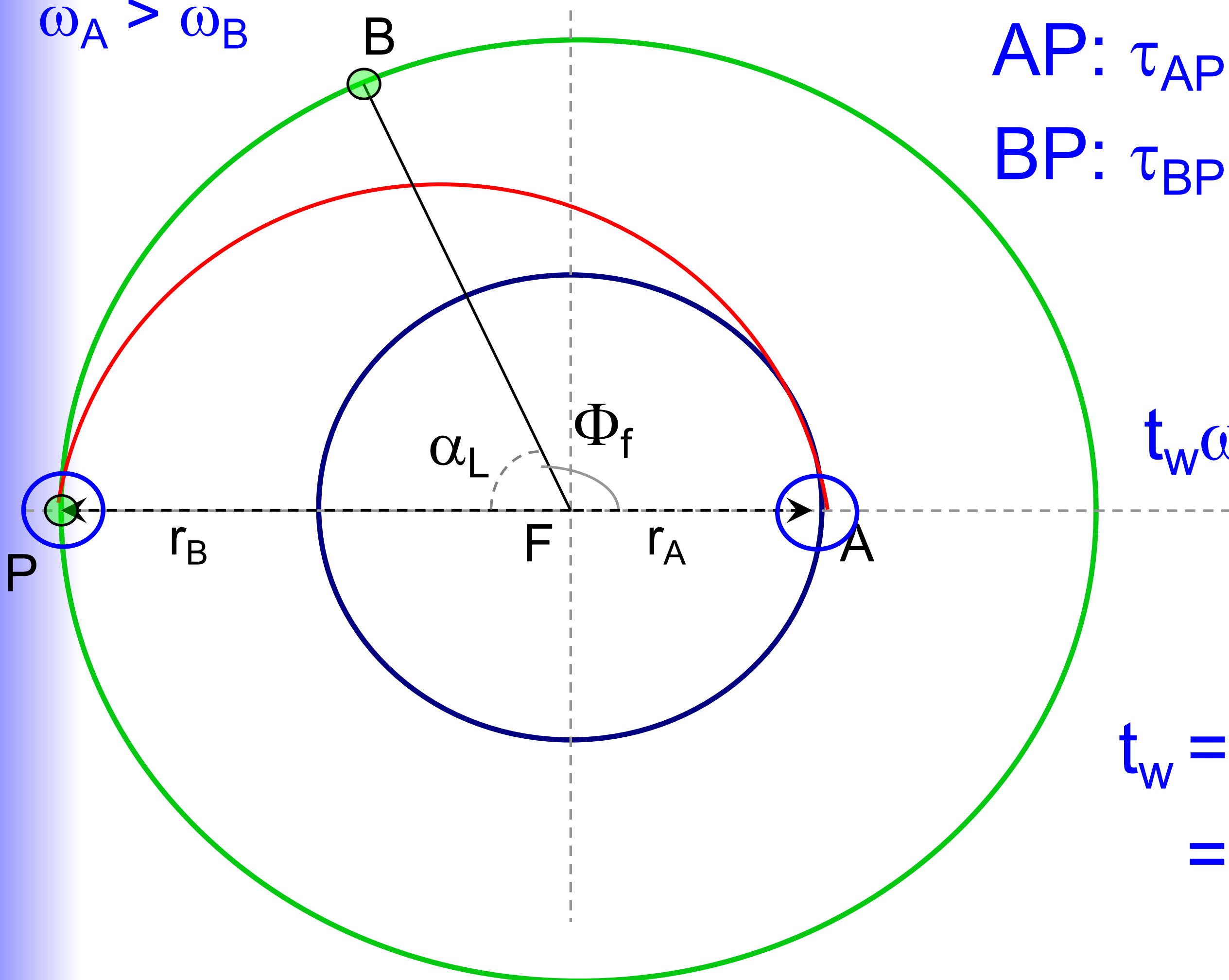
Forze Gravitazionali del Sole e della Luna

$$\Delta v_{Luna,anno} = 102.67 \cos \alpha \sin \alpha \text{ m/s /anno} \qquad \sim 36.93 \text{ m/s /anno}$$

$$\Delta v_{Sole,anno} = 40.17 \cos \gamma \sin \gamma \text{ m/s /anno} \qquad \sim 14.45 \text{ m/s /anno}$$

Appuntamenti in orbita

$\omega_A > \omega_B$



$$AP: \tau_{AP} = \pi \sqrt{a^3/\mu} = \tau_{BP} = \tau_H$$

$$BP: \tau_{BP} = \alpha_L/\omega_B = \alpha_L \sqrt{r_B{}^3/\mu}$$

$$\alpha_L = \pi - \Phi_f$$

$$t_w\omega_A - t_w\omega_B = \Phi_i - \Phi_f \ (+2k\pi)$$

$$(\Phi_i = 0)$$

$$t_w = (\alpha_L - \pi + 2k\pi) / (\omega_A - \omega_B)$$

$$= (\tau_H\omega_B - \pi + 2k\pi) / (\omega_A - \omega_B)$$

$$t_{tot} = t_w + \tau_H + t_o$$

Parametri Ellisse

$v_t = v \cos \gamma$ (*)

$v_r = v \sin \gamma$

γ=0 per orbite circolari ($v=v_t$, v_r=0)