$$T(n) = \begin{cases} T(n/2) + b & n > 1 \\ c & n \leq 1 \end{cases}$$

ANALISI PER LIVELLI

$$T(n) = b + T(n/2) =$$
$$= b + b + T(n/4) =$$
$$= b + b + b + T(n/8) =$$
$$= \underbrace{b + \dots + b}_{\log(n)} + T(1)$$

ABBIAMO ASSUNTO PER SEMPLICITA' $n = 2^k$
$\Rightarrow k = \log(n)$

$$T(n) = b \cdot \log(n) + c = \Theta(\log(n))$$

$$T(n) = \begin{cases} 4 \cdot T(n/2) + n & n > 1 \\ 1 & n \leq 1 \end{cases}$$

$$T(n) = n + 4T(n/2) =$$
$$= n + 4 \cdot n/2 + 16 \cdot T(n/4) =$$
$$= n + 2n + 16/4\, n + 64T(n/8) =$$
$$\vdots$$
$$= n + 2n + 4n + \dots + 2^{\log n - 1} n + 4^{\log n} T(1)$$
$$= n \cdot \sum_{j=0}^{\log n - 1} 2^j + 4^{\log n}$$

$$\boxed{\sum_{j=0}^{k} x^j = \frac{x^{k+1} - 1}{x - 1}}$$

$$= n \cdot \frac{2^{\log n} - 1}{2 - 1} + 4^{\log n} =$$
$$= n(n-1) + 4^{\log n} =$$
$$= n(n-1) + n^{\log 4} =$$
$$= n(n-1) + n^2 = \Theta(n^2)$$

METODO SOSTITUZIONE

$$T(n) = \begin{cases} T(\lfloor n/2 \rfloor) + n & n > 1 \\ 1 & n \leq 1 \end{cases} \quad \Rightarrow \boxed{T(n) \leq cn} \quad \exists c$$

SECONDO ME È ... $O(n)$ $\quad \boxed{T(n) \in \Theta(n)}$?

DIMOSTRIAMOLO INDUZIONE

1) CASO BASE $\quad T(1) = 1 \leq 1 \cdot c \quad \forall c \geq 1$

2) CASO INDUTTIVO

$$\begin{aligned} T(n) &= T(\lfloor n/2 \rfloor) + n \leq \\ &\leq c \lfloor n/2 \rfloor + n \leq \\ &\leq c \cdot \frac{n}{2} + n = \quad ? \\ &= n\left(1 + \frac{c}{2}\right) \leq cn \iff \frac{c}{2} + 1 \leq 1 \\ &\qquad\qquad\qquad\qquad \iff c \geq 2 \end{aligned}$$

$\boxed{c = 2} \Rightarrow T(n) \in \Theta(n)$

$T(n) \in \Omega(n) \qquad \exists d>0 \mid T(n) \geq dn$

- CASO BASE $T(1) = 1 \geq 1 \cdot d \iff d \leq 1$
- PASSO INDUTTIVO

$$T(n) = T(\lfloor n/2 \rfloor) + n \geq$$

$$\geq d\lfloor n/2 \rfloor + n \geq \quad ?$$

$$\geq d \cdot \frac{n}{2} - 1 + n = \left(\frac{d}{2} - \frac{1}{n} + 1\right) \cdot n \geq dn$$

$$\iff \frac{d}{2} - \frac{1}{n} + 1 \geq d$$

$$\iff d \leq 2 - 2/n$$

$$\boxed{d=1} \implies T(n) \in \Omega(n) \implies T(n) \in \Theta(n)$$

$T(n) = 3T(n/4) + n \cdot \log n$

$a = 3$

$b = 4$ $\quad \Rightarrow (3) \Rightarrow T(n) = \Theta(n \log n)$

$\log_b a \approx 0.79$

$f(n) = n \cdot \log(n) = \Omega(n^{\log_4 3 + \varepsilon})$ con $\varepsilon < 1 - \log_4 3 \approx 0{,}208$

dobbiamo mostrare che:

$\exists c \leq 1 \ \exists m \geq 0 \quad a \cdot f(n/b) \leq c \cdot f(n) \quad \forall n \geq m$

$$
\begin{aligned}
a \cdot f(n/b) &= 3 \cdot (n/4 \log(n/4)) \\
&= 3/4 \cdot n (\log n - \log 4) \\
&\leq 3/4\, n \cdot \log n \overset{?}{\leq} c \cdot n \cdot \log n
\end{aligned}
$$

$\Leftrightarrow 3/4 \leq c$

$\boxed{c = 3/4}$ ok

$T(n) = 2T(n/2) + n \log n$

$a = 2$

$b = 2$

$\log_b a = 1$

(1) $f(n) = n \cdot \log n \notin O(n^{1-\varepsilon})$

(2) $f(n) = n \cdot \log n \notin \Theta(n)$

(3) $f(n) = n \cdot \log n \notin \Omega(n^{1+\varepsilon})$

BISOGNA UTILIZZARE ALTRI METODI