$|x-x_0| < \varepsilon$ se $|y-y_0| < \delta$ — se $\underline{y_0-\delta < y < y_0+\delta}$

$x_0-\varepsilon < x < x_0+\varepsilon$ stiamo supponendo che $x_0-\varepsilon, x_0+\varepsilon \in I$

f è strett. crescente

$$f(x_0-\varepsilon) < f(x) < f(x_0+\varepsilon) \qquad y_0 = f(x_0)$$

$$f(x_0-\varepsilon) - y_0 < \underset{y}{f(x)} - y_0 < f(x_0+\varepsilon) - y_0 \qquad f(x)=y \quad x = f^{-1}(y)$$

$$-\underbrace{[y_0 - f(x_0-\varepsilon)]}_{>0} < y - y_0 < \underbrace{[f(x_0+\varepsilon) - y_0]}_{>0}$$

$$y_0 - \underbrace{[y_0 - f(x_0-\varepsilon)]} < y < y_0 + \underbrace{[f(x_0+\varepsilon) - y_0]}$$

x_0

Prendiamo $\delta = \min\{y_0 - f(x_0-\varepsilon), f(x_0+\varepsilon) - y_0\}$. Allora se $|y-y_0| < \delta$

Se $I = [a,b]$ $\quad x_0 = a \quad$ so però che $\quad \underbrace{x_0-\varepsilon <} x < x_0+\varepsilon$

$$x_0-\varepsilon = a-\varepsilon < \underline{\underline{a \le x}}$$

$x_0 = b \qquad x \le \underbrace{b < b+\varepsilon}$

Conseguenze: sono funzioni continue le radici $\sqrt[n]{x}$ i logaritmi

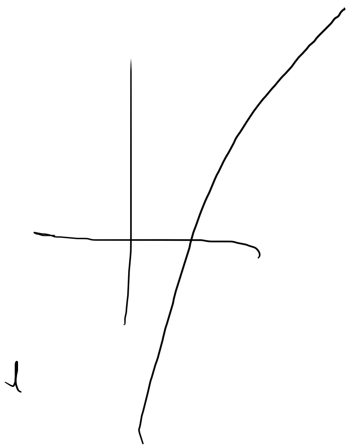
$\log_a x$ $\quad$ $\arcsin x$ $\quad$ $\arccos x$ $\quad$ $\operatorname{arctg} x$

$f^{-1}(y) = a^y$ è l'inversa di $f(x) = \log_a x$

definita su $]0,+\infty[$ e

crescente

$\Rightarrow a^y$ è continua.



Calcolo dei limiti

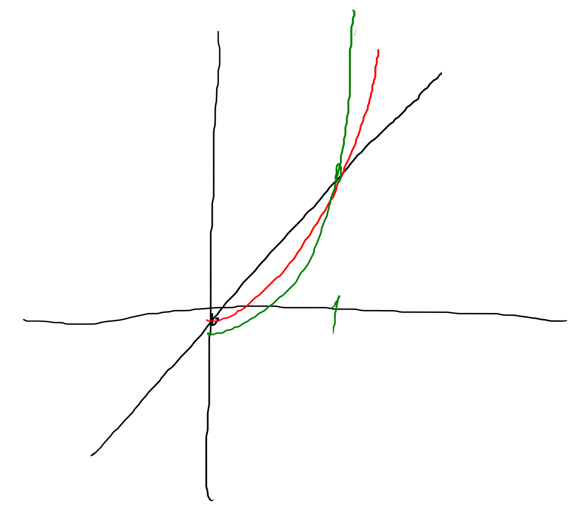
Es: $\lim_{x \to 1} \arctg(x) = \arctg(1) = \frac{\pi}{4}$ $\qquad \arctg(1) = \frac{\pi}{4}$

Es: funzioni razionali $f(x) = \frac{p(x)}{q(x)}$ p, q polinomi $\qquad n \in \mathbb{N}^+$

se $x_0 \in \mathbb{R}$ $q(x_0) \neq 0$ $\lim_{x \to x_0} f(x) = f(x_0)$ $\qquad \lim_{x \to +\infty} x^n = +\infty$

$a_n \neq 0 \; b_m \neq 0$

$x_0 = +\infty$ $\lim_{x \to +\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0} =$



$= \lim_{x \to +\infty} \underbrace{\left(\frac{x^n}{x^m}\right)}_{x^{n-m}} \frac{\left(a_n + a_{n-1} \frac{1}{x} + \dots + a_1 \frac{1}{x^{n-1}} + \frac{a_0}{x^n}\right)}{\left(b_m + b_{m-1} \frac{1}{x} + \dots + b_1 \frac{1}{x^{m-1}} + \frac{b_0}{x^m}\right)} =$

$\frac{1}{x} \to 0$

$\to \frac{a_n}{b_m} \in \mathbb{R}$

$$\lim_{x\to+\infty} x^{n-m} = \begin{cases} +\infty & \text{se } n>m \\ 1 & \text{se } n=m \\ 0 & \text{se } n<m \end{cases}$$

$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = \begin{cases} 0 & \text{se } n<m \\ \dfrac{a_n}{b_m} & \text{se } n=m \\ +\infty \text{ se } \dfrac{a_n}{b_m}>0, \quad -\infty \text{ se } \dfrac{a_n}{b_m}<0 & \text{se } n>m \end{cases}$$

$x^{n-m} \to +\infty$

$\left(\dfrac{a_n}{b_m} \right) \to$

$$\lim_{x\to-\infty} x^{n-m} = \begin{cases} +\infty \text{ se } n-m \text{ è pari}, \quad -\infty \text{ se } n-m \text{ è dispari} & n>m \\ 1 & \text{se } n=m \\ 0 & \text{se } n<m \end{cases}$$

$\dfrac{a_n}{b_m}$

$$\lim_{x\to-\infty} \frac{a_n x^n + \dots + a_0}{b_m x^m + \dots + b_0} = \lim_{x\to-\infty} x^{n-m} \left(\frac{a_n + \frac{a_{n-1}}{x} + \dots + \frac{a_0}{x^n}}{b_m + \frac{b_{m-1}}{x} + \dots + \frac{b_0}{x^m}} \right)$$

$$\lim_{x \to -\infty} \frac{a_n x^n + \dots}{b_m x^m + \dots} = \begin{cases} 0 \\ \dfrac{a_n}{b_m} \\ \text{se } n > m \end{cases}$$

se $n > m$ $n-m$ pari $\frac{a_n}{b_m} > 0$ $+\infty$

$\frac{a_n}{b_m} < 0$ $-\infty$

$n-m$ dispari $\frac{a_n}{b_m} > 0$ $-\infty$

< 0 $+\infty$

$$\lim_{x \to -\infty} \left(3x^3 + 2x^2 + 1\right) = \lim_{x \to -\infty} \underbrace{x^3}_{\to -\infty} \Big(\underbrace{3}_{>0} + \underbrace{\frac{2}{x} + \frac{1}{x^2}}_{\to 0}\Big) = -\infty$$

$x_0 \in \mathbb{R}$ $\quad q(x_0) = 0$ $\quad p(x) =$ dividiamo per $(x-x_0)^k$

$$\lim_{x\to x_0} \frac{p(x)}{q(x)} = \lim_{x\to x_0} \frac{(x-x_0)^k \, p_1(x)}{(x-x_0)^j \cdot q_1(x)}$$

$k \geq 0 \quad p_1(x_0) \neq 0$

$j \geq 1 \quad q_1(x_0) \neq 0$

$$= \lim_{x\to x_0} (x-x_0)^{k-j} \cdot \frac{p_1(x)}{q_1(x)} = \begin{cases} \frac{p_1(x_0)}{q_1(x_0)} & \text{se } k=j \\ 0 & \text{se } k>j \\ \nexists & \text{se } j-k \text{ dispari} \\ +\infty & \text{se } j-k \text{ pari } \frac{p_1(x_0)}{q_1(x_0)} > 0 \\ -\infty & \text{"} \quad \text{"} < 0 \end{cases}$$

$\frac{p_1(x_0)}{q_1(x_0)} \in \mathbb{R}$

se $k = j$

se $k > j$

se $k < j$
$j-k$ dispari

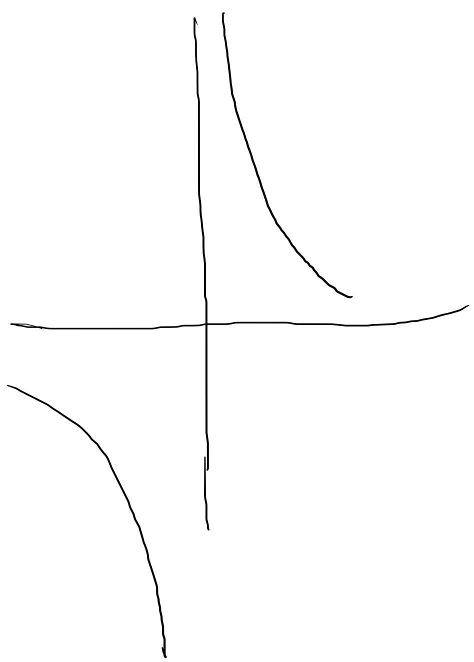
$$\lim_{x\to x_0^-} (x-x_0)^{k-j} = -\infty \qquad \lim_{x\to x_0^+} (x-x_0)^{k-j} = +\infty$$

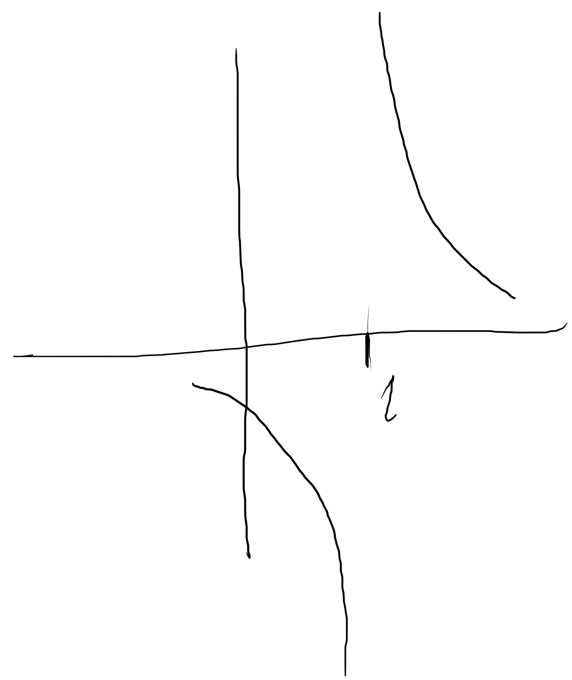
$j-k$ pari $+\infty$ $\qquad +\infty$

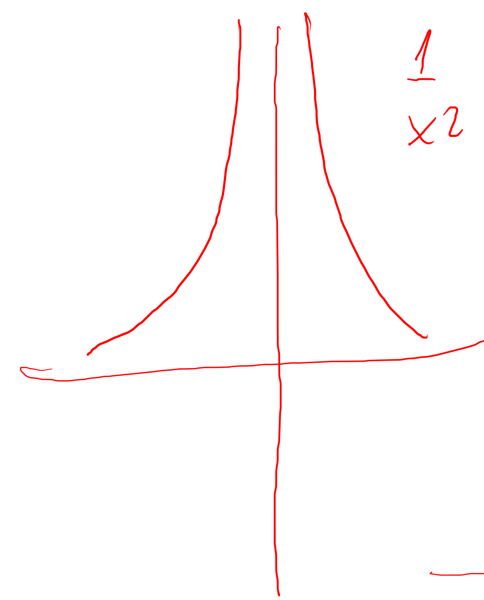
Es. $\lim_{x \to 1} \frac{x^3-1}{(x-1)^2(x^2-1)} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)^3(x+1)} =$

$= \lim_{x \to 1} \frac{1}{(x-1)^2} \cdot \left(\underbrace{\;\text{---}\;}_{3/2} \right) = \nexists$ $= +\infty$

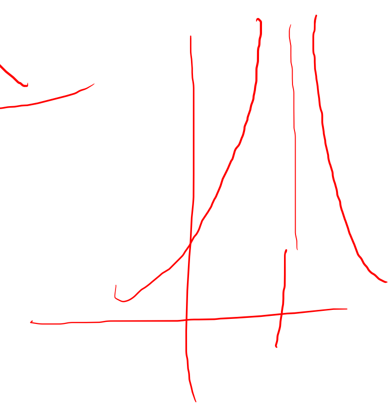
$\lim_{x \to 1^-} f(x) = -\infty \qquad \lim_{x \to 1^+} f(x) = +\infty$







$\frac{1}{x^2}$



$$\lim_{x\to+\infty} \frac{(3x-1)^3 + 2x^2 - \operatorname{sen} x}{x^4+2x-1} = \lim_{x\to+\infty} \frac{x^3}{x^4} \cdot \frac{\left(3-\frac{1}{x}\right)^3 + \frac{2}{x} - \frac{\operatorname{sen} x}{x^3}}{1+\frac{2}{x^3}-\frac{1}{x^4}}$$

$$\frac{1}{x} \qquad 3^3$$

$$= 0$$

$$(3x-1)^3 = \left(x\left(3-\frac{1}{x}\right)\right)^3 = x^3\left(3-\frac{1}{x}\right)^3$$

$$\lim_{x\to\pi/2} \underbrace{\log(\overset{f(x)}{\cos x})}_{g(f(x))} = -\infty \qquad \lim_{x\to\pi/2} \overset{f(x)}{\cos x} = 0 = y_0$$

$$\lim_{y\to y_0=0} \log(y) = -\infty$$

$$\lim_{x\to x_0} f(x) = y_0$$

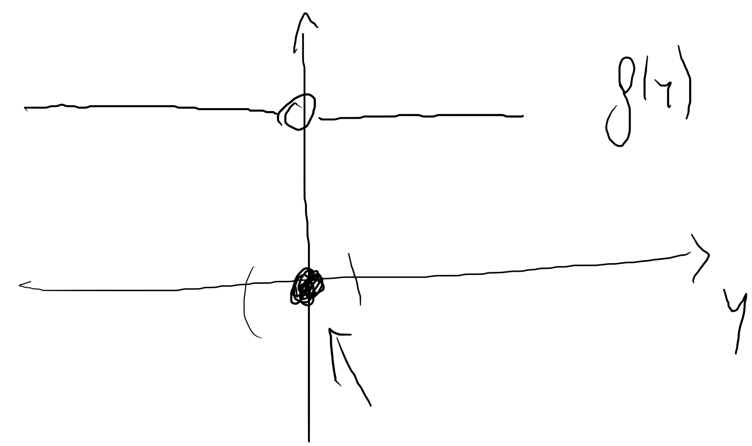
$$\lim_{y\to y_0} g(y) = \alpha \quad \Longrightarrow \quad \lim_{x\to x_0} g(f(x)) = \alpha$$

Es:

$f(x) = 0$ costante

$$g(y) = \begin{cases} 1 & \text{se } y \neq 0 \\ 0 & \text{se } y = 0 \end{cases}$$

$$\lim_{\substack{x \to 0 \\ \| \\ x_0}} f(x) = \underset{\substack{\| \\ y_0}}{0}$$



$$\lim_{y \to 0} g(y) = 1 \neq 0 = g(0)$$

$$\lim_{x \to 0} \underbrace{g(\underset{\substack{\| \\ 0}}{f(x)})}_{\substack{\| \\ 0}} = 0 \neq \lim_{y \to 0} g(y)$$

Teorema (limite della funzione composta)

$f: E \subseteq \mathbb{R} \to \mathbb{R}$ $\quad g: F \subseteq \mathbb{R} \to \mathbb{R}$ $\quad F \subseteq f(E)$ $\quad x_0$ punto di accumulazione per E

($x_0 = -\infty, +\infty$ ...)

sia $\lim_{x \to x_0} f(x) = y_0$ $\quad y_0$ punto di accumulazione per F

(oppure $-\infty, +\infty$ ...)

sia $\lim_{y \to y_0} g(y) = \alpha$

Valga inoltre l'ipotesi (*) Esiste un intorno U_0 di x_0 tale che $\forall x \in U_0 \cap E$ $x \neq x_0$

sia $f(x) \neq y_0$

Allora $\lim_{x \to x_0} g(f(x)) = \alpha$.

Dim: Sia W intorno di α; poichè $\boxed{\lim_{y \to y_0} g(y) = \alpha}$

esiste un intorno V di y_0 tale che $\forall y \in F \cap V$ $y \neq y_0$ si ha $g(y) \in W$.

poichè $\lim_{x \to x_0} f(x) = y_0$ esiste un intorno U di x_0 tale che $\forall x \in U \cap E$ $x \neq x_0$ si ha

$f(x) \in V$.

Sia $x \in U \cap E$ $x \neq x_0$ allora $f(x) = y \in V$ quindi $g(f(x)) \in W$.

Sia $U_1 = U_0 \cap U$; se $x \in U_1 \cap E$ $x \neq x_0$ allora $f(x) \in V$ e $\boxed{f(x) \neq y_0}$, quindi

$g(f(x)) \in W$.

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x}$$

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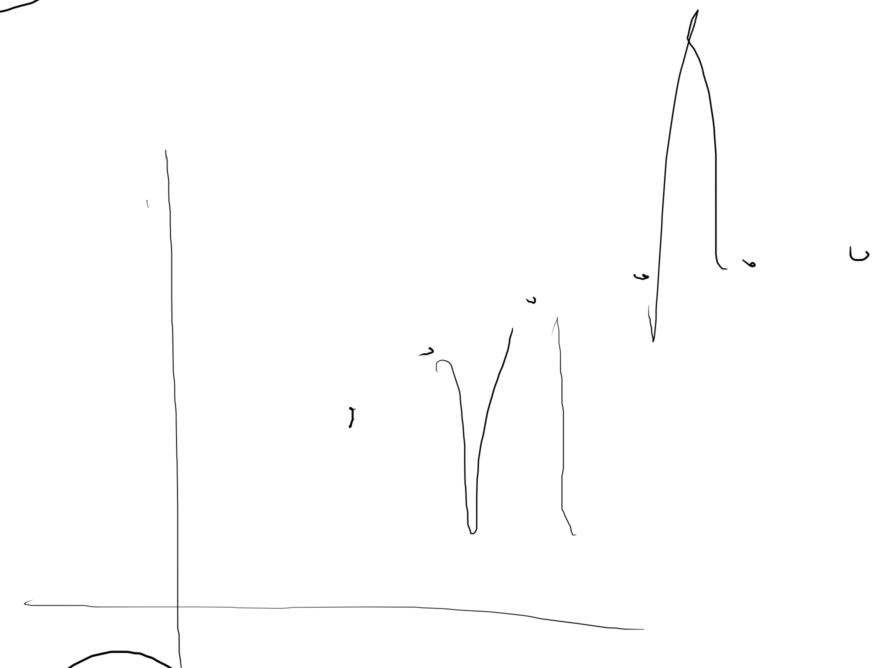
$$\boxed{\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{x} = e}$$



Pihoch 1 $\quad \lim_{x \to 0} \frac{(1+x)^{\alpha} - 1}{x}$ $\quad \alpha \in \mathbb{R}$

Pihoch 2 $\quad \lim_{x \to 0} \frac{\log_a (x+1)}{x} =$

Renato $\quad \lim_{x \to 0} \frac{a^{x} - 1}{x}$

Mystery Teorema di esistenza degli zeri di Bolzano

[metodi di bisezione]

Ghibli Teorema di Weierstrass