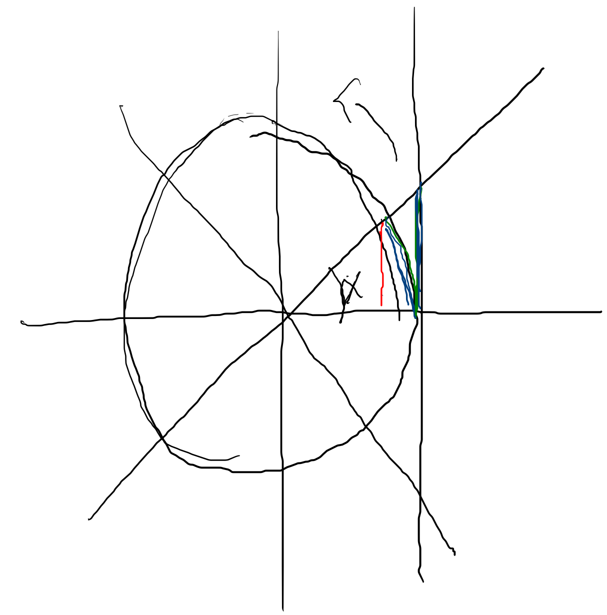
$$\lim_{x \to x_0} f(x) = f(x_0) \quad f \text{ continua}$$

Teoremi algebrici + composti ...

$$\lim_{x \to 0} \frac{\operatorname{sen} x \to 0}{x \to 0}$$

$$\lim_{x \to 0} \frac{\sin x}{x}$$



$$\sin \alpha \leq \alpha \leq \operatorname{tg} \alpha$$

$$\alpha \in \left]0 ; \frac{\pi}{2}\right[$$

$$\sin x \leq x \leq \operatorname{tg} x$$

$$\sin x \leq x \leq \frac{\sin x}{\cos x} \quad \Big/ \frac{1}{\sin x}$$

$$1 \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

$$\begin{matrix} 1 & \geq & \dfrac{\sin x}{x} & \geq & \cos x \\ \downarrow & & \downarrow & & \downarrow \\ 1 & & & & 1 \end{matrix}$$

$$\lim_{x \to 0^+} \cos x = 1$$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 1} \frac{x-1}{\arcsin(x-1)} = 1$$

$$\lim_{y \to 0} \frac{\operatorname{sen} y}{y} = 1$$

$$\lim_{x \to x_0} f(x) = y_0 \qquad \lim_{y \to y_0} g(y) = \alpha$$

$$[\text{localmente } f(x) \neq y_0] \Rightarrow \lim_{x \to x_0} g(f(x)) = \alpha$$

$$y = f(x) = \arcsin(x-1)$$

$$\lim_{x \to 1} f(x) = 0$$

$x_0 = 1$, $y_0 = 0$

$$g(y) = \frac{\operatorname{sen} y}{y}$$

$$g(f(x)) = \frac{\operatorname{sen}(\arcsin(x-1))}{\arcsin(x-1)} = \frac{x-1}{\arcsin(x-1)}$$

$$y = \arcsin(x-1)$$

$x \to 1 \leadsto y \to 0$

$$\lim_{x \to 1} \frac{x-1}{\arcsin(x-1)} = \lim_{y \to 0} \frac{\operatorname{sen} y}{y}$$

$$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = \lim_{x \to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\downarrow 1} \cdot \underbrace{\frac{1}{\cos x}}_{\downarrow 1} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{arcsen} x}{x} = \lim_{y \to 0} \frac{y}{\operatorname{sen} y} = 1$$

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$\operatorname{sen}^2 x = 1 - \cos^2 x$$

$y = \operatorname{arcsen} x$

$x \to 0 \Rightarrow y \to 0$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x)}{x^2 \cdot (1+\cos x)} = \lim_{x \to 0} \overbrace{\frac{\operatorname{sen}^2 x}{x^2}}^{1} \cdot \overbrace{\frac{1}{1+\cos x}}^{\frac{1}{2}} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to \pi} \frac{tg\, x}{1+\cos x} = \lim_{y \to 0} \frac{tg(y+\pi)}{1+\cos(y+\pi)} = \lim_{y \to 0} \frac{tg(y)}{1-\cos(y)} = *$$

$$y = x-\pi \qquad \lim_{x \to \pi} y = 0$$

$$x = y+\pi$$

$$* = \lim_{y \to 0} \underbrace{\frac{tg(y)}{y}}_{1} \cdot \underbrace{\frac{y^2}{1-\cos(y)}}_{2} \cdot \frac{1}{y} = \text{NON esiste}$$

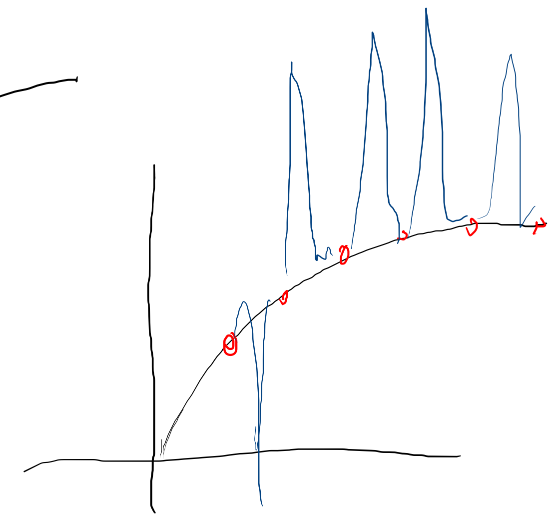
$$\lim_{x \to \pi^-} (\;) = -\infty$$

$$\lim_{x \to \pi^+} (\ldots) = +\infty$$

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x} \overset{?}{=} e$$

$$\left[\ \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n} = e\ \right]$$

Discretizzazione



$x \geq 1$

parte intera

$$0 < [x] \leq x < [x]+1$$

$$\frac{1}{[x]+1} < \frac{1}{x} \leq \frac{1}{[x]}$$

$$1 \leq \frac{x}{[x]} < \frac{[x]+1}{[x]} = 1+\frac{1}{[x]}$$

$$\left(1+\frac{1}{x}\right)^{x} \leq \left(1+\frac{1}{[x]}\right)^{x} < \left(1+\frac{1}{[x]}\right)^{[x]+1} = \underbrace{\left(1+\frac{1}{[x]}\right)^{[x]}}_{\to e} \cdot \underbrace{\left(1+\frac{1}{[x]}\right)}_{\to 1}$$

$$\left(1+\frac{1}{n}\right)^{n+1}$$

$$[x] \le x < [x]+1$$

$$\frac{1}{[x]+1} < \frac{1}{x}$$

$$\left(1+\frac{1}{x}\right)^{x} > \left(1+\frac{1}{[x]+1}\right)^{\cancel{x}} \geq \left(1+\frac{1}{[x]+1}\right)^{[x]+1-1} =$$

$$= \left(1+\frac{1}{[x]+1}\right)^{[x]+1} \cdot \left(1+\frac{1}{[x]+1}\right)^{-1}$$

$$1+\frac{1}{x} > 1+\frac{1}{[x]+1}$$

$$\left(1+\frac{1}{n+1}\right)^{n+1} \to e \qquad \left(1+\frac{1}{[x]+1}\right)^{-1} \to 1$$

$$n = [x]$$

$$\underbrace{\left(1+\frac{1}{n+1}\right)^{-1}}_{\downarrow\, 1} \cdot \underbrace{\left(1+\frac{1}{n+1}\right)^{n+1}}_{\downarrow\, e} \leq \left(1+\frac{1}{x}\right)^{x} < \underbrace{\left(1+\frac{1}{n}\right)^{n}}_{\downarrow\, e} \cdot \underbrace{\left(1+\frac{1}{n}\right)}_{\downarrow\, 1}$$

2 Carabinieri //

$$\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x = e$$

$$\boxed{\lim_{x \to -\infty} \left(1+\frac{1}{x}\right)^x = e}$$

$1-x-1=-x$

$$\left(1+\frac{1}{x}\right)^x = \left(\frac{x+1}{x}\right)^x = \left(\frac{x}{x+1}\right)^{-x} = \left(\frac{1-(x+1)}{-(x+1)}\right)^{-x} = \left(1+\frac{1}{-(x+1)}\right)^{-(x+1)+1} =$$

$$= \left(1+\frac{1}{-(x+1)}\right)^{-(x+1)} \cdot \left(1+\frac{1}{-(x+1)}\right) \to e$$

$\downarrow 1$

$y=-(x+1)$

$\lim_{x\to-\infty} y = +\infty$

$$\lim_{x \to -\infty} \left(1+\frac{1}{-(x+1)}\right)^{-(x+1)} = \lim_{y \to +\infty} \left(1+\frac{1}{y}\right)^y = e$$

$$\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = \lim_{y \to +\infty} \left(1+\frac{1}{y}\right)^y = e$$

$$y = \frac{1}{x} \qquad \lim_{x \to 0^+} y = +\infty$$

$$\lim_{x \to 0^-} (1+x)^{\frac{1}{x}} = \lim_{y \to -\infty} \left(1+\frac{1}{y}\right)^y = e$$

$$y = \frac{1}{x} \qquad \lim_{x \to 0^-} y = -\infty$$

$$\Rightarrow \boxed{\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e}$$

Dimostriamo $\lim_{x \to 0} \frac{\log_a(x+1)}{x} = \frac{1}{\log a}$

- $\lim_{x \to 0} \frac{1}{x}\left(\log_a(x+1)\right)$
- $\lim_{x \to 0} \log_a (x+1)^{\frac{1}{x}}$

- $\lim_{x \to 0} (x+1)^{\frac{1}{x}}$ $\quad x = \frac{1}{t}$
- $\lim_{t \to +\infty} \left(\frac{1}{t}+1\right)^t = e$

$$\lim_{y \to e} \log_a y \Rightarrow \log_a e = \frac{\log e}{\log a} = \frac{1}{\log a}$$

Premesso $\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x = e$

Per il teorema

$\lim_{x \to x_0} f(x) = y_0$

$\lim_{y \to y_0} g(y) = \alpha$ $\quad f(x) \neq y_0$

Allora $\lim_{x \to x_0} g(f(x)) = \alpha$

$$\frac{\log_a(x+1)}{x} = \underbrace{\log_a \left((x+1)^{\frac{1}{x}}\right)}_{\log_a e}$$

$(x+1)^{\frac{1}{x}} \to e$

$a \in \mathbb{R} \quad a > 0$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \lim_{y \to 0} \frac{y}{\log_a(y+1)} = \log a$$

$$\underline{y = a^x - 1} \qquad \lim_{x \to 0} y = 0$$

$$a^x = y + 1 \qquad x = \log_a(y+1)$$

$$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x}$$

$$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \qquad \alpha \in \mathbb{R}$$

$$y = \log(x+1)$$

$$x \to 0 \quad y \to 0$$

$$e^y = x + 1$$

$$\Rightarrow x = e^y - 1$$

$$\alpha = 0$$

$$\alpha \in \mathbb{R}$$

$$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \lim_{x \to 0} \frac{e^{\log(1+x)^\alpha} - 1}{x} = \lim_{x \to 0} \frac{e^{\alpha \log(1+x)} - 1}{x} =$$

$$= \lim_{y \to 0} \frac{e^{\alpha y} - 1}{e^y - 1} \cdot \frac{\alpha}{\alpha} = \lim_{y \to 0} \frac{e^{\alpha y} - 1}{\alpha} \cdot \frac{\alpha}{e^y - 1} \cdot \frac{y}{y} =$$

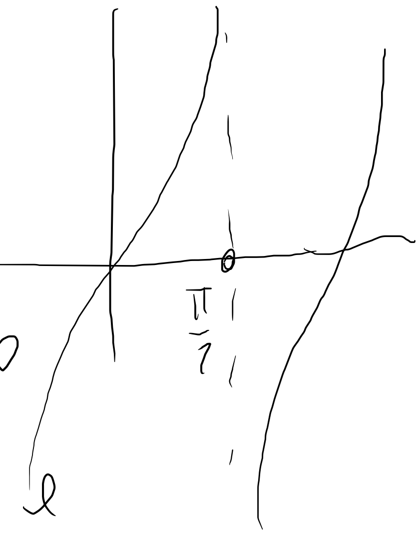
$$= \lim_{y \to 0} \alpha \cdot \underbrace{\frac{e^{\alpha y} - 1}{\alpha y}}_{\to 1} \cdot \underbrace{\frac{y}{e^y - 1}}_{\to 1} = \lim_{y \to 0} \alpha = \alpha$$

$$\lim_{x \to \pi/2} \left(1 + \frac{\cos x}{\operatorname{sen} x}\right)^{\operatorname{tg} x} \leftarrow \quad \to +\infty \text{ se } x \to \frac{\pi}{2}^-$$

$$-\infty \text{ se } x \to \frac{\pi}{2}^+$$

$\frac{\cos x}{\operatorname{sen} x} \to 0$

$$\left(1 + \to 0\right)^{\to \infty}$$



$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

$x \to +\infty$, $\frac{1}{x} \to 0$

$$\lim_{x \to \frac{\pi}{2}^+} \left(\quad\right) \ldots$$

$$y = \operatorname{tg} x$$

$$\lim_{x \to \frac{\pi}{2}^+} y = -\infty$$

$$= \lim_{y \to -\infty} \left(1 + \frac{1}{y}\right)^{y} = e$$

$$\lim_{x \to \frac{\pi}{2}^-} \left(\ldots\right) = \lim_{y \to +\infty} \left(1 + \frac{1}{y}\right)^{y} = e$$

$$\lim_{x \to 0} \frac{\sqrt{3x+2} - \sqrt{2}}{\sin x} = \frac{3}{4}\sqrt{2}$$

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$= \lim_{x \to 0} \frac{x}{\sin x} \cdot \frac{\sqrt{3x+2} - \sqrt{2}}{x}$$

$$\frac{x}{\sin x} \to 1$$

$$\frac{\sqrt{3x+2} - \sqrt{2}}{x} = \frac{\sqrt{2}\left[\frac{1}{\sqrt{2}}\sqrt{3x+2} - 1\right]}{x} = \frac{\sqrt{2}\left[\sqrt{\frac{3}{2}x+1} - 1\right]}{x} = \sqrt{2}\,\frac{\sqrt{1+\frac{3}{2}x} - 1}{x}$$

$$\lim_{x \to 0} \frac{(1+x)^{\alpha} - 1}{x} = \alpha$$

$$\alpha = 1/2$$

$$\frac{\sqrt{1+x} - 1}{x} \to \frac{1}{2}$$

$$\lim_{y \to 0} \sqrt{2}\,\frac{\sqrt{1+y} - 1}{\frac{2}{3}y}$$

$$y = \frac{3}{2}x$$

$$= \sqrt{2} \cdot \frac{3}{2} \cdot \frac{1}{2}$$

$$\lim_{x \to -1} \frac{\log_{\pi}(x+2)}{\operatorname{arcsen}(x+1)} = \lim_{x \to -1} \frac{x+1}{\operatorname{arcsen}(x+1)} \cdot \frac{\log_{\pi}(x+2)}{x+1} =$$

$$y = x+1$$

$$\lim_{x \to -1} y = 0$$

$$= \lim_{y \to 0} \underbrace{\frac{y}{\operatorname{arcsen} y}}_{1} \cdot \underbrace{\frac{\log_{\pi}(y+1)}{y}}_{\frac{1}{\log \pi}} = = \frac{1}{\log \pi}$$

$$\lim_{x \to 0} \left(\frac{x^2+x-1}{x-1} \right)^{2/x^2}$$

$$\lim_{y \to 0} (1+y)^{\frac{1}{y}}$$

$$y = \frac{x^2}{x-1}$$

$$= \lim_{x \to 0} \exp\left[\frac{2}{x^2} \log\left(\frac{x^2+x-1}{x-1} \right) \right] = \lim_{x \to 0} \exp\left[2\,\frac{1}{x-1} \cdot \frac{\log(1+y)}{y} \right]$$

$$y+1 = \frac{x^2}{x-1}+1$$

$$2 \cdot \frac{x-1}{x^2} \, \frac{1}{x-1} \log(1+y)$$

$$\frac{x-1}{x^2} = \frac{1}{y}$$

$$\lim_{x \to 0} e^{2(x-1) \cdot \log \frac{1+y}{y}}$$

$$2(x-1) \to 2$$

$$\lim_{x \to 0} y = -1$$

$$\lim_{x\to 0}\left(\frac{x^2+x-1}{x-1}\right)^{2/x^2} \qquad \frac{x^2+x-1}{x-1} = 1+\frac{x^2}{x-1}$$

$$\| $$

$$\exp\left[\frac{2}{x^2}\cdot\log\left(1+\frac{x^2}{x-1}\right)\right] = \exp\left[2\cdot\frac{\log\left(1+\frac{x^2}{x-1}\right)}{\frac{x^2}{x-1}}\cdot\underbrace{(x-1)}_{\to -1}\right]$$

1

$$y=\frac{x^2}{x-1} \qquad \lim_{x\to 0} y = 0 \qquad \lim_{y\to 0}\frac{\log(1+y)}{y} = 1$$

$$= e^{-2}$$

$$\lim_{x \to +\infty} x^x \left(2^{\left(x^{-x}\right)} - 1 \right) = \lim_{y \to +\infty} y \left(2^{1/y} - 1 \right) = *$$

$$\frac{1}{y} = z \qquad \lim_{y \to +\infty} z = 0^+$$

$$\lim_{x \to +\infty} x^x = +\infty$$

$$* \lim_{z \to 0^+} \frac{1}{z} \left(2^z - 1 \right) = \log 2$$

$$y = x^x \qquad \lim_{x \to +\infty} y = +\infty$$