Progettazione di Materiali e Processi

Modulo 1 – Lezione 5

Progettazione e selezione di materiali e processi

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Shape efficiency

- When materials are loaded in bending, in torsion, or are used as slender columns, section shape becomes important

- ”Shape” = cross section formed to a

  tubes
  I-sections
  tubes
  hollow box-section
  sandwich panels
  ribbed panels

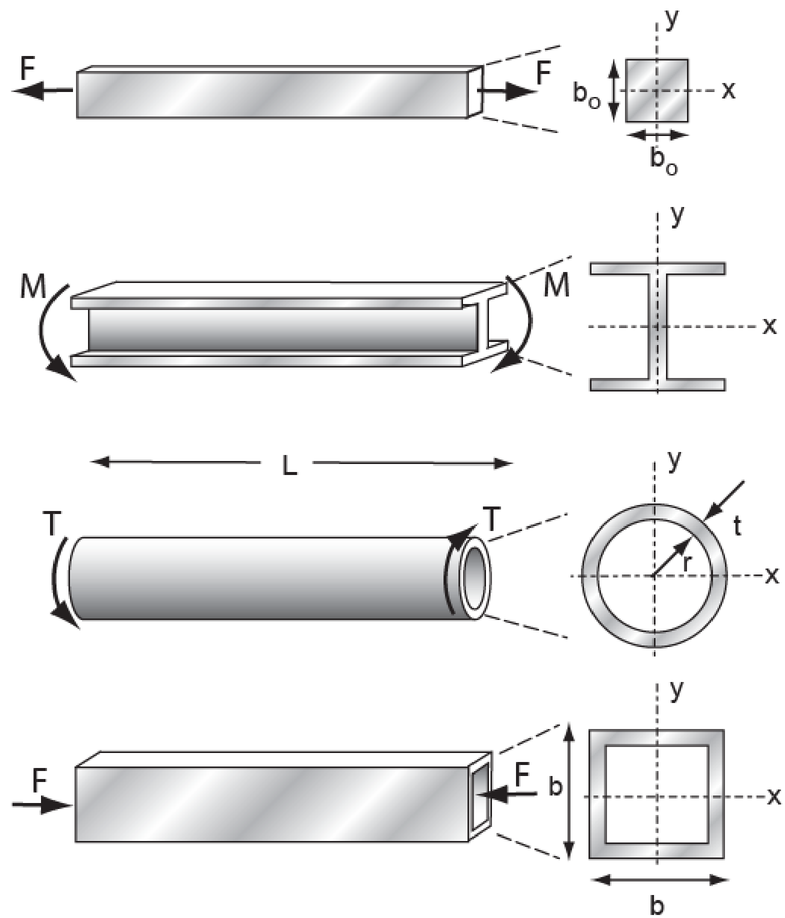
- “Efficient” = use least material for given stiffness or strength

- Shapes to which a material can be formed are limited by the material itself

- Goals: understand the limits to shape
  develop methods for co-selecting material and shape

Shape and mode of loading

Standard structural members



Tie — Area A matters, not shape

Beam — Area A and shape I_{XX}, I_{YY} matter

Torsion bar — Area A and shape J matter

Column — Area A and shape I_{min} matter

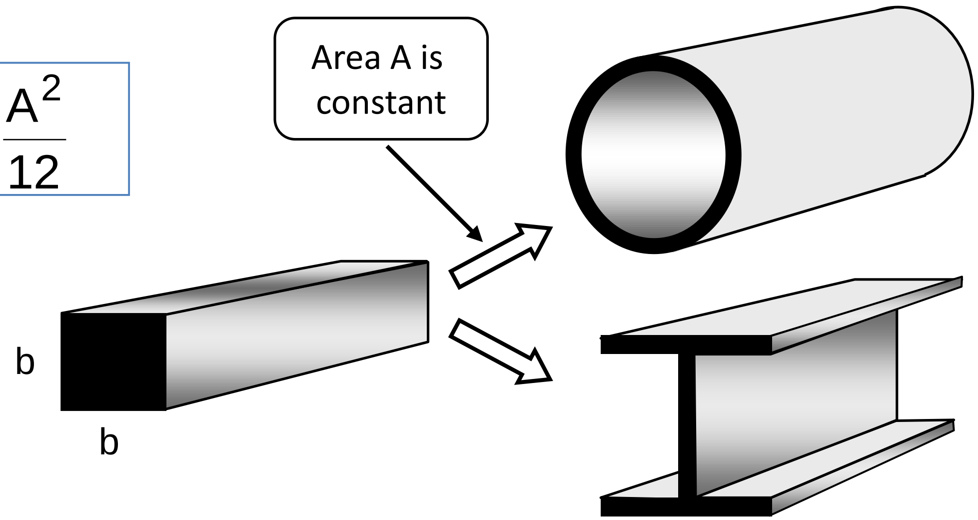
Certain materials can be made to certain shapes: what is the best combination?

Shape efficiency: bending stiffness

- Take ratio of bending stiffness S of shaped section to that (S_o) of a neutral reference section of the same cross-section area
- Define a standard reference section: a solid square with area $A = b^2$
- Second moment of area is I; stiffness scales as EI.

$$I_o = \frac{b^4}{12} = \frac{A^2}{12}$$

Area $A = b^2$



$$I = \int y^2 b(y) dy$$

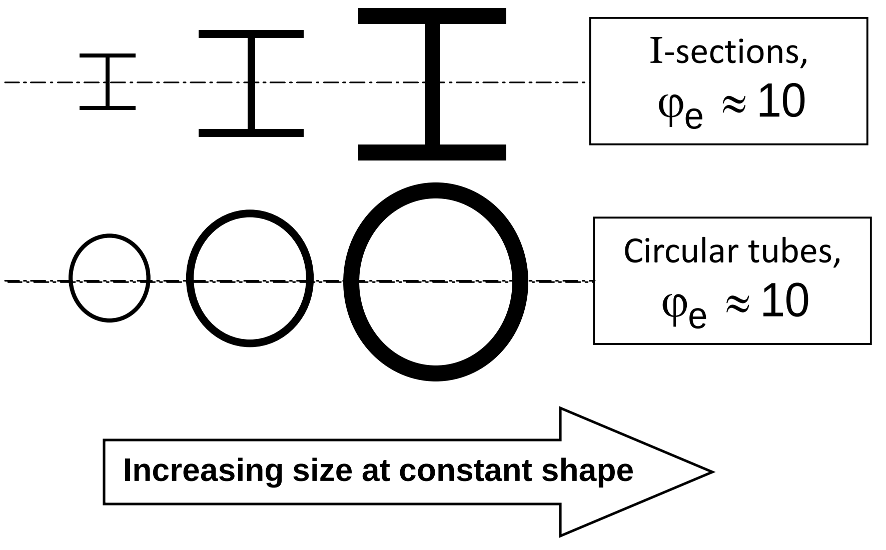
Area A and modulus E unchanged

- Define **shape factor for elastic bending,** measuring efficiency, as

$$\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12\frac{I}{A^2}$$

Properties of the shape factor

- The shape factor is dimensionless -- a pure number.
- It characterizes shape.



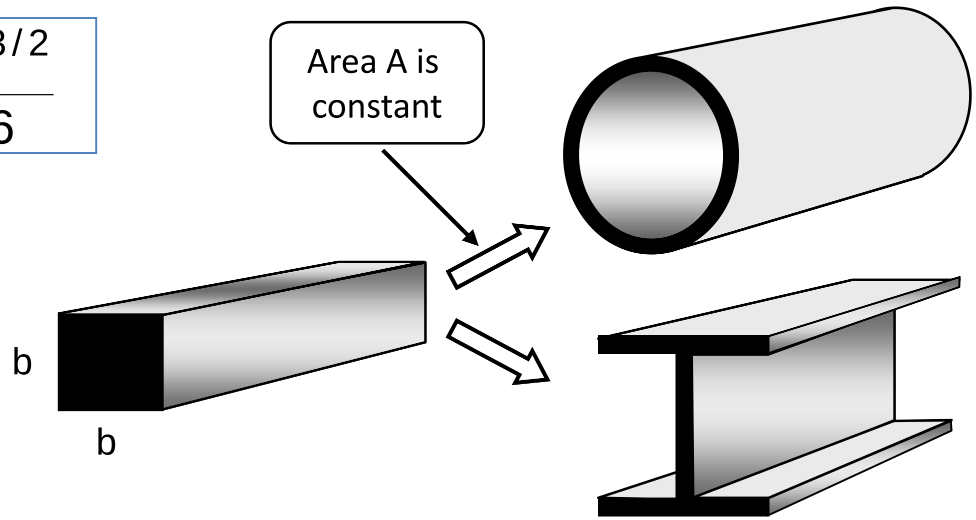
- Each of these is roughly 10 times stiffer in bending than a solid square section of the same cross-sectional area

Shape efficiency: bending strength

- Take ratio of bending strength F_f of shaped section to that ($F_{f,o}$) of a neutral reference section of the same cross-section area
- Section modulus of area is Z; strength scales as $\sigma_y Z$

$$Z_o = \frac{b^3}{6} = \frac{A^{3/2}}{6}$$

Area $A = b^2$



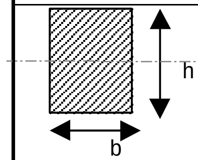
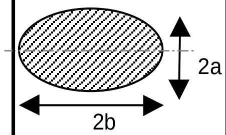
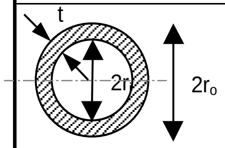
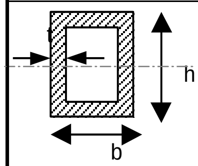
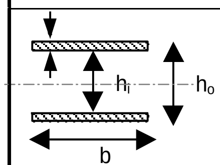
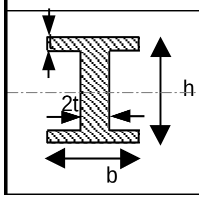
$$Z = \frac{I}{y_{max}}$$

Area A and yield strength σ_y unchanged

- Define **shape factor for onset of plasticity (failure),** measuring efficiency, as

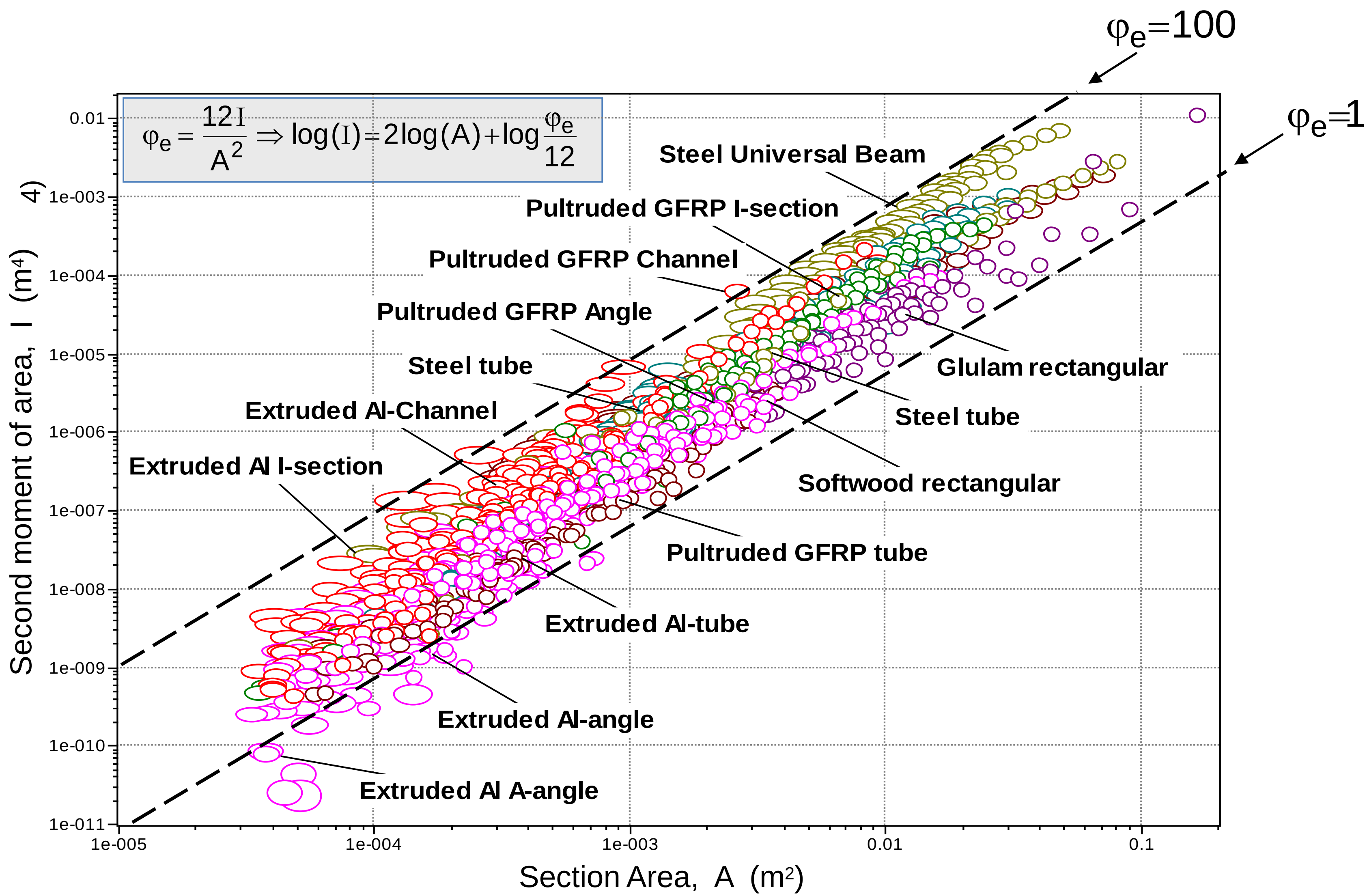
$$\varphi_f = \frac{F_f}{F_{fo}} = \frac{\sigma_y Z}{\sigma_y Z_o} = 6\frac{Z}{A^{3/2}}$$

Tabulation of shape factors

Section shape	Area A m	Second moment I, m^4	Elastic shape factor
	bh	$\frac{bh^3}{12}$	$\frac{h}{b}$
	πab	$\frac{\pi}{4}a^3 b$	$\frac{3}{\pi}\frac{a}{b}$
	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi r t$	$\frac{\pi}{4}(r_o^4 - r_i^4)$ $\approx \pi r^3 t$	$\frac{3}{\pi}\left(\frac{r}{t}\right)$ $(r >> t)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t(1+3\frac{b}{h})$	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b >> t$
	$b(h_o - h_i)$ $\approx 2bt$ $(h, b >> t)$	$\frac{b}{12}(h_o^3 - h_i^3)$ $\approx \frac{1}{2} b t h_o^2$	$\frac{3}{2}\frac{h_o^2}{bt}$ $(h, b >> t)$
	$2t(h+b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t(1+3\frac{b}{h})$	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b >> t)$

What values of φ_e exist in reality?

- Data for structural steel, 6061 aluminium, pultruded GFRP and timber



Limits for Shape Factors φ_e and φ_f

- There is an upper limit to shape factor for each material

Material	Max φ_e	Max φ_f
Steels	65	13
Aluminium alloys	44	10
GFRP and CFRP	39	9
Unreinforced polymers	12	5
Woods	8	3
Elastomers	<6	-
Other materials	..can calculate	

- Limit set by: (a) manufacturing constraints
  (b) local buckling

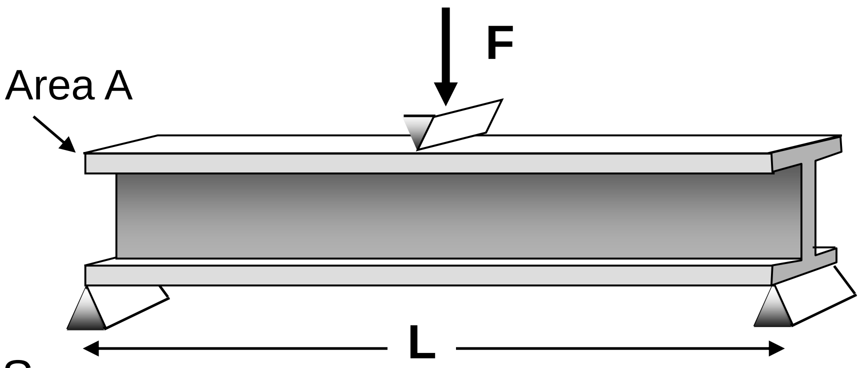
- Theoretical limit:

$$\varphi_e \approx 2\sqrt{\frac{E}{\sigma_y}}$$

Modulus (E)

Yield strength (σ_y)

Indices that include shape



Function *Beam* (shaped section).

Objective *Minimise mass*, m, where:

$$m = A L \rho$$

Constraint *Bending stiffness* of the beam S:

$$S = \frac{CEI}{L^3}$$

I is the second moment of area:

$$\varphi_e = 12 \frac{I}{A^2} \qquad A = \left(\frac{12 I}{\varphi_e}\right)^{1/2}$$

m = mass
A = area
L = length
ρ = density
b = edge length
S = stiffness
I = second moment of area
E = Youngs Modulus

Combining the equations gives:

$$m = \left(\frac{12\, S\, L^5}{C}\right)^{1/2} \left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$$

Chose materials with smallest $\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$

Selecting material-shape combinations

- Materials for stiff, *shaped* beams of minimum weight
- Fixed shape (φ_e fixed): choose materials with low $\frac{\rho}{E^{1/2}}$
- Shape φ_e a variable: choose materials with low $\frac{\rho}{(\varphi_e E)^{1/2}}$

Material	ρ, Mg/m³	E, GPa	$\varphi_{e,max}$	$\rho / E^{1/2}$	$\rho / (\varphi_{e,max} E)^{1/2}$
1020 Steel	7.85	205	65	0.55	0.068
6061 T4 Al	2.70	70	44	0.32	0.049
GFRP	1.75	28	39	0.35	0.053
Wood (oak)	0.9	13	8	0.25	0.088

- Commentary: Fixed shape (up to $\varphi_e = 8$): wood is best
  Maximum shape ($\varphi_e = \varphi_{e,max}$): Al-alloy is best
  Steel recovers some performance through high $\varphi_{e,max}$

Shape on selection charts

- Note that $\dfrac{\rho}{(\varphi_e E)^{1/2}} = \dfrac{\rho/\varphi_e}{(E/\varphi_e)^{1/2}}$ New material with $\begin{cases} \rho^* = \rho/\varphi_e \\ E^* = E/\varphi_e \end{cases}$