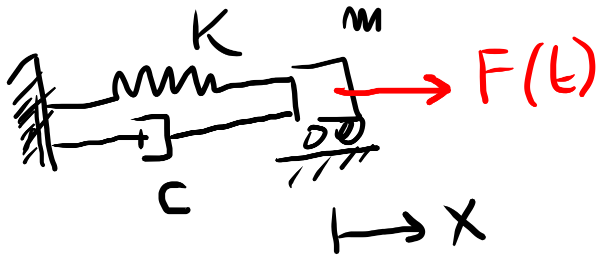
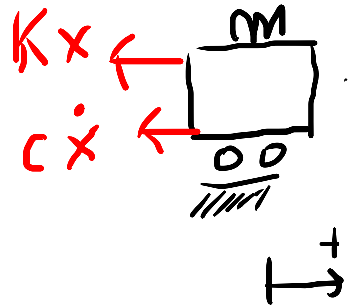
OSCILLATORE CON SMORZAMENTO



OSCILLAZIONI LIBERE

$F(t) = 0$



c : coefficiente di smorzamento viscoso

$c\dot{x}$: FORZA

$[c] = \left[F \frac{T}{L}\right] \sim \frac{Ns}{m}$

SISTEMA DISSIPATIVO

EQ. DELLA DINAMICA $\quad m\ddot{x} = -Kx - c\dot{x}$

$$m\ddot{x} + c\dot{x} + Kx = 0 \rightsquigarrow \ddot{x} + 2\frac{c}{2m\omega}\omega\dot{x} + \omega^2 x = 0 \rightsquigarrow \ddot{x} + 2\nu\omega\dot{x} + \omega^2 x = 0 \quad (*)$$

$\nu = \frac{c}{c_{cr}}$: Rapporto di smorzamento ; $c_{cr} = 2m\omega = 2\sqrt{Km}$: smorzamento critico

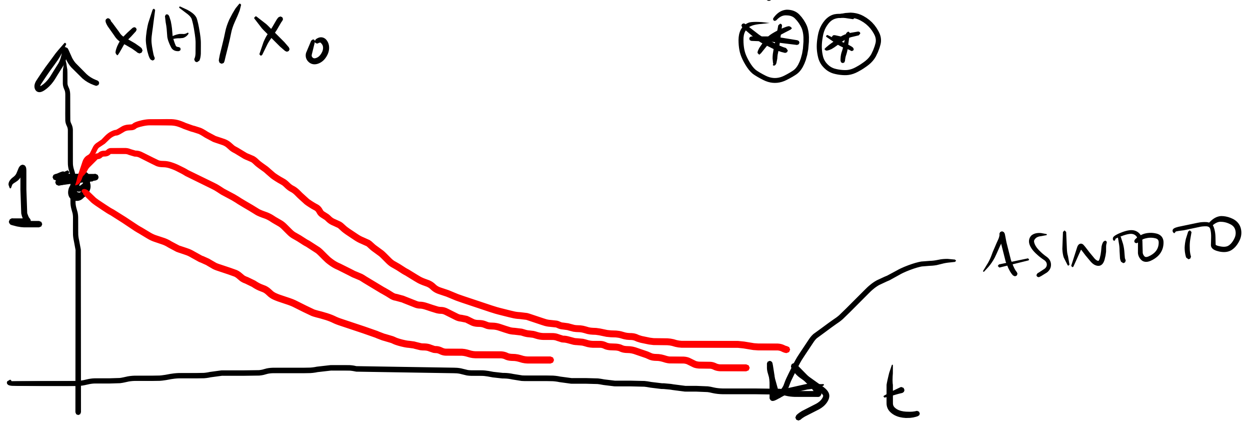
$\nu \sim 0.03 \div 0{,}07$ nelle strutture civili.

$$x(t) = e^{\lambda t}, \quad \dot{x} = \lambda e^{\lambda t}, \quad \ddot{x} = \lambda^2 e^{\lambda t} \Rightarrow (*) \quad (\lambda^2 + 2\nu\omega\lambda + \omega^2)\underbrace{e^{\lambda t}}_{\neq 0} = 0$$

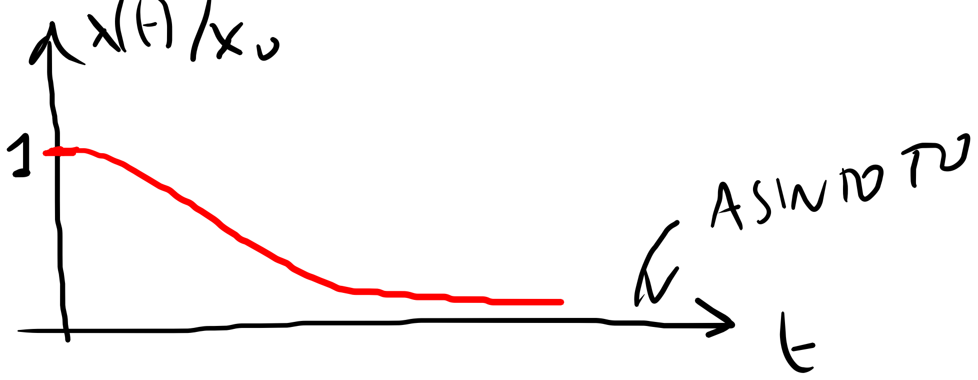
$$\left.\begin{matrix}\lambda_1 \\ \lambda_2\end{matrix}\right\} = -\nu\omega \pm \sqrt{\nu^2\omega^2 - \omega^2} = \omega\left(-\nu \pm \sqrt{\nu^2 - 1}\right)$$

- $\nu > 1$ SMORZ. SUPERCRITICO
- $\nu = 1$ SMORZ. CRITICO
- $\nu < 1$ " SOTTOCRITICO

$$\nu > 1 : \quad x(t) = \underbrace{A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}}_{(*)(*)} = \underbrace{e^{-\nu\omega t}}_{\text{MODULANTE}} \Big[\underbrace{A_1 e^{+\omega\sqrt{\nu^2-1}\, t} + A_2 e^{-\omega\sqrt{\nu^2-1}\, t}}_{\text{CONDIZ. INIZIALI}} \Big]$$



$\nu = 1$: $\lambda_1 = \lambda_2$, la soluz. (*)(*) NON VA PIÙ BENE $\rightarrow x(t) = A_1 e^{-\nu\omega t} + A_2\, t\, e^{-\nu\omega t}$



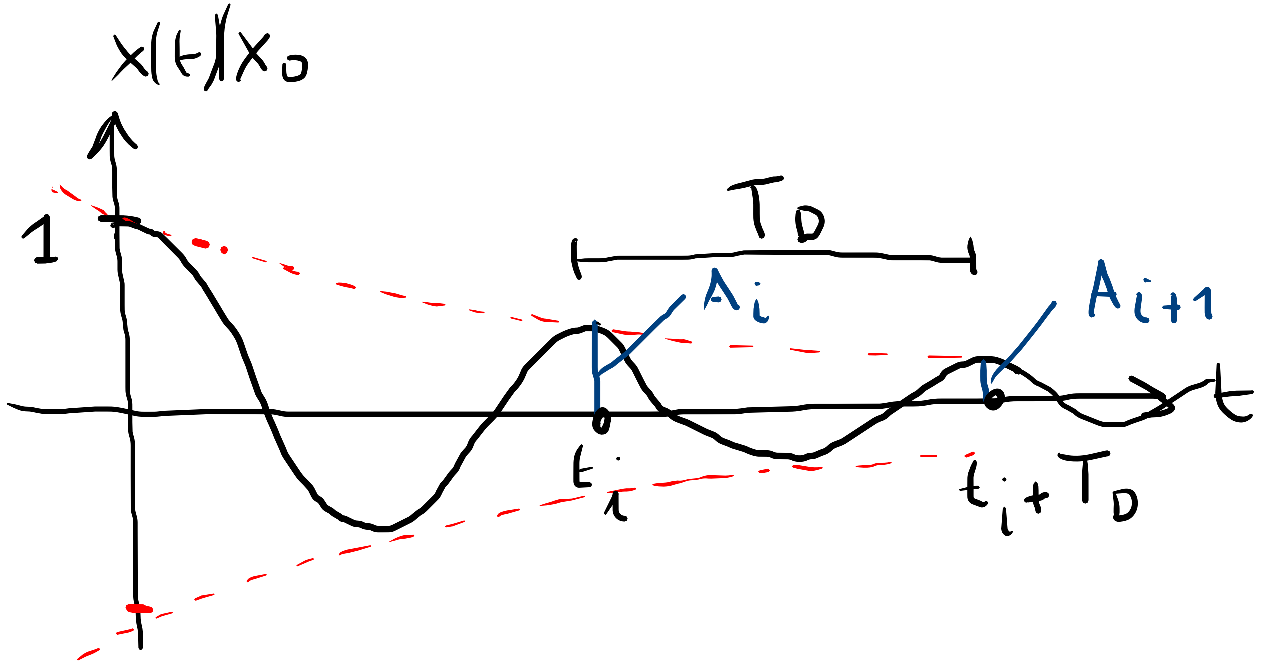
$$\nu < 1: \quad x(t) = A_1 \underbrace{e^{-\nu\omega t}}\, e^{+i\underbrace{\omega\sqrt{1-\nu^2}}_{\omega_D}\, t} + A_2 \underbrace{e^{-\nu\omega t}}\, e^{-i\omega\sqrt{}\, t}$$

$$\omega_D = \omega\sqrt{1-\nu^2} \quad \text{è} \simeq \omega_0$$

per $\nu \ll 1$

$$= \underbrace{e^{-\nu\omega t}}_{\text{MODULANTE}} \big(\underbrace{B_1 \sin\omega_D t + B_2 \cos\omega_D t}_{\text{OSCILLANTI}}\big)$$

$$T_D = \frac{2\pi}{\omega_D} \quad : \text{periodo "ridotto"}$$



ASINTOTIC. A ZERO

$\omega_D < \omega$; $T_D > T$

$\nu \to 0$, $\omega_D = \omega$, $T_D = T$

$\nu \to 1$; $\omega_D \to 0$, $T_D \to \infty$

Ad ogni ciclo, la perdita di E_{mecc} TOTALE è PARI A: $\Delta E_{mecc} = \frac{1}{2} K \left(A_{i+1}^2 - A_i^2\right) < 0$

$$A_i = e^{-\nu\omega t_i}$$

$$A_{i+1} = e^{-\nu\omega(t_i + T_D)} = e^{-\nu\omega t_i} \cdot e^{-\nu\omega T_D} = A_i\, e^{-\nu\omega T_D}$$

$$A_i / A_{i+1} = e^{\nu\omega T_D}$$

$$\frac{A_i}{A_{i+1}} = e^{\nu\omega T_D} = e^{\delta}$$; $$\delta = \nu\omega\frac{2\pi}{\omega\sqrt{1-\nu^2}} = \frac{2\pi\nu}{1-\nu^2}$$ nel caso $\nu << 1$ $\boxed{\delta \simeq 2\pi\nu}$

δ: DECREMENTO LOGARITMICO

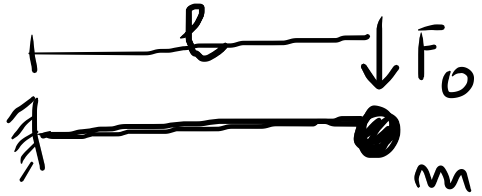
$$\delta = \ln\frac{A_i}{A_{i+1}}$$

$$e^{\delta} \simeq e^{2\pi\nu} \simeq 1 + 2\pi\nu \quad (\nu << 1)$$

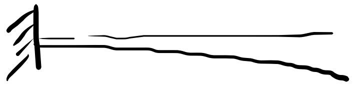
$$\boxed{\frac{A_i}{A_{i+1}} \simeq 1 + 2\pi\nu} \quad (1)$$

si ottiene facilmente ν attraverso il rapporto tra 2 ampiezze successive.

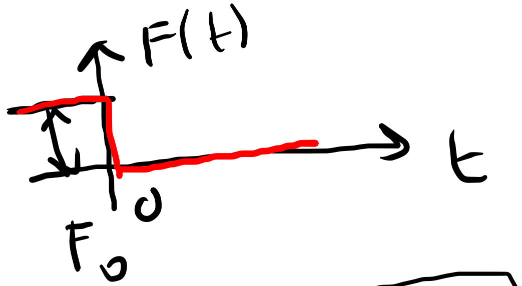
IDENTIFICAZ. STRUTTURALE studio le oscillazioni libere $\longrightarrow$ il coefficiente ν

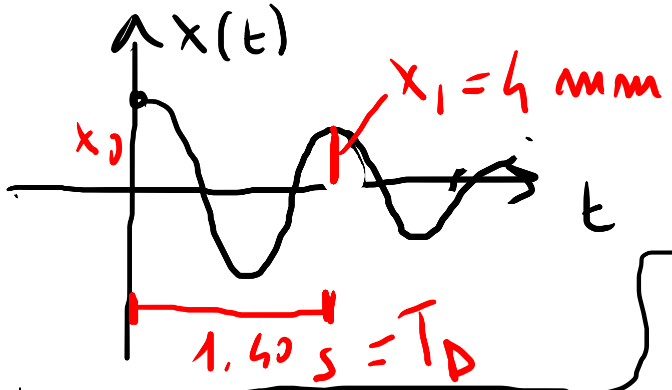


F_0: FORZA STATICA = 90 KN



$x_0 = 5$ mm $\boxed{K x_0 = F_0 + mg}$





K? c? m?

DA (1) ottengo ν

$$\omega = \frac{\omega_D}{\sqrt{1-\nu^2}} = \frac{2\pi}{T_D}\frac{1}{\sqrt{1-\nu^2}} \quad \checkmark$$

$\boxed{\omega^2 = \frac{K}{m}}$ $\boxed{K x_0 = F_0 + mg}$ $\Rightarrow$ 2 EQ. NELLE INCOGNITE m, K

$$K = \frac{3EI}{l^3}$$