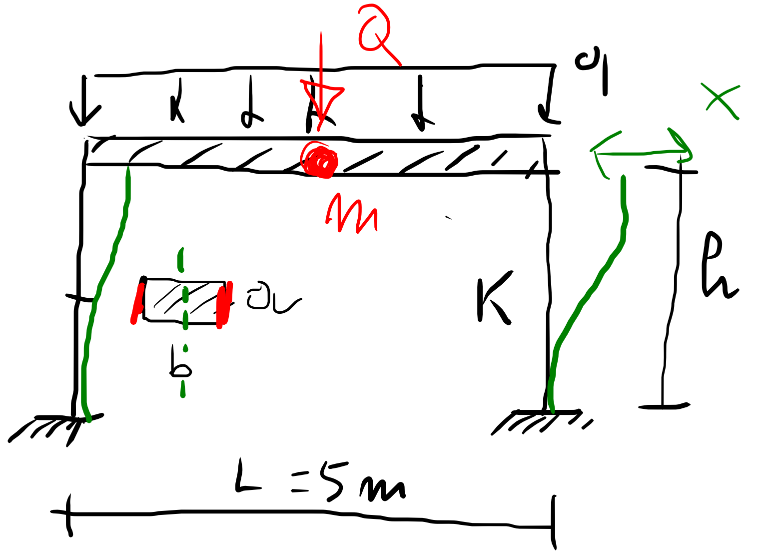
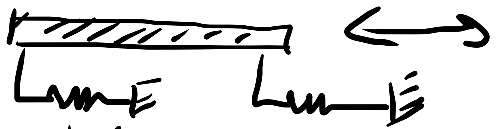
ES

9/11/21

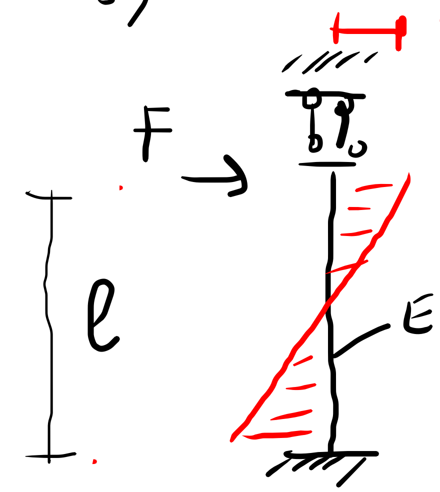


1) T ? $h = 3\,m$, $b = 30\,cm$, $a = 30\,cm$

$q = 40\,KN/m$, $E = 25000\,\frac{N}{mm^2}$ (cls)



$$K_1 = 12\frac{EI}{h^3}$$



$$\delta = \frac{F\,l^3}{12EI} \qquad \frac{1}{K}$$

$$K = \frac{12EI}{l^3}$$

$$Q = qL = 200\,KN$$

$$m = \frac{Q}{g} = \frac{200000\,N}{9.81\,m/s^2} = 2\cdot 10^4\,Kg$$

$$\omega\ ;\ T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$K = 2K_1 = 24\frac{EI}{h^3}$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{24\,E\,\frac{1}{12}ab^3\,\frac{1}{h^3}\cdot\frac{1}{m}} = \hat{f}\left(\sqrt{\left(\frac{b}{h}\right)^3}\right)$$

$$I = \frac{ab^3}{12}$$

$$\omega = 27.39\ rad/s$$

$$T = 0.23\,s\ ;\ f = \frac{1}{T} = 4.35\,Hz$$

Osservo che $T = \frac{2\pi}{\omega} = \hat{g}\left(\sqrt{\frac{h^3}{b^3}}\right) = \bar{g}\left(h^{3/2}\right)$

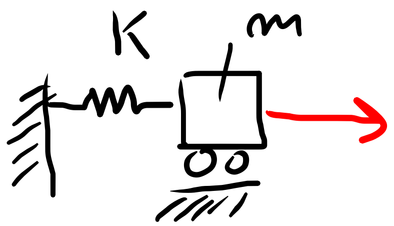
"Normative"

$$T \sim C_1 H^{3/4}$$

$C_1 \sim 0.05$ dipende dal materiale

2) Se $b = 30$ cm, $h = 4$ m, $T = 0.35$ s
5 m $= 0.49$ s

OSCILLATORE SEMPLICE CON FORZANTE ARMONICA



$F \sin \bar{\omega} t = F(t)$

Ω : PULSAZ. DELLA FORZANTE

$$m\ddot{x} + Kx = F \sin \bar{\omega} t$$

$$\ddot{x} + \omega^2 x = \frac{F}{m} \sin \bar{\omega} t = \frac{F}{K}\frac{K}{m} \sin \bar{\omega} t = \frac{F}{K}\omega^2 \sin \bar{\omega} t$$

$$x(t) = x^{hom}(t) + x^{part}(t)$$

$$x^p(t) = X \sin \bar{\omega} t$$

$$\ddot{x}^p(t) = -\bar{\omega}^2 X \sin \bar{\omega} t$$

$$-m\bar{\omega}^2 X \cancel{\sin \bar{\omega} t} + K X \cancel{\sin \bar{\omega} t} = F \cancel{\sin \bar{\omega} t}$$

$$X (K - m\bar{\omega}^2) = F \qquad X \text{ determ.!!}$$

$$X\left(1 - \frac{m}{K}\bar{\omega}^2\right) = \frac{F}{K} \; ; \quad X\left(1 - \frac{\bar{\omega}^2}{\omega^2}\right) = \frac{F}{K}$$

$$x(t) = B_1 \sin \omega t + B_2 \cos \omega t + \frac{F}{K}\frac{1}{1 - \frac{\bar{\omega}^2}{\omega^2}} \cdot \sin \bar{\omega} t$$

Per avere le soluz. effettive si applicano le c. iniziali:

$x(0) = x_0$; $\dot{x}(0) = \dot{x}_0 \longrightarrow$ avremo da un contributo della velocità anche ... $\sin \bar{\omega} t$

$$\boxed{B_2 = x_0}$$

$$\boxed{B_1 = \frac{\dot{x}_0}{\omega} - \frac{\bar{\omega}/\omega}{1 - (\bar{\omega}/\omega)^2}\frac{F}{K}}$$

$$\frac{F}{K} = X_{st}$$

studiamo il probl con $x_0 = \dot{x}_0 = 0$

$$x(t) = x_{st}\,\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\left[\sin\bar\omega t - \frac{\bar\omega}{\omega}\sin\omega t\right]$$

SOLUZ. NON VALE $\bar\omega = \omega$

$\bar\omega \to \omega$, $x(t) \to \infty$ <u>RISONANZA</u>

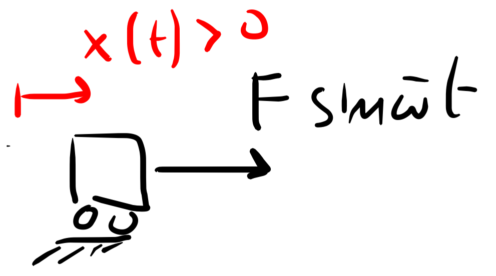
concentriamoci su

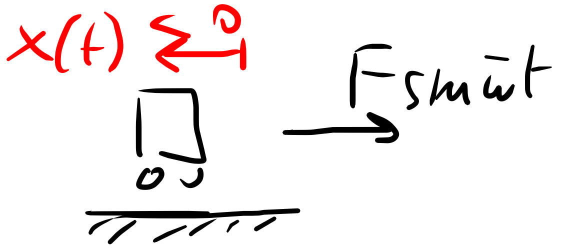
$$x(t) \simeq x_{st}\,\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\sin\bar\omega t$$

se $\bar\omega < \omega$ allora $x(t) > 0$ quando $\sin\bar\omega t > 0$

$\bar\omega > \omega$ " $x(t) < 0$

IN FASE





IN OPPOSIZ. DI FASE

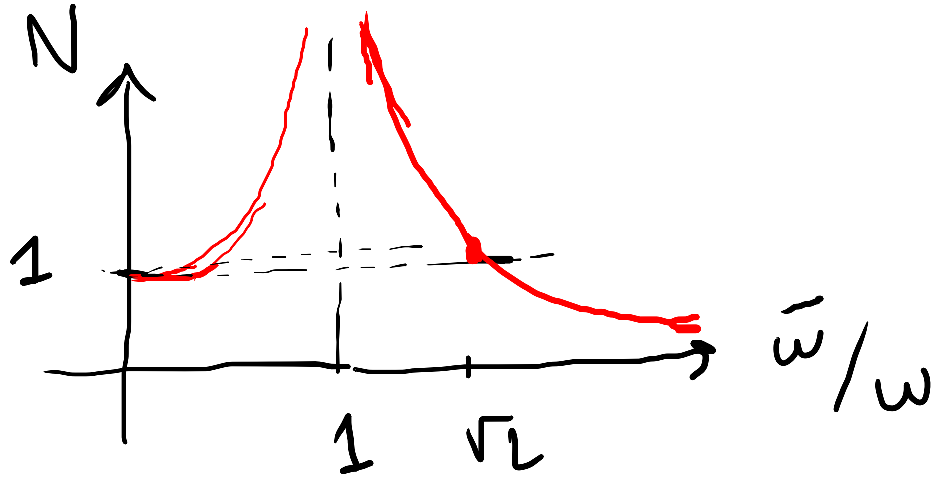
$$x(t) = N\, x_{st} \sin(\bar\omega t - \xi)$$

$\xi = 0$ se $\bar\omega < \omega$

$\xi = \pi$ " $\bar\omega > \omega$

$$N = \frac{1}{\left|1-\left(\frac{\bar\omega}{\omega}\right)^2\right|}$$; FATTORE DI AMPLIFICAZ. DINAMICA

$$N = \frac{x(t)_{max}}{x_{st}} \to \sin(\dots) = 1$$



ASPETTI ENERGETICI

$$E_{mecc} = E_{el} + E_{cin}$$

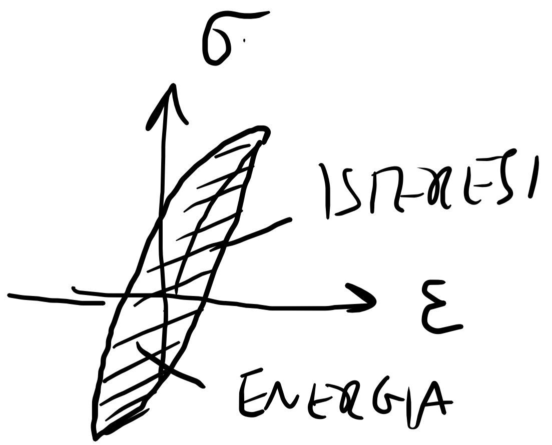
SIST. CONSERVATIVO $\frac{d}{dt} E_{mecc} = 0$

" NON CONSERVATIVO : ENERGIA IMMESSA DALLA FORZANTE ESTERNA $F(t)$

$$\underbrace{\frac{d}{dt} E_{mecc}}_{\text{POTENZA}} = \underbrace{F(t) \cdot \dot{x}(t)}_{\text{POTENZA DELLA FORZA AL TEMPO } t}$$

$F, \dot{x}$ CONCORDI $\frac{d}{dt} E_{mecc} > 0$

" " DISCORDI $\frac{d}{dt} E_{mecc} < 0$



DISSIPATA PER UNITA' DI VOLUME

Posso scrivere

$$\frac{d}{dt} E_{mecc} = P_F - P_D$$

P_F : POTENZA DELLA FORZANTE

P_D : " DISSIPATA (—⊐—) $P_D = c\dot{x}^2$