1 GDL GENERALIZZATO

16/11/2021

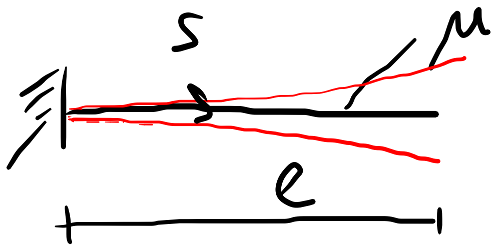
- $\psi(s)$ PUÒ ESSERE SCELTA IN PRIMA APPROSSIM. COME LA DEF. ELASTICA QUANDO IL CARICO q AGISCE STATICAMENTE

$$EI\,\psi^{IV} = q(s)$$ LINEA ELASTICA DEL IV ORDINE

C.C. $\psi(0)=\psi(\ell)=0 \;;\quad \psi''(0)=\psi''(\ell)=0$

$$EI\,v'' = -M$$

- ESEMPIO



(μ DENSITÀ PER UNITÀ DI LUNGHEZZA.)

SOLUZ. ESATTA $\omega_e = \dfrac{3.516}{\ell^2}\sqrt{\dfrac{EI}{\mu}}$

IL MODO DI VIBRARE ESATTO È

$$\psi_e = f\left(\sinh, \cosh\left(\frac{s}{\ell}\right)\right)$$

STUDIO LE OSCILL. LIBERE

STIMO LA PULSAZ. $\omega^2 = \dfrac{K_{eq}}{m_{eq}}$

1) APPROSSIMAZ. $\psi_1(s) = \frac{3 s^2}{2\ell^2} - \frac{s^3}{2\ell^3}$, $\psi_1(0) = 0 = \psi_1'(0)$ CONDIZ. DI VINCOLO SODDISFATTE

$m_{eq}^{(1)} = 0{,}236\, \mu \ell$

$K_{eq}^{(1)} = \frac{3 EI}{\ell^3}$

$\omega^{(1)} \simeq 3.568 \frac{}{\ell^2} \sqrt{\frac{EI}{\mu}}$ APPROSSIMA BENE ω_e, errore 1.5%

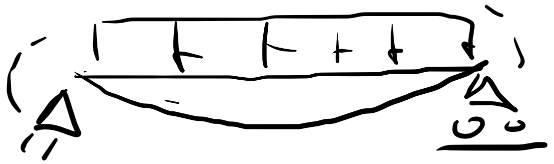
OSSERV.

LE FUNZ DI FORMA $\psi_K(s)$ $\psi''(\ell) = 0$

APPROSS. DOVREBBERO SODDISF. ANCHE LE CONDIZ. DI TIPO STATICO

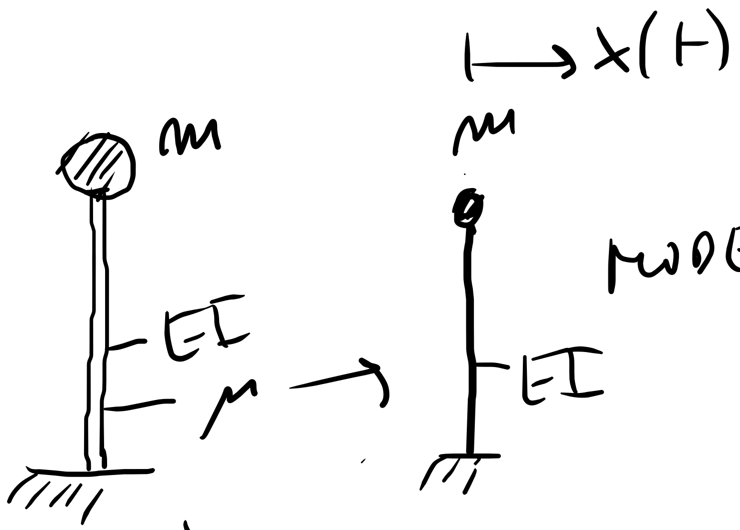
2) APPROSS. $\psi_2(s) = 1 - \cos \frac{\pi s}{2\ell}$ $\psi_2(0) = \psi_2'(0) = 0$

$\omega^{(2)} \simeq 3.664 \frac{}{\ell^2} \sqrt{\frac{EI}{\mu}}$ errore 4%.

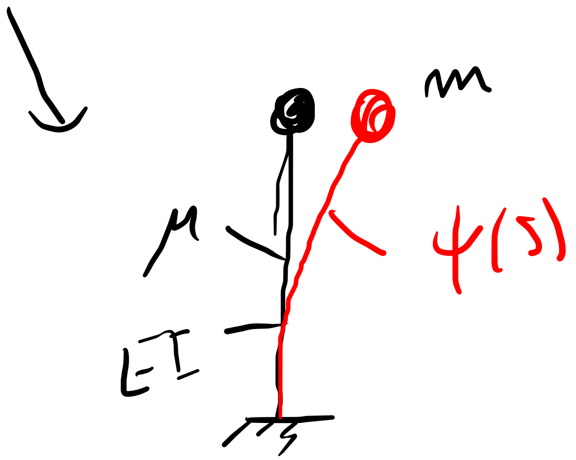


$\psi''(0) = \psi''(\ell) = 0$

OSS CHE $\omega^{(1)}, \omega^{(2)} > \omega_e$



MODELLO PIÙ SEMPLICE: OSCILLATORE CLASSICO



OSCILLATORE SEMPLICE CON GDL GENERALIZZATO

TENGO CONTO DELLA MASSE DEL PILASTRO.

DINAMICA DEI SISTEMI A PIÙ G.D.L.

1 GDL $\quad m\ddot{x} + c\dot{x} + Kx = F(t)$

N GDL $\quad \underset{\sim}{M}\,\underline{\ddot{x}} + \underset{\sim}{C}\,\underline{\dot{x}} + \underline{K}\,\underline{x} = \underline{F}(t)$

matrice delle masse ... delle smorzamento ... di rigidezza

stiffness.

$\underline{x}$: vettore dei gdl.

$\underset{\sim}{M}\,\underline{\ddot{x}} + \underset{\sim}{K}\,\underline{x} = \underline{0}$ oscill libere

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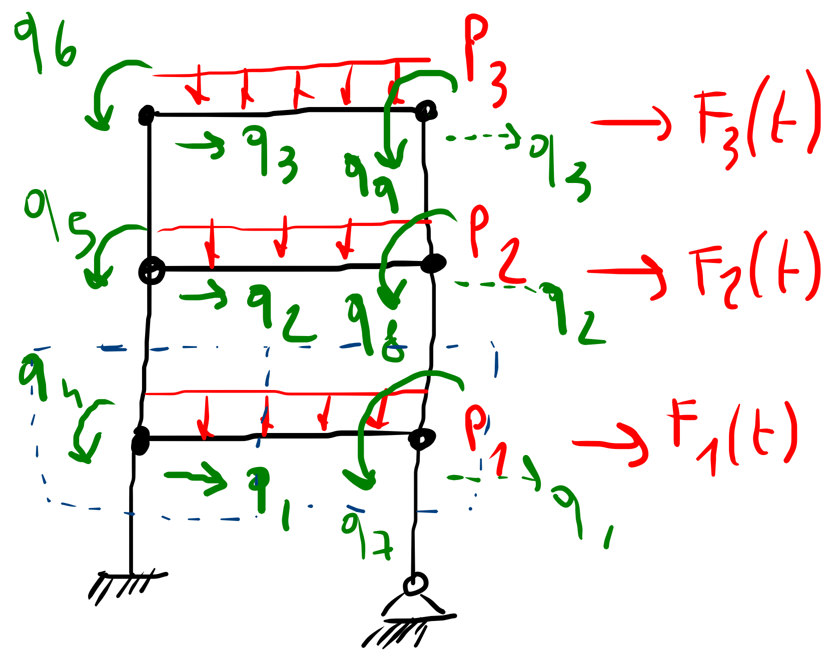
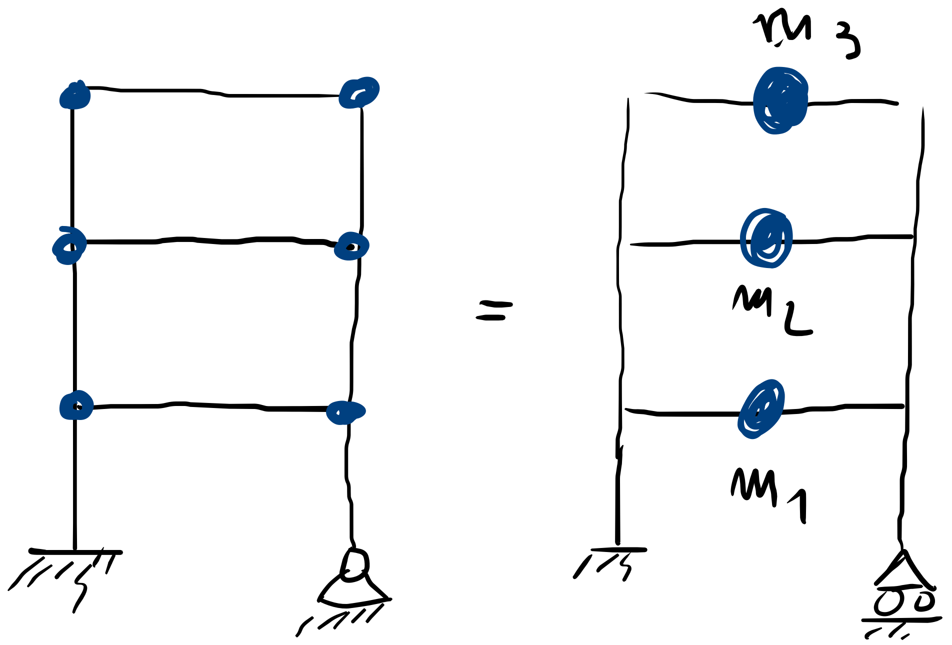
$\underset{\sim}{K}\,\underline{x} = \underline{F}$ significato della matrice di rigidezza in termini statici

Scelta dei gdl.



CONCENTR. MASSE



$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \vdots \\ q_9 \end{bmatrix} = \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix}$$

TELAIO ELASTICO PIANO

TRAVI E PILASTRI

INDEFORM. ASSIALMENTE

$q_1 \cdots\cdots q_9$

$q_4 --- q_9$ SONO LE ROTAZ.

DEI NODI

Le masse nei nodi

sviluppano energia cinetica

per le velocità di traslazione

dei "traversi"

$$E_{cin} = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2$$

Con le scomposiz dei gdl in $\underline{q}_A$ e $\underline{q}_B$, il sist. $\underset{\sim}{M}\ddot{\underline{q}} + \underset{\sim}{K}\underline{q} = \underline{F}(t)$ si può scrivere:

$$\begin{bmatrix} \underset{\sim}{M}_A & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} \end{bmatrix} \begin{bmatrix} \ddot{\underline{q}}_A \\ \ddot{\underline{q}}_B \end{bmatrix} + \begin{bmatrix} \underset{\sim}{K}_{AA} & \underset{\sim}{K}_{AB} \\ \underset{\sim}{K}_{BA} & \underset{\sim}{K}_{BB} \end{bmatrix} \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix} = \begin{bmatrix} \underline{F}_A(t) \\ \underline{0} \end{bmatrix} \begin{matrix} \} \text{ 3 EQ.} \\ \} \text{ 6 EQ.} \end{matrix}$$

CONDENSAZ. STATICA

TRASFORMO IL SIST ~~AB~~ IN UN SIST. RIDOTTO DI SOLE INCOGNITE $\underline{q}_A$!

$$\overset{\underset{\sim}{M}}{\underset{\sim}{M}_A} = \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{bmatrix}$$

$$\left\{ \underset{\sim}{M}_A \ddot{\underline{q}}_A + \underset{\sim}{K}_{AA}\underline{q}_A + \underset{\sim}{K}_{AB}\underline{q}_B = \underline{F}_A(t) \right. \qquad \text{3 EQL}$$

$$\underset{\sim}{K}_{BA}\underline{q}_A + \underset{\sim}{K}_{BB}\underline{q}_B = \underline{0} \; \}$$

$$\underline{q}_B = -\underset{\sim}{K}_{BB}^{-1}\underset{\sim}{K}_{BA}\underline{q}_A$$

$$\underset{\sim}{M}_A \ddot{\underline{q}}_A + \underset{\sim}{K}_{AA}\underline{q}_A - \underset{\sim}{K}_{AB}\underset{\sim}{K}_{BB}^{-1}\underset{\sim}{K}_{BA}\underline{q}_A = \underline{F}_A(t)$$

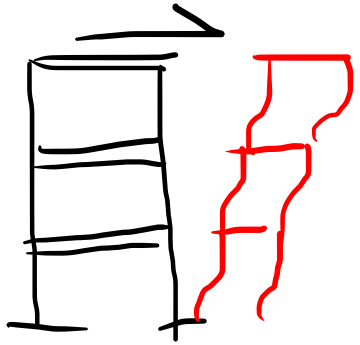
SIST CON I "SOLI" GDL "DINAMICI

$$\underset{\sim}{M}_A \ddot{\underline{q}}_A + \Big[\underbrace{\underset{\sim}{K}_{AA} - \underset{\sim}{K}_{AB}\underset{\sim}{K}_{BB}^{-1}\underset{\sim}{K}_{BA}}_{\underset{\sim}{K}_A \text{ MAT. CONDENS.}}\Big]\underline{q}_A = \underline{F}_A(t)$$

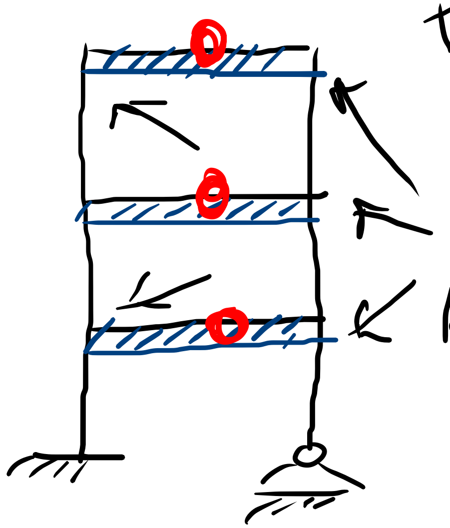
MOTO IMPRESSO ALLA BASE $(\ddot{y})$

$$\underline{F}(t) = -\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \ddot{y}(t) + \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\underset{\sim}{M}\,\underline{1}\,\ddot{y}(t) + \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$



UNA SCELTA DEI GDL MENO ACCURATA
È LA SEGUENTE



TELAI SHEAR TYPE

ROTAZ. NULLE

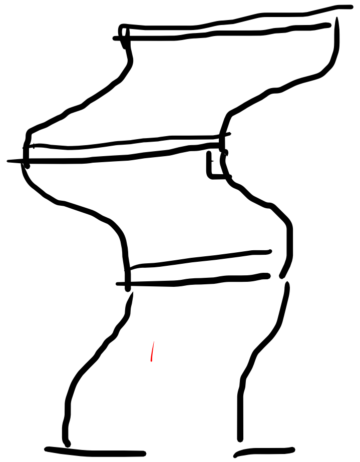
$\rightarrow q_3$

$\rightarrow q_2$

$\rightarrow q_1$

SIST. 3X3

$$\underset{\sim AA}{M}\,\underline{\ddot{q}}_A + \underset{\sim AA}{\hat{K}}\,q_A = \underline{F}(t)$$

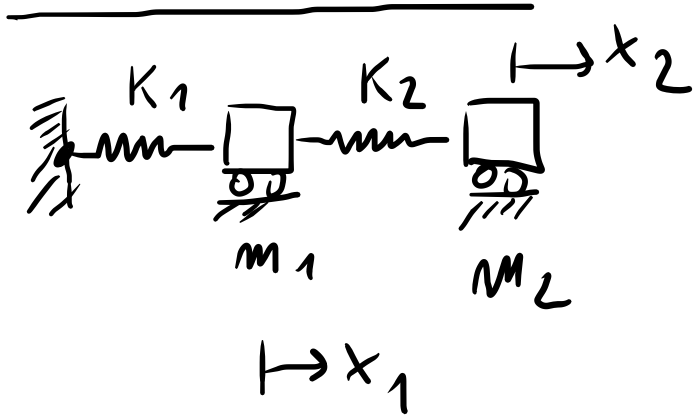


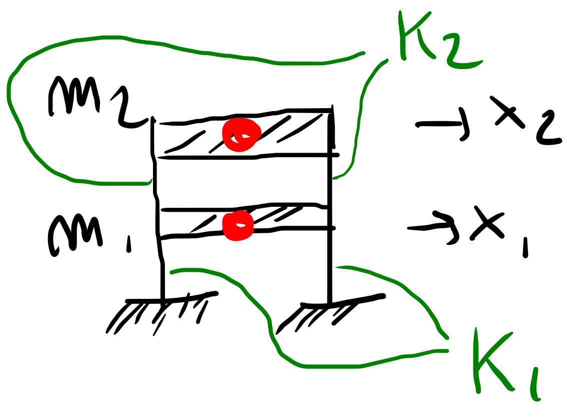
MENSOLA A TAGLIO

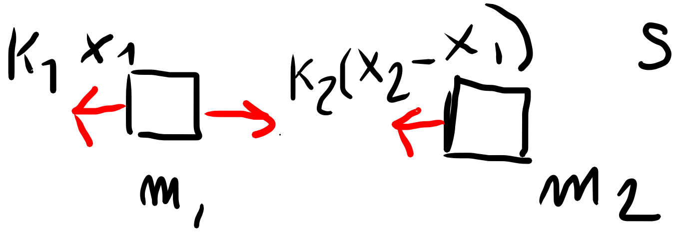
TRAVERSI "INFINITI" RIGIDI FLESSIONALMENTE

$EI_{travi} \gg EI_{pile}$.

SIST. 2 GDL







S.C.L.

$$\begin{cases} m_1\ddot{x}_1 = K_2(x_2-x_1) - K_1x_1 \\ m_2\ddot{x}_2 = -K_2(x_2-x_1) \end{cases}$$

OSCILLAZ. LIBERE $\rightarrow$ 2 PULSAZIONI DEL SISTEMA

$$\begin{cases} m_1\ddot{x}_1 + K_1x_1 + K_2(x_1-x_2) = 0 \\ m_2\ddot{x}_2 + K_2(x_2-x_1) = 0 \end{cases}$$

$$\underset{\underset{\sim}{M}}{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}} \underset{\underline{\ddot{x}}}{\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}} + \underset{\underset{\sim}{K}}{\begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}} \underset{\underline{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{\underline{0}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

Immagino che $x_1, x_2 \rightarrow x_1 = U_1 \sin\omega t$
$x_2 = U_2 \sin\omega t$

$$\left[m_1U_1(-\omega^2) + K_1U_1 + K_2(U_1-U_2)\right] \cancel{\sin}\,\omega t = 0$$

$$\left[m_2U_2(-\omega^2) + K_2(U_2-U_1)\right] \sin\cancel{\omega t} = 0$$

ω^2 si ottengono dalle solut. del det della matrice associata al sistema

$$\begin{vmatrix} K_1+K_2-m_1\omega^2 & -K_2 \\ -K_2 & K_2-m_2\omega^2 \end{vmatrix} = 0$$

$\Rightarrow \omega_1^2, \omega_2^2$; DUE AUTOVALORI