COORDINATE PRINCIPALI $(\underline{z}(t))$

$$\underset{\sim}{M}\,\underline{\ddot{q}} + \underset{\sim}{K}\,\underline{q} = \underline{0}$$

$$\underset{N}{\underline{q}(t)} = \underset{N\times N}{\underset{\sim}{\Phi}}\;\underset{N}{\underline{z}(t)}$$

$$\underline{\dot{q}}(t) = \underset{\sim}{\Phi}\,\underline{\dot{z}}$$

$$\underline{\ddot{q}}(t) = \underset{\sim}{\Phi}\,\underline{\ddot{z}}$$

$$\underset{\sim}{M}\,\underset{\sim}{\Phi}\,\underline{\ddot{z}} + \underset{\sim}{K}\,\underset{\sim}{\Phi}\,\underline{z} = \underline{0}$$

$$\underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{M}\,\underset{\sim}{\Phi}}_{\hat{\underset{\sim}{M}}}\,\underline{\ddot{z}} + \underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{K}\,\underset{\sim}{\Phi}}_{\hat{\underset{\sim}{K}}}\,\underline{z} = \underline{0}$$

$$\hat{\underset{\sim}{M}}\,\underline{\ddot{z}} + \hat{\underset{\sim}{K}}\,\underline{z} = \underline{0}$$

Si può dimostrare che $\hat{\underset{\sim}{M}}$ e $\hat{\underset{\sim}{K}}$ sono DIAGONALI.

$$\hat{\underset{\sim}{M}} = \begin{bmatrix} \hat{m}_1 & & & 0 \\ & \hat{m}_2 & & \\ & & \ddots & \\ 0 & & & \hat{m}_N \end{bmatrix} ; \quad \hat{\underset{\sim}{K}} = \begin{bmatrix} \hat{k}_1 & & & 0 \\ & \hat{k}_2 & & \\ & & \ddots & \\ 0 & & & \hat{k}_N \end{bmatrix}$$

$$N \begin{cases} \hat{m}_1 \ddot{z}_1 + \hat{k}_1 z_1 = 0 \quad (*) \\ \hat{m}_2 \ddot{z}_2 + \hat{k}_2 z_2 = 0 \\ \vdots \\ \hat{m}_N \ddot{z}_N + \hat{k}_N z_N = 0 \end{cases}$$

SIST. DISACCOPPIATO DI N EQUAZIONI.

N OSCILL. SEMPLIC.

$$\hat{m}_i = \underline{\phi}^{(i)T}\,\underset{\sim}{M}\,\underline{\phi}^{(i)}$$

$$\hat{k}_i = \underline{\phi}^{(i)T}\,\underset{\sim}{K}\,\underline{\phi}^{(i)}$$

$$(*) \quad \ddot{z}_1 + \omega_1^2 z_1 = 0$$

$$\left[\omega_1^2 = \frac{\hat{k}_1}{\hat{m}_1} = \frac{\underline{\phi}^{(1)T}\,\underset{\sim}{K}\,\underline{\phi}^{(1)}}{\underline{\phi}^{(1)T}\,\underset{\sim}{M}\,\underline{\phi}^{(1)}}\right]$$

RAPPORTO O QUOZIENTE DI RAYLEIGH

per OGNI OSCILLATORE

– Risposta con forzanti

$$\underset{\sim}{M}\,\ddot{\underline{q}} + \underset{\sim}{K}\,\underline{q} = \underline{F}(t)$$

$$\underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{M}\, \underset{\sim}{\Phi}}\, \ddot{\underline{z}} + \underbrace{\underset{\sim}{\Phi}^T \underset{\sim}{K}\, \underset{\sim}{\Phi}}\, \underline{z} = \underbrace{\underset{\sim}{\Phi}^T \underline{F}(t)}_{\hat{\underline{F}}(t)}$$

$$\hat{\underset{\sim}{M}}\,\ddot{\underline{z}} + \hat{\underset{\sim}{K}}\,\underline{z} = \hat{\underline{F}}(t)$$

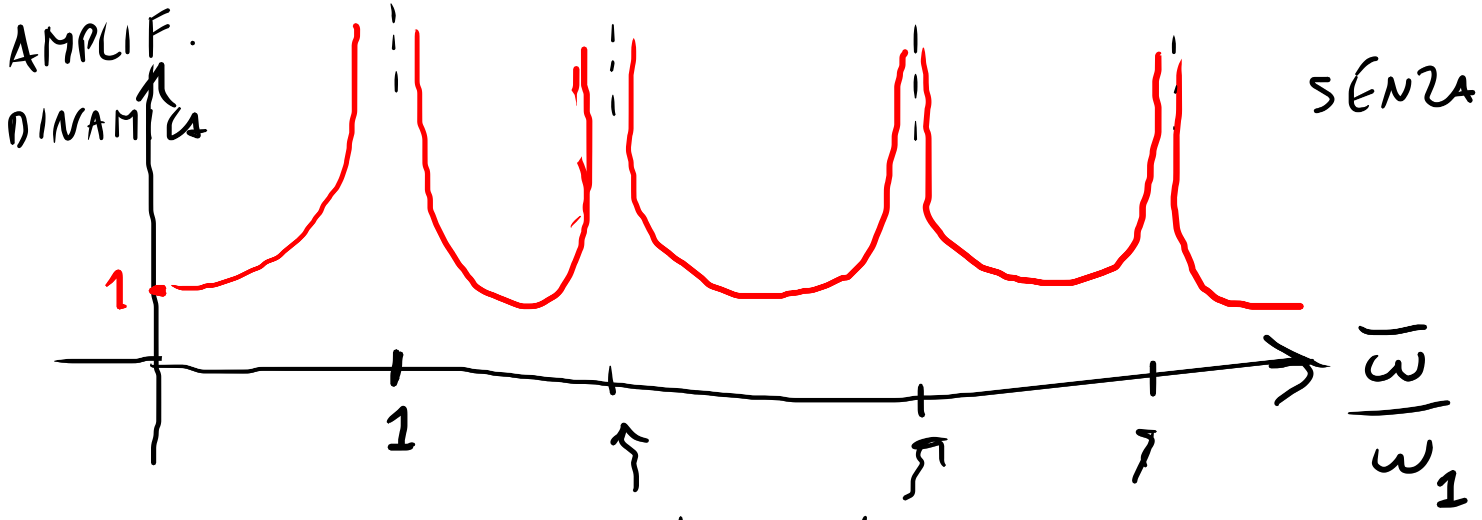
$$N\begin{cases} \hat{m}_1 \ddot{z}_1 + \hat{k}_1 z_1 = \hat{F}_1(t) \\ \vdots \\ \hat{m}_N \ddot{z}_N + \hat{k}_N z_N = \hat{F}_N(t) \end{cases}$$

$$\begin{cases} \ddot{z}_1 + \omega_1^2 z_1 = \dfrac{1}{\hat{m}_1}\hat{F}_1(t) \\ \vdots \\ \ddot{z}_N + \omega_N^2 z_N = \dfrac{1}{\hat{m}_N}\hat{F}_N(t) \end{cases}$$

CON FORZANTI SINUSOIDALI

$$\underline{F}(t) = \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} \sin \bar{\omega} t$$

ALLORA IL SIST. MOSTRA LA PRESENZA DI N CONDIZ. DI RISONANZA:

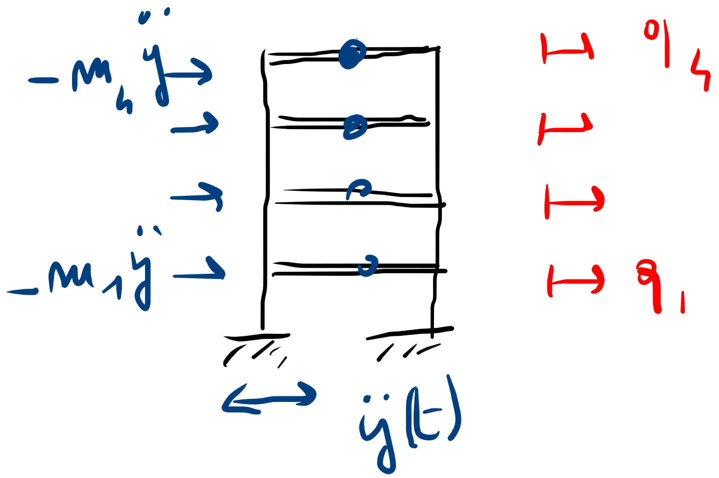


SENZA SMORZAM.

corrispondenza con $\omega_2, \omega_3, \omega_4$

RISONANZA $\bar{\omega} = \omega_i$

$\underline{F}(t)$ ottenuta da moto impresso alla base:



VETTORE UNITARIO

$$\underline{F}(t) = -\underset{\sim}{M}\,\underline{1}\,\ddot{y}(t)$$

$$\hat{\underline{F}}(t) = -\underset{\sim}{\Phi}^T\,\underset{\sim}{M}\,\underline{1}\,\ddot{y}(t)$$

Nel sist. principali l'eq. dell'oscill. diventa

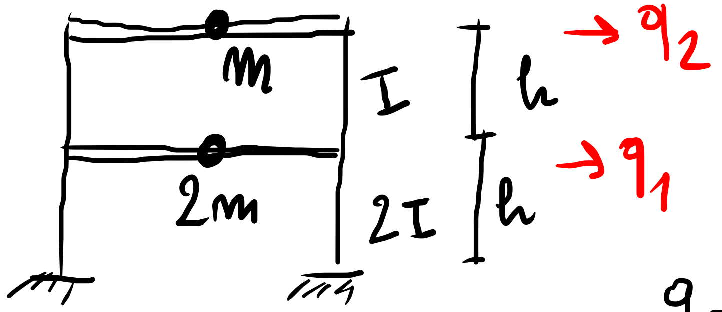
$$\hat{m}_1\,\ddot{z}_1 + \hat{K}_1 z_1 = -\underline{\phi}^{(1)}\cdot\underset{\sim}{M}\,\underline{1}\,\ddot{y}(t)$$

$$\ddot{z}_1 + \omega_1^2 z_1 = -\underbrace{\frac{\underline{\phi}^{(1)}\cdot\underset{\sim}{M}\,\underline{1}}{\hat{m}_1}}_{g_1}\,\ddot{y}(t)$$

g_1: COEFFICIENTE DI PARTECIPAZIONE MODALE

$$g_1 = \frac{m_1\phi_1^{(1)} + m_2\phi_2^{(1)} + m_3\phi_3^{(1)} \ldots}{m_1\phi_1^{(1)}\phi_1^{(1)} + m_2\phi_2^{(1)}\phi_2^{(1)}}$$ se $\underset{\sim}{M}$ è DIAGONALE

ES



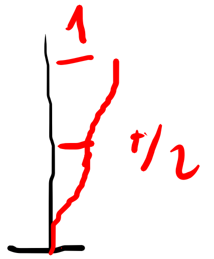
TELAIO SHEAR-TYPE

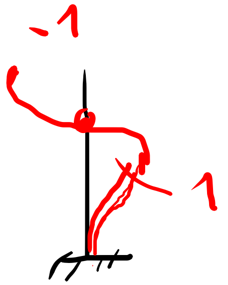
$$\underset{\sim}{M} = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

$$\underset{\sim}{K} = \underbrace{\frac{24\,EI}{h^3}}_{K} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{cases} \omega_1^2 = 1/2 \; K/m \\ \omega_2^2 = 2 \; K/m \end{cases}$$

$$\underset{\sim}{\phi}^{(1)} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \qquad \underset{\sim}{\phi}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$





$$\underline{q} = \underset{\sim}{\Phi}\, \underline{z}$$

$$\begin{rcases} q_1 = \frac{z_1}{2} + z_2 \\ q_2 = z_1 - z_2 \end{rcases}$$

$$\underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} = \underset{\sim}{\Phi}^T$$ (CASO PARTICOLARE)

$$\underset{\sim}{\hat{M}} = \underset{\sim}{\Phi}^T \underline{M}\, \underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 & 0 \\ 0 & 3 \end{bmatrix} m$$

$$\begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} m & 2m \\ m & -m \end{bmatrix} = \begin{bmatrix} 3/2\, m & 0 \\ 0 & 3m \end{bmatrix}$$ OK

$$\underset{\sim}{\hat{K}} = \underset{\sim}{\Phi}^T \underset{\sim}{K}\, \underset{\sim}{\Phi} = K \begin{bmatrix} 3/4 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\begin{cases} \overbrace{\tfrac{3}{2} m}^{\hat{m}_1} \ddot{z}_1 + \overbrace{\tfrac{3}{4} K}^{\hat{K}_1} z_1 = 0 \\ 3m\, \ddot{z}_2 + 6K\, z_2 = 0 \end{cases}$$ IN COORD PRINC.

$$\omega_1^2 = \frac{3}{4} K \frac{2}{3m} = \frac{K}{m} \, 1/2$$ OK

$$\omega_2^2 = 6K/3m = 2\, K/m$$ OK

moto impresso alla base:

$$\underline{F}(t) = -\begin{bmatrix} 2 \\ 1 \end{bmatrix} m\, \ddot{y}(t)$$

OCCHIO $\underset{\sim}{\Phi} = \underset{\sim}{\Phi}^T$ IN QUESTO ESEMPIO

$$\hat{\underline{F}}(t) = \underset{\sim}{\Phi}^T \underline{F}(t) = -\begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} m\, \ddot{y}(t) = -\begin{bmatrix} 2 \\ 1 \end{bmatrix} m\, \ddot{y}(t)$$

$$\begin{cases} \frac{3}{2} m \ddot{z}_1 + \frac{3}{4} K z_1 = -2 m \ddot{y}(t) \\ 3 m \ddot{z}_2 + 6 K z_2 = -m \ddot{y}(t) \end{cases}$$

$$\begin{cases} \ddot{z}_1 + \omega_1^2 z_1 = -\underbrace{4/3}_{g_1}\, \ddot{y}(t) \\ \ddot{z}_2 + \omega_2^2 z_2 = -\underbrace{1/3}_{g_2}\, \ddot{y}(t) \end{cases}$$

ORTOGONALITÀ DEI MODI DI VIBRARE

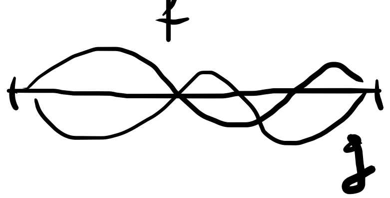
$\underline{u} \perp \underline{v}$ se $\underline{u} \cdot \underline{v} = 0$

$$\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(j)} = \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} = 0 \quad \text{se } i \neq j$$

(ORTOGONALITÀ GENERALIZZ.)

$\underline{u} \cdot \underset{\sim}{I}\, \underline{v} = 0$

DIM. $\quad (\underset{\sim}{K} - \omega^2 \underset{\sim}{M})\, \underline{\phi} = \underline{0}$



$$(\underset{\sim}{K} - \omega_i^2 \underset{\sim}{M})\, \underline{\phi}^{(i)} = \underline{0} \Rightarrow \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2 \underset{\sim}{M}\, \underline{\phi}^{(i)} \Rightarrow \underline{\phi}^{(j)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2\, \underline{\phi}^{(j)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \quad (*)$$

$$(\underset{\sim}{K} - \omega_j^2 \underset{\sim}{M})\, \underline{\phi}^{(j)} = \underline{0} \Rightarrow \underset{\sim}{K}\, \underline{\phi}^{(j)} = \omega_j^2 \underset{\sim}{M}\, \underline{\phi}^{(j)} \Rightarrow \underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(j)} = \omega_j^2\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)}$$

$\int f g \, dx = 0$

$f \perp g$

$\underset{\sim}{K}$ e $\underset{\sim}{M}$ SIMMETRICHE $\Rightarrow \quad 0 = (\omega_i^2 - \omega_j^2)\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)}$

se $\omega_i \neq \omega_j$ allora $\underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(j)} = 0 \quad (i \neq j)$

Se ~~a~~desso considero $(*)$, se $i \neq j$ allora anche il primo membro è nullo $\Rightarrow \underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(j)} = \underline{0} \quad (i \neq j)$ ∎

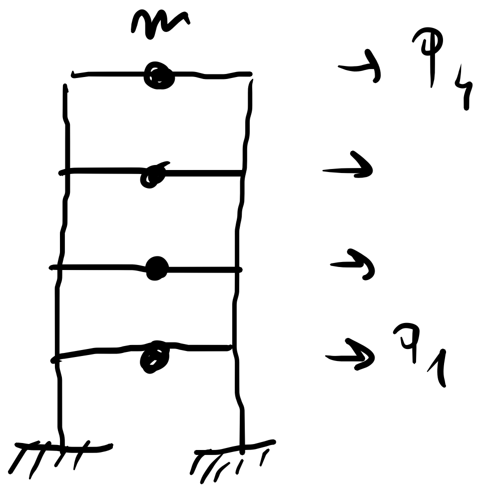
Attenzione. Considero la (*) della pagina precedente con $i=j$:

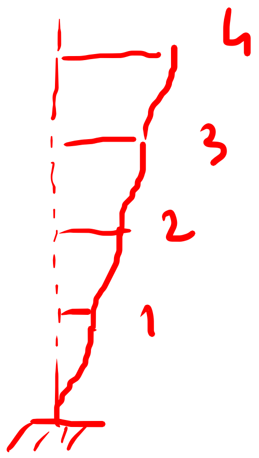
$$\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \Rightarrow \omega_i^2 = \frac{\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)}}{\underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)}}$$

RAPPORTO DI RAYLEIGH

ATTRAVERSO LE PROPRIETÀ DI ORTOGONALITÀ DEI MODI DI VIBRARE POSSO DEFINIRE LE COORDINATE PRINCIPALI.

Posso stimare ω_i^2 scegliendo un vettore $\underline{\phi}^{(i)}$ DI TENTATIVO se NON CONOSCO ESATTAMENTE IL MODO i-SIMO.





$\underline{\phi}$ DI TENTATIVO PER IL CALCOLO DI ω_1^2