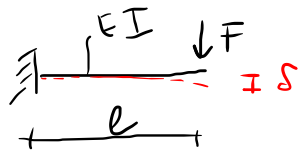


RIPASSO SU COEFFICIENTI DI RIGIDEZZA

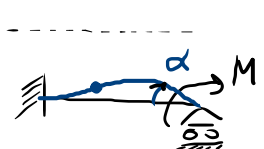
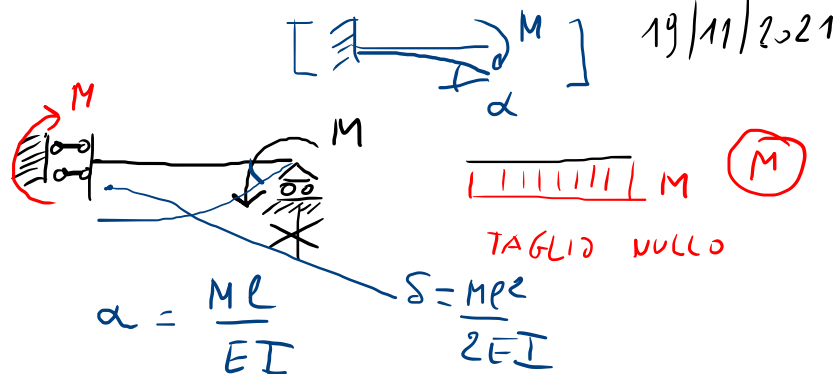
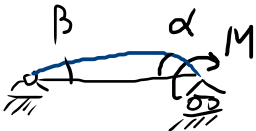


$$\delta = \frac{F l^3}{3 E I}$$

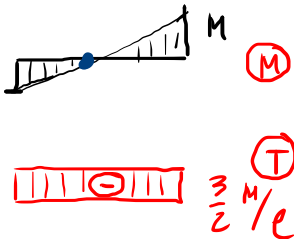
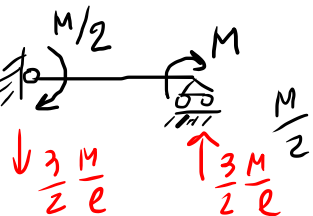
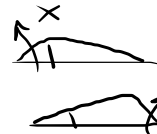


$$\alpha = \frac{M l}{3 E I}$$

$$\beta = \frac{M l}{6 E I}$$

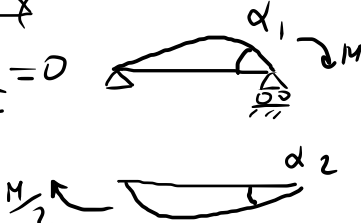


$$\alpha = \frac{M l}{4 E I}$$

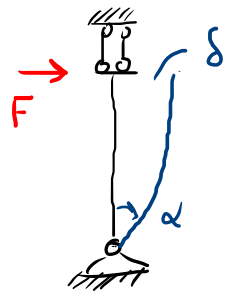


$$\sum \phi_A = 0 : \frac{X l}{3 E I} + \frac{M l}{6 E I} = 0$$

$$X = -M/2$$

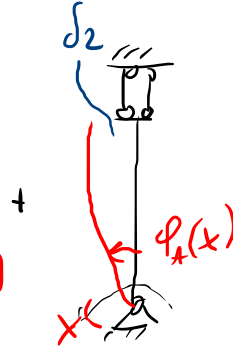
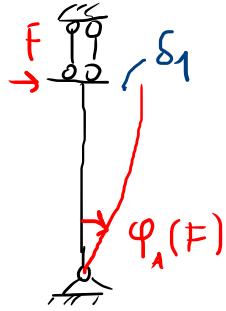
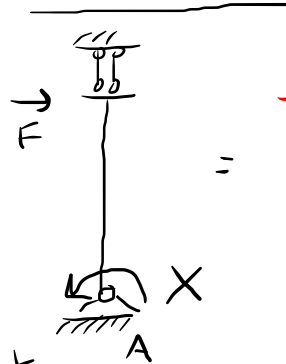
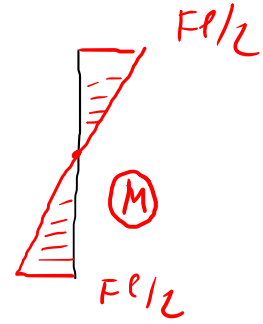
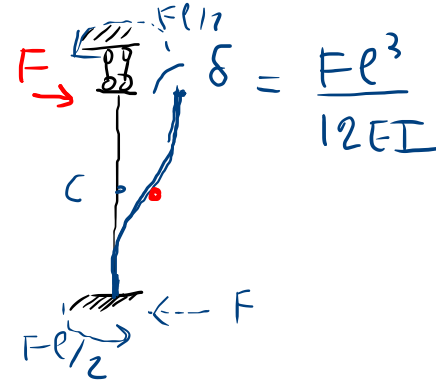
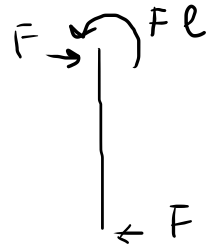


$$\alpha = \frac{M l}{3 E I} - \frac{M}{2} \frac{l}{6 E I} = \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4} M l / E I$$



$$\delta = \frac{F l^3}{3EI}$$

$$\alpha = \frac{F l^2}{2EI}$$

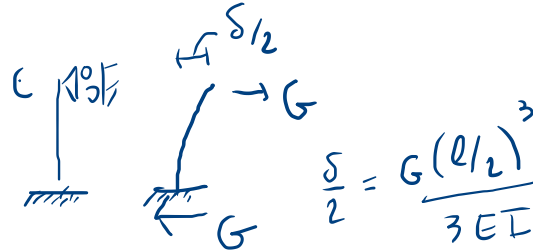


$$\delta = \frac{F l^3}{3EI} - \frac{F l}{2} \frac{l^2}{2EI} = \frac{4-3}{12} = \frac{1}{12} \frac{F l^3}{EI}$$

$$\varphi_A = 0$$

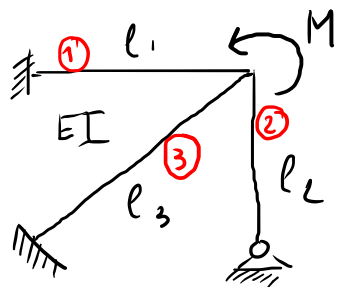
$$X = \frac{F l}{2}$$

$$\varphi_A = \frac{F l^2}{2EI} - \frac{X l}{EI} = 0$$



$$\frac{\delta}{2} = \frac{G (l/2)^3}{3EI}$$

1°) ESEMPIO ; RIGIDEZZA DI UN MODO

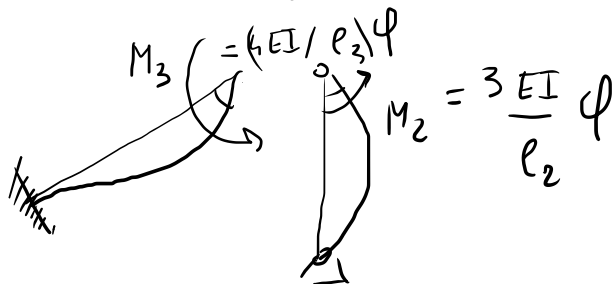


$$M \leftrightarrow \varphi$$

METODO
SPOSTAM.



$$M_1 = 4 \frac{EI}{l_1} \varphi$$



$$M_3 = 4 \frac{EI}{l_3} \varphi$$

$$M_2 = 3 \frac{EI}{l_2} \varphi$$

$$M = M_1 + M_2 + M_3$$

$$= \left(4 \frac{EI}{l_1} + 3 \frac{EI}{l_2} + 4 \frac{EI}{l_3} \right) \varphi$$

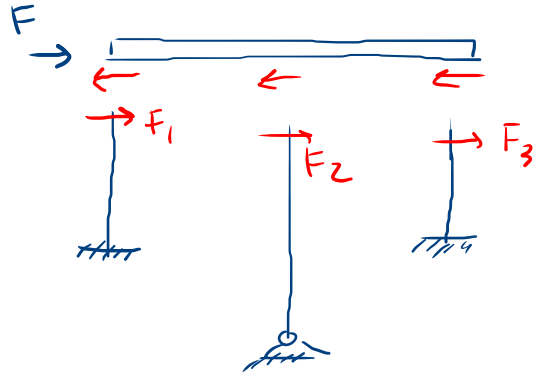
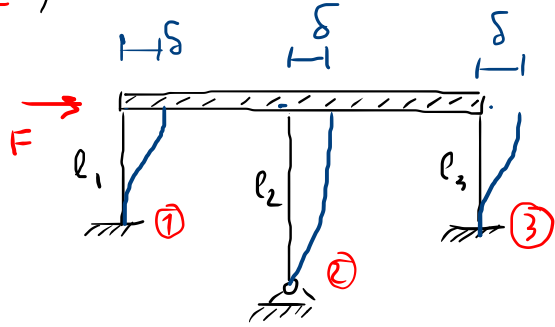
$$\varphi = \frac{M}{\left(\right)}$$

RIGIDEZZA TOTALE DEL
MODO.

$$M = K \varphi$$

↑
RIGIDEZZA
ELASTICA
ROTAZ.

2°) ESEMPIO



$$F_1 = 12 \frac{EI}{l_1^3} \delta$$

$$F_2 = 3 \frac{EI}{l_2^3} \delta$$

$$F_3 = 12 \frac{EI}{l_3^3} \delta$$

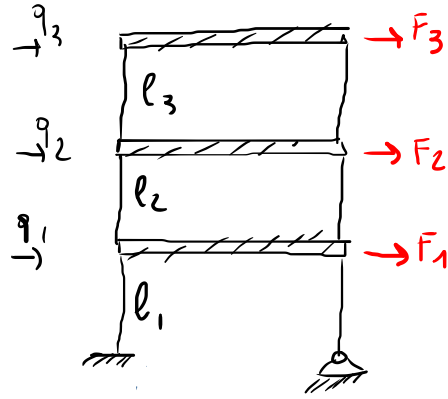
$$F = F_1 + F_2 + F_3$$

$$F = \left(12 \frac{EI}{l_1^3} + 3 \frac{EI}{l_2^3} + 12 \frac{EI}{l_3^3} \right) \delta$$

RIGIDEZZA DI PIANO

$$F = K_T \delta$$

3° ESEMPIO



$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} & & \\ & \tilde{K} & \\ & & \end{bmatrix}_{3 \times 3} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

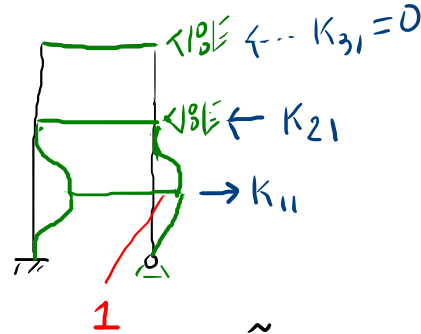
$$\underline{F} = \underline{K} \underline{q}$$

$$\underline{q} = \underline{K}^{-1} \underline{F}$$

$$\underline{K} = \underline{K}^T \text{ PER IL T.D. DI BETTI.}$$

K_{ij} : È LA FORZA CORRETTA ALLO SPOST. i -SIMO QUANDO $q_j = 1$ e tutti gli altri q sono NULLI.

$$[q_1 = 1, q_2 = q_3 = 0]$$

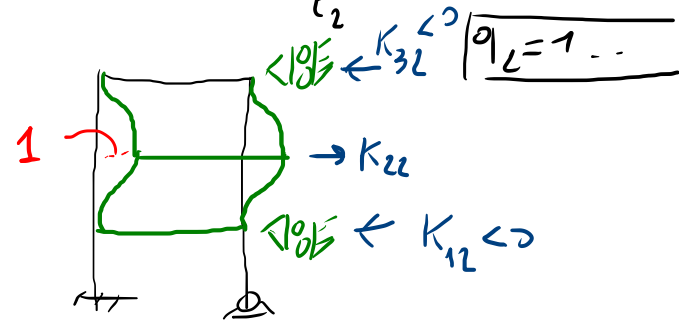


$$K_{21} = -\frac{24EI}{l_2^3}$$

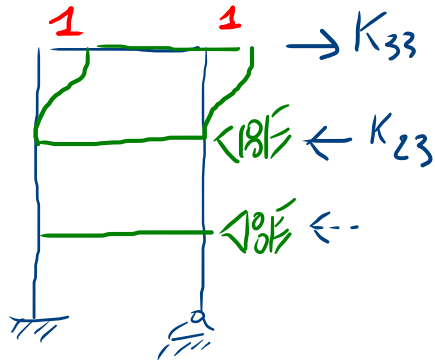
$$K_{11} = \sum_{\alpha=1}^n \tilde{K}_{\alpha} = \frac{15EI}{l_1^3} + \frac{24EI}{l_2^3} \quad (*)$$

$$\tilde{K}_1 = \frac{12EI}{l_1^3}; \quad \tilde{K}_2 = \frac{3EI}{l_1^3}; \quad \tilde{K}_3 = \tilde{K}_4 = \frac{12EI}{l_2^3}$$

$$\begin{bmatrix} (*) & -\frac{24EI}{l_2^3} & 0 \\ -\frac{24EI}{l_2^3} & \tilde{K}_{22} & -\frac{24EI}{l_2^3} \\ 0 & -\frac{24EI}{l_2^3} & \frac{24EI}{l_2^3} \end{bmatrix}$$



$$q_3 = 0$$

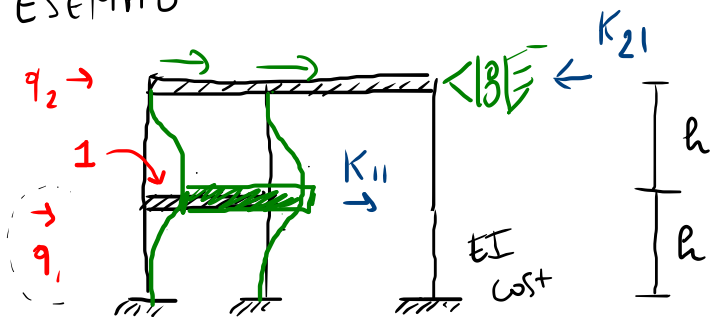


$$K_{33} = \frac{24 EI}{l_3^3}$$

$$K_{23} = - \frac{24 EI}{l_3^3}$$

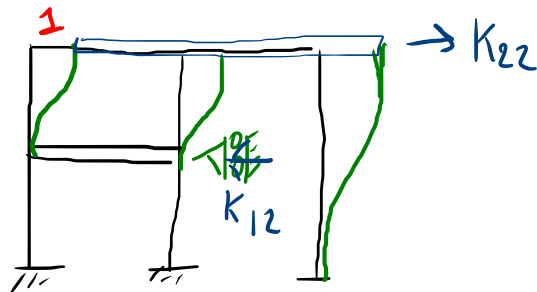
Diagram illustrating the stiffness coefficients K_{33} and K_{23} for a frame structure. The top beam is labeled with a red '1' at both ends. The left column is fixed at the bottom, and the right column is pinned at the bottom. Green curved arrows indicate moments at the joints. Labels include K_{33} pointing right, K_{23} pointing left, and a green arrow pointing left.

ESEMPIO



$$K_{11} = 48 \frac{EI}{h^3}$$

$$K_{21} = -24 \frac{EI}{h^3}$$



$$K_{22} = 24 \frac{EI}{h^3} + 12 \frac{EI}{(2h)^3} = \left(24 + \frac{3}{2}\right) \frac{EI}{h^3}$$

$$K_{12} = -24 \frac{EI}{h^3}$$