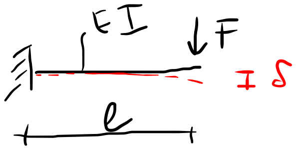
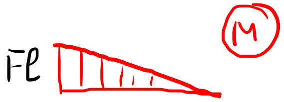
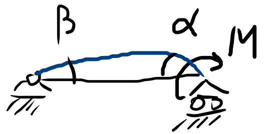
RIPASSO SU COEFFICIENTI DI RIGIDEZZA

19/11/2021

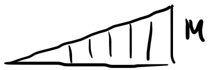


$$\delta = \frac{F l^3}{3EI}$$

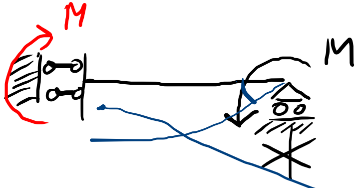


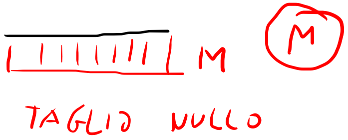


$$\alpha = \frac{M l}{3EI}$$

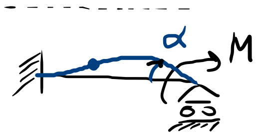


$$\beta = \frac{M l}{6EI}$$

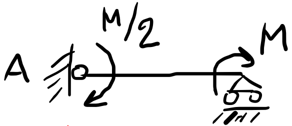




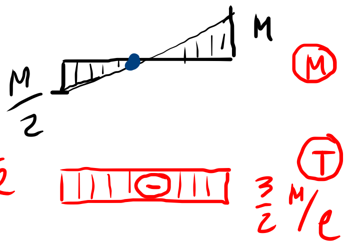
$$\alpha = \frac{M l}{EI} \qquad \delta = \frac{M l^2}{2EI}$$

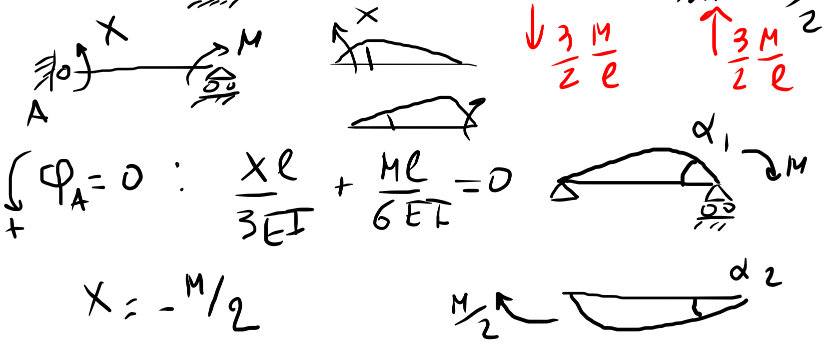


$$\alpha = \frac{M l}{4EI}$$



$$\downarrow \frac{3}{2}\frac{M}{l} \qquad \uparrow \frac{3}{2}\frac{M}{l}$$

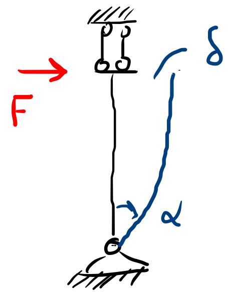




$$\varphi_A = 0 : \quad \frac{X l}{3EI} + \frac{M l}{6EI} = 0$$

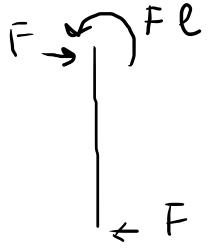
$$X = -M/2$$

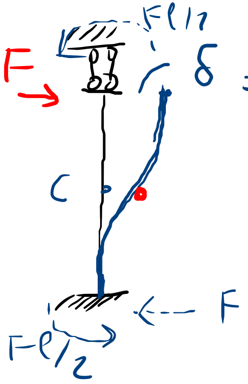
$$\alpha = \frac{M l}{3EI} - \frac{M}{2}\frac{l}{6EI} = \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}\, M l / EI$$



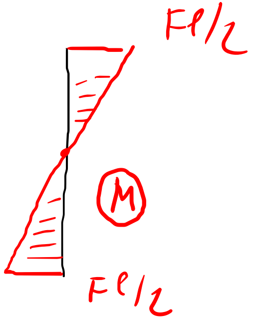
$$\delta = \frac{F\ell^3}{3EI}$$

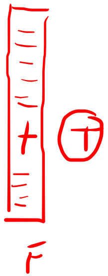
$$\alpha = \frac{F\ell^2}{2EI}$$

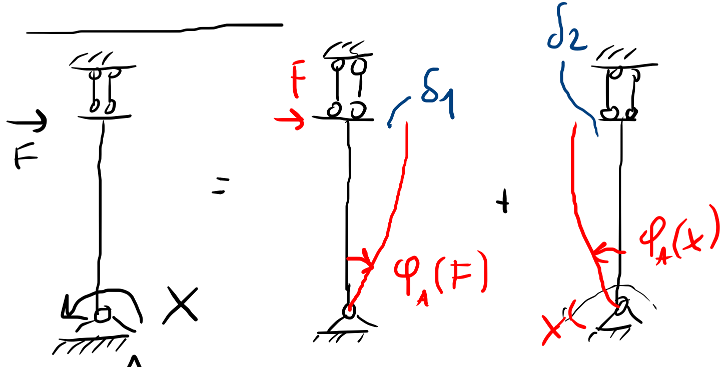




$$\delta = \frac{F\ell^3}{12EI}$$





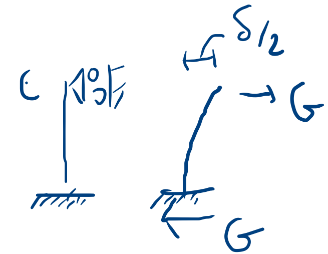


$+\ \varphi_A = 0$

$$\varphi_A = \frac{F\ell^2}{2EI} - \frac{X\ell}{EI} = 0$$

$$X = \frac{F\ell}{2}$$

$$\delta = \frac{F\ell^3}{3EI} - \frac{F\ell}{2}\,\frac{\ell^2}{2EI} = \frac{4-3}{12} = \frac{1}{12}\,\frac{F\ell^3}{EI}$$

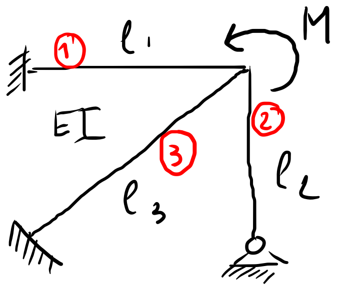


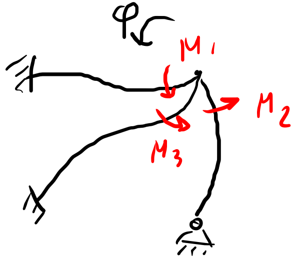
$$\frac{\delta}{2} = \frac{G(\ell/2)^3}{3EI}$$

1°) ESEMPIO ; RIGIDEZZA DI UN NODO

$$M = K\varphi$$

RIGIDEZZA ELASTICA ROTAZ.



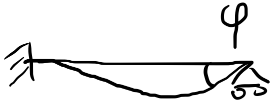


$$M = M_1 + M_2 + M_3$$

$$= \underbrace{\left(4\frac{EI}{\ell_1} + 3\frac{EI}{\ell_2} + 4\frac{EI}{\ell_3}\right)}_{\text{RIGIDEZZA TOTALE DEL NODO.}}\varphi$$

$M \leftrightarrow \varphi$ METODO SPOSTAM.

$$\varphi = \frac{M}{(\quad)}$$

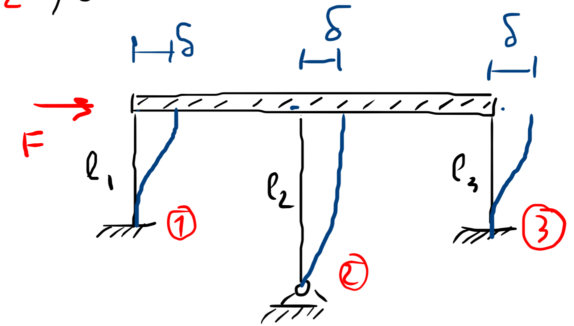


$$M_1 = 4\frac{EI}{\ell_1}\varphi$$

$$M_3 = (4EI/\ell_3)\varphi$$

$$M_2 = 3\frac{EI}{\ell_2}\varphi$$

2°) ESEMPIO

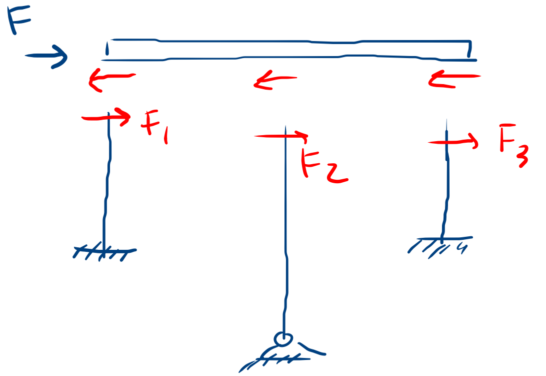


$$F_1 = 12\,\frac{EI}{l_1^3}\,\delta$$

$$F_2 = 3\,\frac{EI}{l_2^3}\,\delta$$

$$F_3 = 12\,EI/l_3^2\;\delta$$

$$F = K_T\,\delta$$

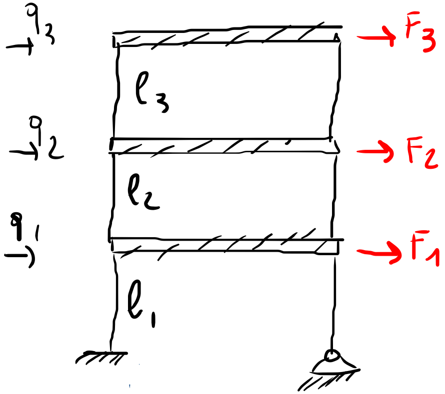


$$F = F_1 + F_2 + F_3$$

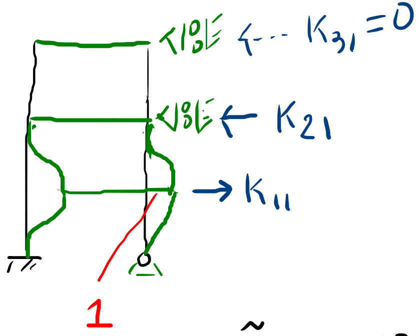
$$F = \underbrace{\left(12\,\frac{EI}{l_1^3} + 3\,\frac{EI}{l_2^3} + 12\,\frac{EI}{l_3^3}\right)}_{\text{RIGIDEZZA DI PIANO}}\delta$$

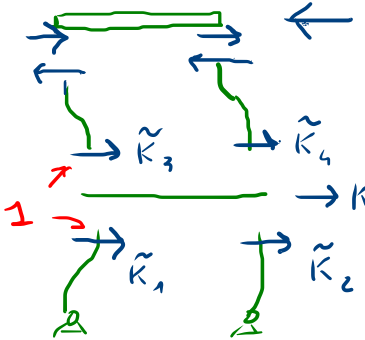
3° ESEMPIO

K_{ij} : È LA FORZA CORRELATA ALLO SPOST. i-SIMO QUANDO $q_j = 1$ e tutti gli altri gdl sono NULLI.



$\boxed{q_1 = 1,\ q_2 = q_3 = 0}$





$K_{21} = -24\,EI/\ell_2^3$

$K_{11} = \sum_{\alpha=1}^{4} \tilde{K}_\alpha = \overbrace{\frac{15\,EI}{\ell_1^3} + \frac{24\,EI}{\ell_2^3}}^{*}$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} & & \\ & \underset{\sim}{K} & \\ & & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$\underline{F}$ 3×3 $\underline{q}$

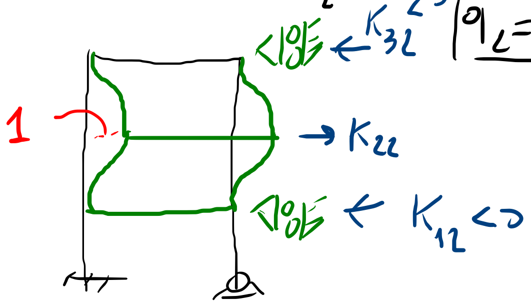
$\underline{q} = \underset{\sim}{K}^{-1}\, \underline{F}$

$\underset{\sim}{K} = \underset{\sim}{K}^T$ PER IL T. DI BETTI.

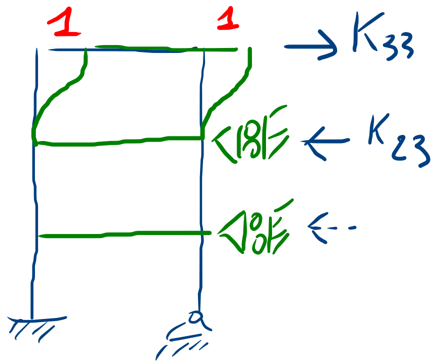
$\tilde{K}_1 = \frac{12\,EI}{\ell_1^3}$; $\tilde{K}_2 = \frac{3\,EI}{\ell_1^3}$; $\tilde{K}_3 = \tilde{K}_4 = \frac{12\,EI}{\ell_2^3}$

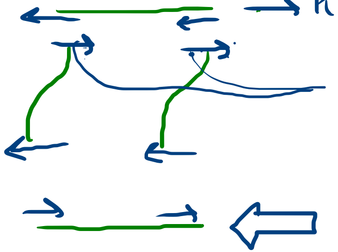
$$\begin{bmatrix} * & -24\frac{EI}{\ell_2^3} & 0 \\ -24\frac{EI}{\ell_2^3} & \sim & -24\,EI/\ell_3^3 \\ 0 & \sim \quad 24\,EI/\ell_3^3 & \end{bmatrix}$$

$\boxed{q_2 = 1 \ldots}$



$q_3 = 0$

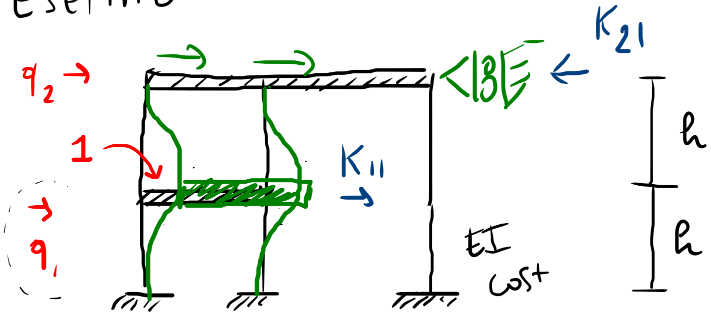


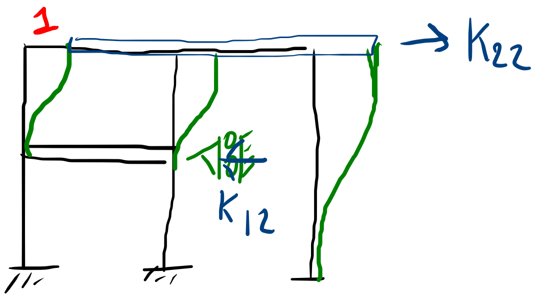


$\rightarrow K_{33} = 24\frac{EI}{l_3^3}$

$\Leftarrow K_{23} = -24\frac{EI}{l_3^3}$

ESEMPIO





$$K_{11} = 48\,\frac{EI}{h^3}$$

$$K_{21} = -24\,\frac{EI}{h^3}$$

$$K_{22} = 24\,\frac{EI}{h^3} + 12\,\frac{EI}{(2h)^3} = \left(24 + \frac{3}{2}\right)\frac{EI}{h^3}$$

$$K_{12} = -24\,\frac{EI}{h^3}$$