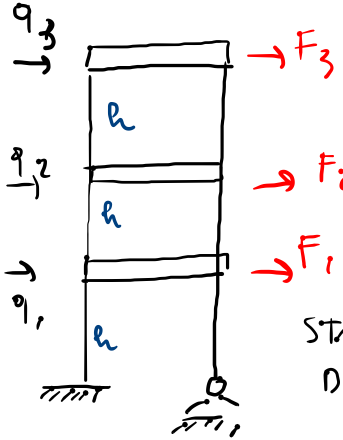
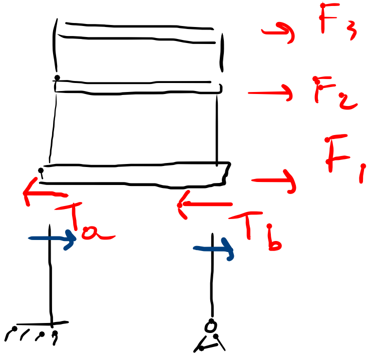
…. MATRICI DI RIGIDEZZA (e METODO DEGLI SPOSTAMENTI) 23/11/21



$$\underset{\sim}{F} = \underset{\approx}{K}\, \underset{\sim}{q}$$

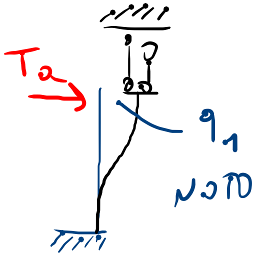
$$\underset{\sim}{q} = \underset{\approx}{K}^{-1} \underset{\sim}{F} \rightarrow q_1, q_2, q_3$$

STATO DI SOLLECIT. (I PIANO) DEI 2 PILASTRI

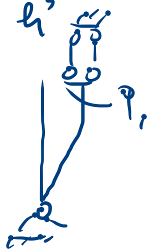


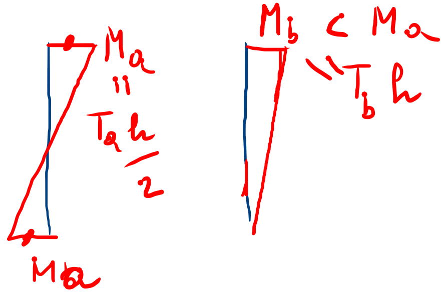
$$T_a + T_b = F_1 + F_2 + F_3$$

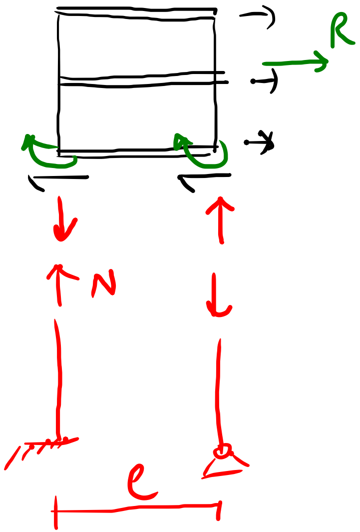
IL TAGLIO AL PIANO I



$$T_a = 12\,\frac{EI}{h^3}\, q_1$$

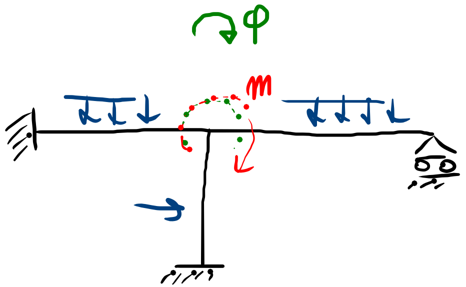
$T_b =$  $= 3\,\dfrac{EI}{\ell^3}\, q_1$





NB: MOM CHE EQUILIBRA IL CORPO RIGIDO

ES

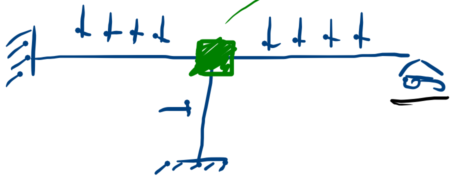


$m = K_{NODO}\,\varphi$

$(\Sigma\, K$ singole aste$)$

PER RISOLV. IL PROBLEMA (M, T, N IN OGNI TRAVE) ..

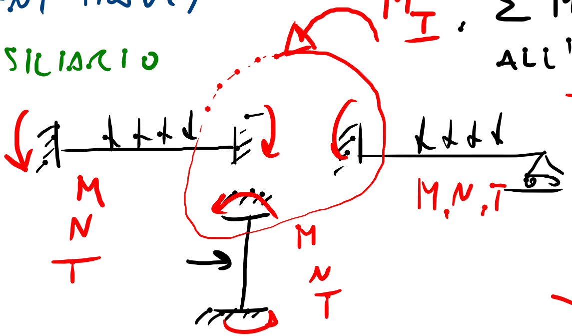
I)



INCASTRO AUSILIARIO

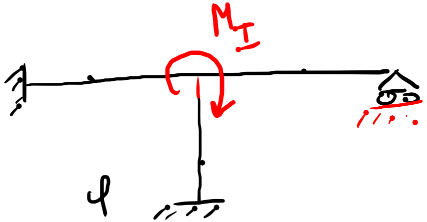
$\boxed{\varphi = 0}$

M_I : Σ Momenti ALL'INCASTRO AUSILIARIO

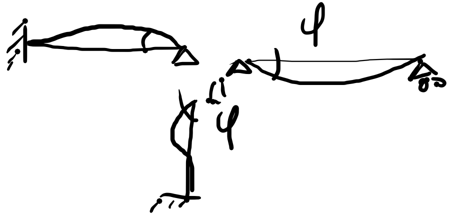


I

II)

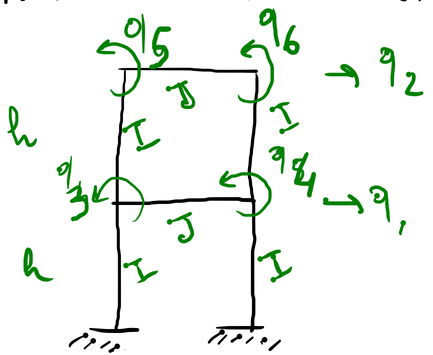


$$\varphi = \frac{M_I}{K_{NODO}}$$



M, T, N schema II

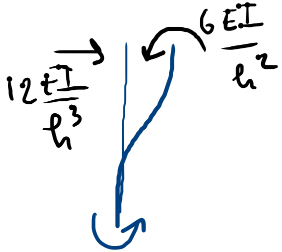
MATRICE DI RIGIDEZZA 6x6

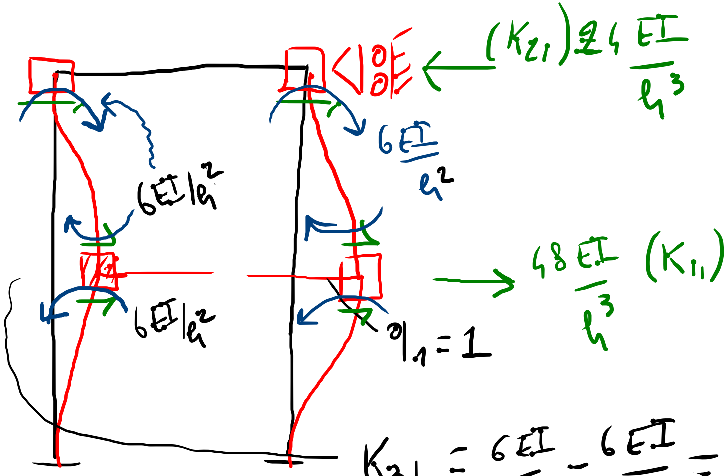


K_{i1}

K_{ij} : FORZA (GENERALIZZ) CORRELATA AL GDL i-SIMO QUANDO IL GDL j-SIMO VALE 1 E I TUTTI GLI ALTRI SONO NULLI

$$\underset{\sim}{K} = \begin{bmatrix} 48\,EI/h^3 \\ -24\,EI/h^3 \\ 0 \\ 0 \\ -6\,EI/h^2 \\ -6\,EI/h^2 \end{bmatrix}$$





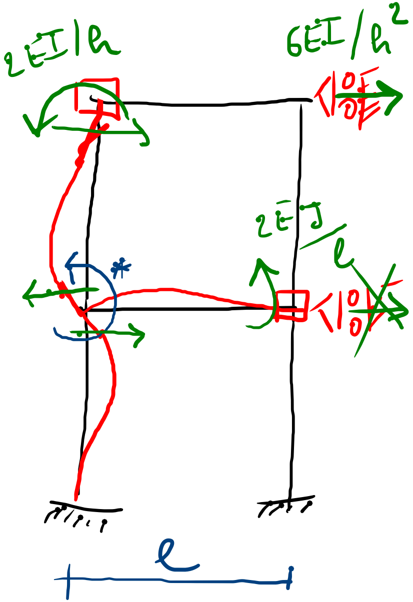
$q_2 = \dots q_6 = 0$

$$K_{31} = \frac{6EI}{h^2} - \frac{6EI}{h^2} = 0$$

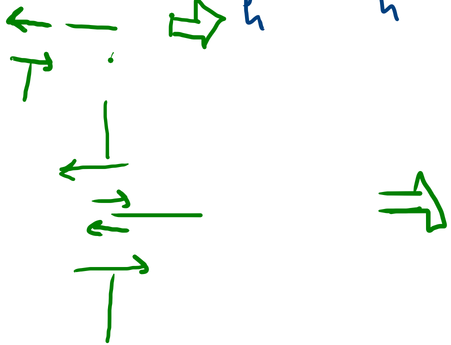
$K_{41} = 0$

$$K_{51} = -\frac{6EI}{h^2}$$

$q_3 = 1,\ q_1 = q_2 = q_4 = \dots = 0$

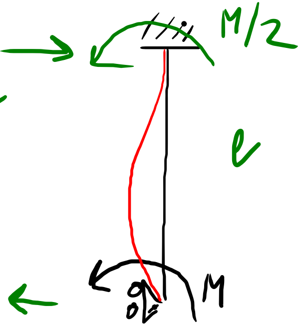


$* = K_{33} = 4\frac{EI}{h} + 4\frac{EI}{h} + 4\frac{EJ}{\ell}$



$\frac{6EI\varphi}{\ell^2} = \frac{4EI}{\ell^2}\frac{3\varphi}{2} = \frac{3}{2}\frac{M}{\ell}$

$\varphi = \frac{M\ell}{4EI}$



$M = \frac{4EI}{\ell}\varphi$

TAGLIO SUI PILASTRI SONO UGUALI ED OPPOSTI

$K_{13} = 0$

$K_{23} = +\frac{6EI}{h^2}$

$K_{43} = +\frac{2EJ}{\ell}$

$K_{53} = +\frac{2EI}{h}$

$K_{63} = 0$

COLONNA 3

$$K = \begin{bmatrix} \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \frac{6EI}{h^2} & \cdot & \cdot \\ \cdot & \cdot & \frac{8EI}{h} + \frac{4EJ}{\ell} & \cdot & \cdot \\ \cdot & \cdot & \frac{2EJ}{\ell} & \cdot & \cdot \\ \cdot & \cdot & 2EI/h & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{bmatrix} = \begin{bmatrix} \\ \\ & K & \\ \\ \\ \end{bmatrix} \begin{array}{l} \begin{bmatrix} q_1 \\ q_2 \\ \hline q_3 \\ \vdots \\ q_6 \end{bmatrix} \end{array} \begin{array}{l} \text{TRASL} \\ \\ \text{ROTAZ.} \end{array}$$

$$M_5 = K_{51} q_1 + K_{52} q_2 + K_{53} q_3 \dots$$

$$= -\frac{6EI}{l^2} q_1 + \frac{6EI}{l^2} q_2 + \frac{2EI}{l} q_3 + \dots\dots$$

$$\left[\frac{EI}{l^2} q_1\right] = \left[\frac{F}{L^2}\frac{L^4}{L^2} L\right] = [FL]$$

$$\left[\frac{EI}{l} q_3\right] = \left[\frac{F}{L^2}\frac{L^4}{L} -\right] = [FL]$$

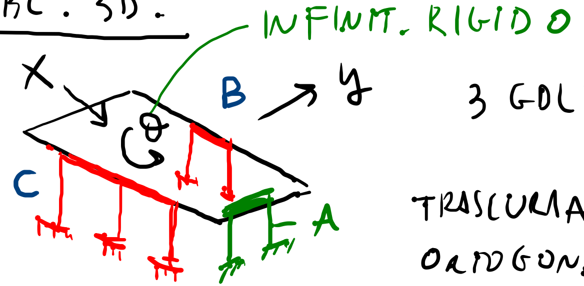
$$F_1 = \underbrace{\frac{48EI}{l^3} q_1}_{\text{FORZE}} + \dots + \underbrace{0\, q_3}_{\text{FORZE}} + \dots$$

MATR. RIGIDEZZA SYM + DEFINITA POSITIVA

$$E_{el} = \frac{1}{2}\, \underline{q} \cdot \underset{\sim}{K}\, \underline{q} \geq 0 \quad \forall \underline{q} \neq 0$$

$$= \frac{1}{2}\, \underline{q} \cdot \underset{\sim}{K}^T \underline{q}$$

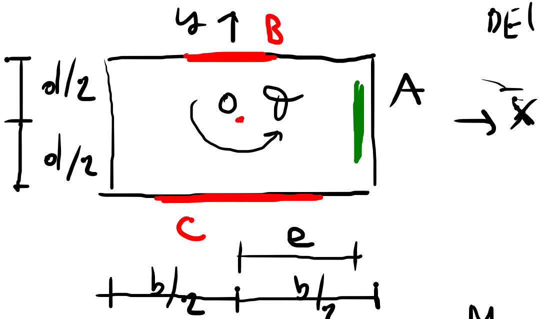
PROBL. 3D.



3 GDL: x, y, ϑ

$$\underline{F} = \underset{\sim}{K}\,\underline{q} \Rightarrow \begin{bmatrix} F_x \\ F_y \\ M_\vartheta \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} & K_{x\vartheta} \\ & K_{yy} & \\ & & K_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} x \\ y \\ \vartheta \end{bmatrix}$$

TRASCURIAMO LE RIGIDEZZE ORTOGONALI AI PIANI "FORTI" DEI TELAI



$$(-\omega^2 \underset{\sim}{M} + \underset{\sim}{K})\,\underline{q} = \underline{0}$$

$$\underset{\sim}{M} = \begin{bmatrix} m & & 0 \\ & m & \\ 0 & & I_0 \end{bmatrix}$$

m: massa del piano

I_0: MOM D'INERZIA POLARE DELLA "LAMINA" $= m\,\dfrac{b^2+d^2}{12}$