

$$\bullet \frac{d^2 m_x}{dx^2} = \frac{dt_x}{dx}$$

$$\bullet \textcircled{*} \frac{d^2 m_x}{dx^2} + \frac{m_\varphi}{R} = -q_z$$

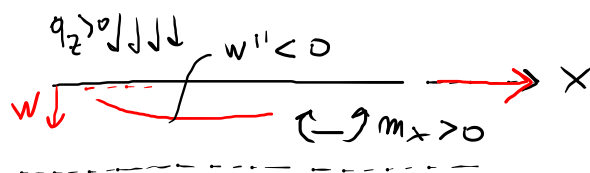
$$-D w^{IV}(x) - \frac{Es}{R^2} w(x) = -q_z$$

$$\left[D w^{IV}(x) + \frac{Es}{R^2} w(x) = q_z(x) \right]$$

"SUOCO ELASTICO": CONTRIBUTO
ALLA WINKLER

$$w^{IV}(x) + \frac{Es}{R^2 D} w(x) = \frac{q_z(x)}{D}$$

$$w^{IV}(x) + 4 \beta^4 w(x) = \frac{q_z(x)}{D}$$



$$m_x = -D w''(x) ;$$

$$m_\varphi = -\frac{Es}{R} w(x)$$

$$D = \frac{Es^3 \cdot 1}{12(1-\nu^2)}$$

PASTRE INFLESSE
COEFF. RIGIDEZZA
PASTRE

$$M = -EI w''(z) \quad 15/12/21$$

$$* (D w'')''$$

$$\beta = \sqrt[4]{\frac{Es}{4R^2 D}} = \sqrt[4]{\frac{3(1-\nu^2)}{R^2 s^2}} \approx \frac{1.3}{\sqrt{R s}} \textcircled{O}$$

$$[\beta] = [L^{-1}] ; \lambda = \frac{2\pi}{\beta} \approx 4.83 \sqrt{R s} \textcircled{O}$$

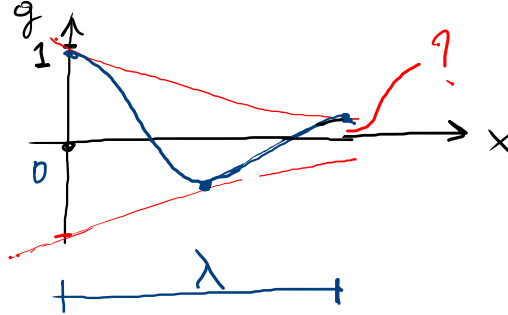
β : NUMERO D'ONDA

λ : LUNGH. D'ONDA.

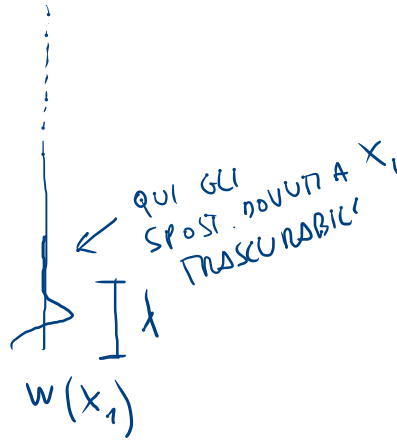
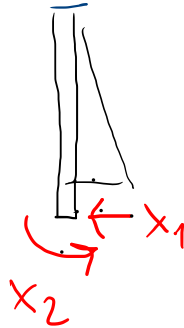
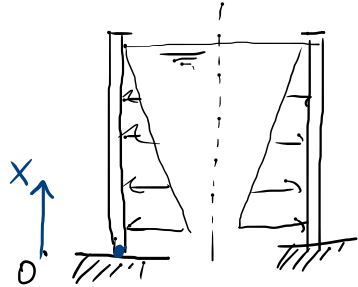
$$w(x) = e^{\beta x} (c_1 \cos \beta x + c_2 \sin \beta x) + \overbrace{e^{-\beta x} (c_3 \cos \beta x + c_4 \sin \beta x)}^{\text{INT. PARTIC.}} + w_p$$

+ 4. CONDIZ. CONFINAMENTO (c_1, c_2, c_3, c_4)

$$g(x) = e^{-\beta x} \cos \beta x ; \quad g(\lambda) ? \approx \frac{1}{535} ; \quad \lambda = \frac{2\pi}{\beta}$$



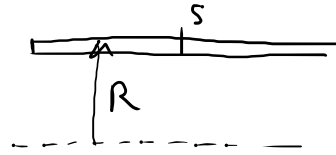
FORTE SMORZAMENTO
DEGLI "EFFETTI" ALL'ORIGINE



$$\begin{cases} \text{EQ. DI CONGA} \\ w(0) = 0 \\ w'(0) = 0 \end{cases}$$

$$w(x) = e^{-\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + w_p$$

i termini in C_1 e C_2 si eliminano.

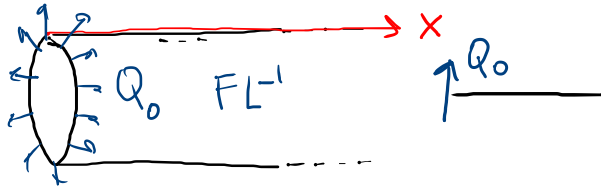


$$\lambda ? \quad R = 1 \text{ m} \\ s = 2 \text{ cm} ; \quad \lambda \approx 68 \text{ cm}$$

$$R = 1 \text{ m} \quad \lambda \approx 48 \text{ cm} \\ s = 1 \text{ cm}$$

Se il cilindro è più lungo
di 2λ allora gli effetti
di bordo non si trasmettono
dall'altra parte.

STUDIO DI 2 PROBLEMI FONDAMENTALI

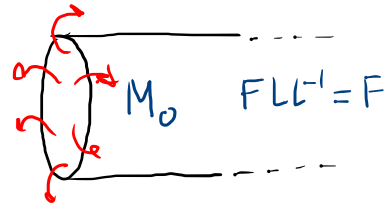


$$\begin{cases} -D w''(0) = 0 \\ -D w'''(0) = +Q_0 \end{cases}$$

$$\begin{cases} C_4 = 0 \\ C_3 = -\frac{Q_0}{2\beta^3 D} \end{cases}$$

$$m_x = -D w''(x) \quad \uparrow \square \uparrow$$

$$t_x = -D w'''(x) \quad \uparrow \square \downarrow$$



$$\begin{cases} -D w''(0) = +M_0 \\ -D w'''(0) = 0 \end{cases}$$

$$\begin{cases} C_3 = -\frac{M_0}{2\beta^2 D} \\ C_4 = +\frac{M_0}{2\beta^2 D} \end{cases}$$

$$\Rightarrow C_3, C_4$$

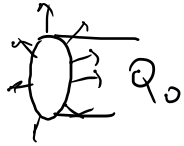
$$w(x) = e^{-\beta x} (C_3 c + C_4 s)$$

$$w'(x) = -\beta e^{-\beta x} (C_3 c + C_4 s) + e^{-\beta x} (C_3 \beta (-s) + C_4 \beta c)$$

$$w''(x) = -$$

$$w'''(x) = -$$

COEFFICIENTI EUSTICI DI BORDO



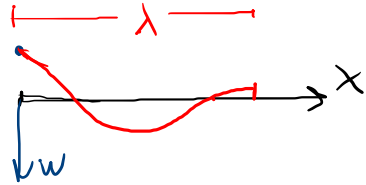
$$w(x) = -\frac{Q_0}{2\beta^3 D} e^{-\beta x} \cos \beta x$$

$$w'(x) = -\frac{Q_0}{2\beta^3 D} (-\beta) e^{-\beta x} \cos \beta x - \frac{Q_0}{2\beta^3 D} e^{-\beta x} (-\beta) \sin \beta x$$

$$= +\frac{Q_0}{2\beta^2 D} e^{-\beta x} (\cos \beta x + \sin \beta x)$$

$$w(0) = -\frac{Q_0}{2\beta^3 D}$$

$$w'(0) = +\frac{Q_0}{2\beta^2 D}$$



COEFF. EUSTICI FORZE APPLICATE

$$\left[|w(0)| = \frac{Q_0}{2\beta^3 D} \quad ; \quad |w'(0)| = \frac{Q_0}{2\beta^2 D} \right]$$

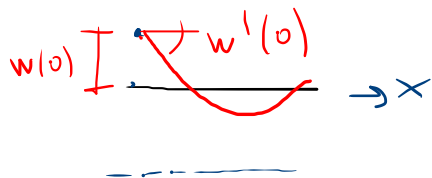
$$v = \frac{Fl^3}{3EI}$$



$$w(x) = \frac{M_0}{2\beta^2 D} e^{-\beta x} (\sin \beta x - \cos \beta x)$$

$$w'(x) = \frac{M_0}{2\beta^2 D} \left[-\beta e^{-\beta x} (\cancel{s-c}) + e^{-\beta x} (\beta \cos \beta x + \beta \cancel{\sin \beta x}) \right]$$

$$= \frac{M_0}{2\beta^2 D} \beta e^{-\beta x} 2 \cos \beta x = \frac{M_0}{\beta D} e^{-\beta x} \cos \beta x$$



$$w(0) = -\frac{M_0}{2\beta^2 D}$$

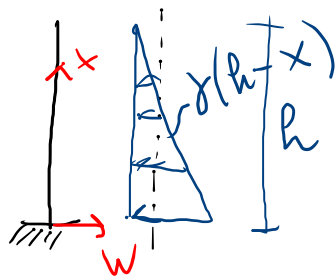
$$w'(0) = +\frac{M_0}{\beta D}$$

$$\Rightarrow \boxed{\begin{aligned} |w(0)| &= \frac{M_0}{2\beta^2 D} \\ |w'(0)| &= \frac{M_0}{\beta D} \end{aligned}}$$

COEFF. ELASTIC

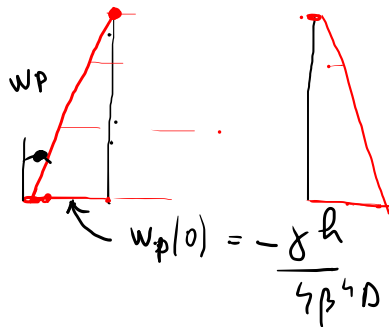
MOMENT APPLICATION

PROBLEMA



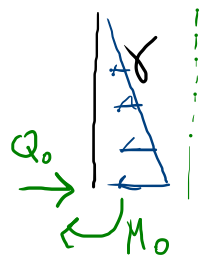
$$q_z(x) = -\gamma(h-x)$$

$$w^{IV}(x) + 4\beta^4 w(x) = -\frac{\gamma(h-x)}{D} \Rightarrow w_p(x) = -\frac{\gamma(h-x)}{4\beta^4 D}$$



$$m_\varphi = -\frac{Es}{R} w(x) \text{ in GENERALE}$$

$$m_\varphi = -\frac{Es}{R} w_p(x) \text{ per la SOLUZ. PARTICOLARE}$$



Q_0, M_0

INCOGNITE

IPERSTATICHE

$$\begin{cases} w(0) = 0 & \text{EQ DI} \\ w'(0) = 0 & \text{CONGR.} \end{cases}$$

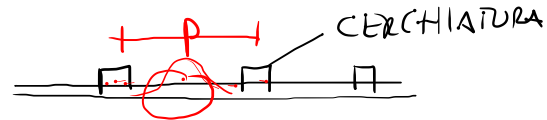
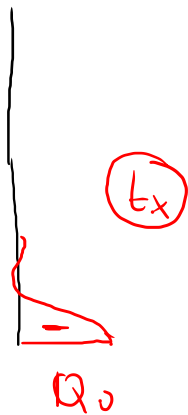
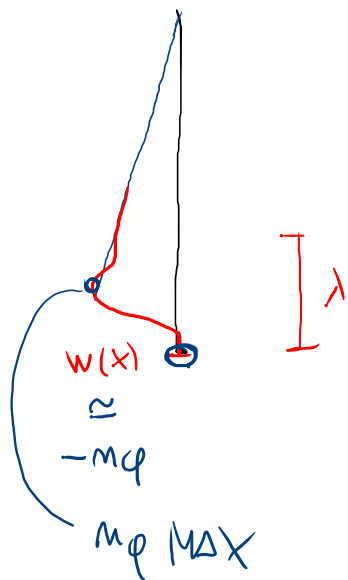
$$\uparrow w : -\frac{\gamma h}{4\beta^4 D} + \frac{Q_0}{2\beta^3 D} - \frac{M_0}{2\beta^2 D} = 0$$

$$\uparrow w' : \frac{\gamma}{4\beta^4 D} - \frac{Q_0}{2\beta^2 D} + \frac{M_0}{\beta D} = 0$$

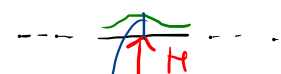
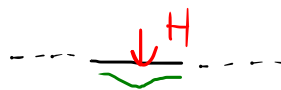
$$\Rightarrow Q_0, M_0 \text{ f } (\gamma, h, D, \beta)$$

$$w(x) = w_p(x) + \frac{Q_0}{2\beta^3 D} e^{-\beta x} \cos \beta x + \frac{M_0}{2\beta^2 D} e^{-\beta x} (\sin \beta x - \cos \beta x)$$

$$\begin{aligned} m_x &= -D w''(x) \\ t_x &= -D w'''(x) \\ m_\varphi &= -\frac{Es}{R} w(x) \end{aligned}$$



$+p$



δ : COEFF ELASTICO

