Un laminato ha la seguente struttura:
$[+45/-45]_T$

E’ costituito da lamine di materiale composito a fibre unidirezionali di carbonio, in matrice epossidica, con spessore pari a 0.25 mm

Le caratteristiche delle lamine sono le seguenti:

- Modulo elastico fibre: 200 GPa
- Modulo elastico matrice: 12 GPa
- Modulo di Poisson matrice: 0.3
- Modulo di Poisson fibre: 0.25
- Modulo di taglio matrice: 4.5 GPa
- Modulo di taglio fibre (ν_{12}): 5 GPa
- Frazione volumetrica fibre: 0.6

Scrivere la relazione costitutiva del laminato.

1. Determinazione delle costanti elastiche della lamina nel sistema di riferimento di ortotropia (modulo 5, pag. 9, pag. 42 ecc.):

$$E_{11} = E_f \cdot V_f + E_m \cdot V_m$$

$$v_{12} = v_{f12} \cdot V_f + v_m \cdot V_m$$

$$\frac{1}{E_{22}} = \frac{V_m}{E_m} + \frac{V_f}{E_f}$$

$$\frac{E_{11}}{E_{22}} = \frac{v_{12}}{v_{21}}$$

$$E = 2G(1 + \nu)$$

$$G_{12} = G_{13} = \frac{G_m}{1 - \sqrt{V_f}(1 - G_m/G_{f12})}$$

$\nu = E/2G - 1$

$\nu_m = 12/(2 * 4.5) - 1 =$

$= 0.33$

$E_1 = 200 * 0.6 + 12 * (1 - 0.6) = 120 + 4.8 = 124.8$ GPa

$E_2 = 1 / (0.4 / 12 + 0.6/200) = 27.5$ GPa

$\nu_{12} = 0.25 * 0.6 + 0.33 * 0.4 = 0.28$

$\nu_{21} = \nu_{12} * E_2/E_1$ (pag. 19, modulo 6) $= 0.28 * (27.5/124.8) = 0.06$

G_{12} (G_6)$= 4.5 / (1 - 0.6^{1/2} * (1 - 4.5 / 5)) = 4.9$ GPa

2. Determinazione della matrice Q, stress piano (modulo 6, pag. 19):

$$Q_{11} = \frac{E_1}{1 - \nu_1 \nu_2} \qquad Q_{22} = \frac{E_2}{1 - \nu_1 \nu_2} \qquad Q_{12} = \frac{\nu_1 E_2}{1 - \nu_1 \nu_2} = \frac{\nu_2 E_1}{1 - \nu_1 \nu_2} \qquad Q_{66} = G_6$$

Q_{11} = 124.8 / (1 – 0.28*0.06) = 122.7 GPa
Q_{22} = 27.5 / (1 – 0.28*0.06) = 27 GPa
Q_{12} = 0.28 * 27.5 / (1 – 0.28*0.06) = 7.6 Gpa
Q_{66} = 4.9 GPa

$$Q = \begin{pmatrix} 122.7 & 7.6 & 0 \\ 7.6 & 27.0 & 0 \\ 0 & 0 & 4.9 \end{pmatrix} \text{[GPa]}$$

3. Determinazione delle matrici di rotazione per gli angoli +45° e -45° (modulo 6, pag. 12):

$$[T]_\sigma^{-1} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \qquad [T]_\varepsilon = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \qquad \begin{aligned} m &= \cos\theta \\ n &= \sin\theta \end{aligned}$$

+45°

$$[T]_\sigma^{-1} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$[T]_\varepsilon = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

-45°

$$[T]_\sigma^{-1} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$[T]_\varepsilon = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

4. Determinazione della matrice $\overline{Q}$, stress piano (modulo 6, pag. 14):

$$
\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})mn^3 \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4)
\end{aligned}
$$

$$
\begin{aligned}
m &= \cos\theta \\
n &= \sin\theta
\end{aligned}
$$

+45°

$$
\overline{Q} = \begin{pmatrix} 46.1 & 36.3 & 23.9 \\ 36.3 & 46.1 & 23.9 \\ 23.9 & 23.9 & 33.6 \end{pmatrix} \text{[GPa]}
$$

-45°

$$
\overline{Q} = \begin{pmatrix} 46.1 & 36.3 & -23.9 \\ 36.3 & 46.1 & -23.9 \\ -23.9 & -23.9 & 33.6 \end{pmatrix} \text{[GPa]}
$$

5. Assemblaggio (modulo 6, da pag. 30 in poi):

$$A_{ij} = \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_k^2 - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_k^3 - h_{k-1}^3)$$

$$[\sigma]_k = \left[\bar{Q}\right]_k [\varepsilon]_k$$

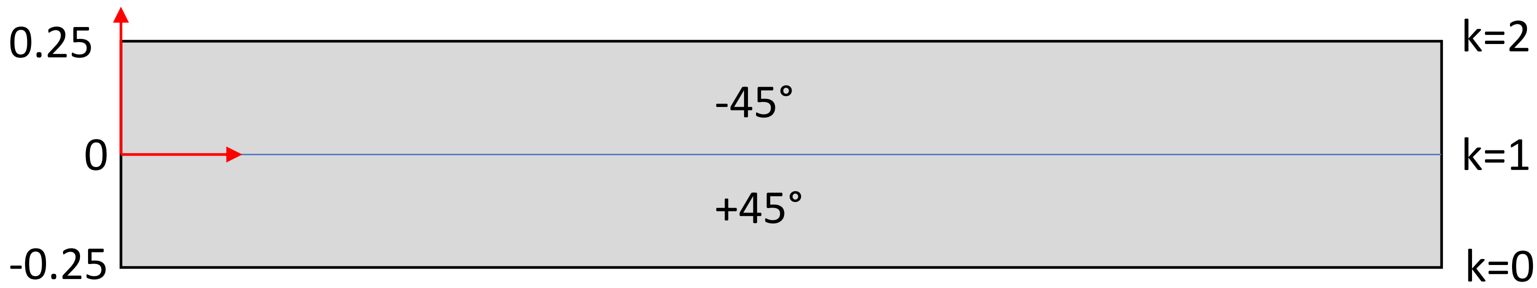
$$\left[\begin{array}{c} N \\ \hline M \end{array}\right] = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \left[\begin{array}{c} \varepsilon^0 \\ \hline K \end{array}\right]$$

$$\left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] = \left(\begin{array}{c|c} \quad & \quad \\ \hline \quad & \quad \end{array}\right)$$

Spessore lamine: 0.25 mm

Layup: $[+45/-45]_T$

$$A_{ij} = \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_k - h_{k-1})$$



$$A = \begin{bmatrix} 46.1 & 36.3 & 23.9 \\ 36.3 & 46.1 & 23.9 \\ 23.9 & 23.9 & 33.6 \end{bmatrix} *(0-(-0.25)) + \begin{bmatrix} 46.1 & 36.3 & -23.9 \\ 36.3 & 46.1 & -23.9 \\ -23.9 & -23.9 & 33.6 \end{bmatrix} *(0.25-0) = \begin{bmatrix} 23.1 & 18.1 & 0 \\ 18.1 & 23.1 & 0 \\ 0 & 0 & 16.8 \end{bmatrix}$$

[GPa]

*1E-3 [m]

*1E-3 [m]

Elimino coeff. 1E-3

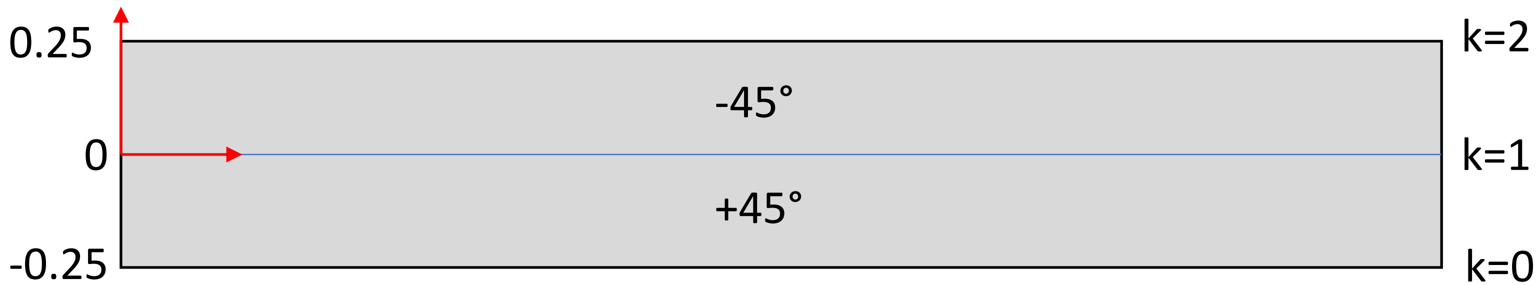
[MN/m]

[kN/mm]

Spessore lamine: 0.25 mm

Layup: $[+45/-45]_T$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k\left(h_k^2 - h_{k-1}^2\right)$$



$$B = \begin{pmatrix} 46.1 & 36.3 & 23.9 \\ 36.3 & 46.1 & 23.9 \\ 23.9 & 23.9 & 33.6 \end{pmatrix} * 0.5 * -0.0625 + \begin{pmatrix} 46.1 & 36.3 & -23.9 \\ 36.3 & 46.1 & -23.9 \\ -23.9 & -23.9 & 33.6 \end{pmatrix} * 0.5 * 0.0625 = \begin{pmatrix} 0 & 0 & -1.5 \\ 0 & 0 & -1.5 \\ -1.5 & -1.5 & 0 \end{pmatrix}$$

[GPa]

*1E-6 [m^2]

*1E-6 [m^2]

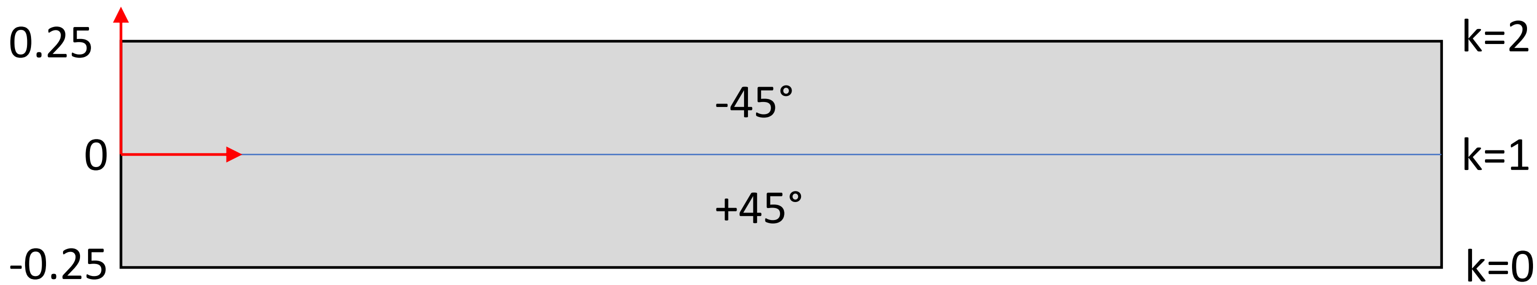
Elimino coeff. 1E-6

[kN]

[N]

Spessore lamine: 0.25 mm
Layup: $[+45/-45]_T$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3)$$



$$D = \begin{pmatrix} 46.1 & 36.3 & 23.9 \\ 36.3 & 46.1 & 23.9 \\ 23.9 & 23.9 & 33.6 \end{pmatrix} *0.33*0.0156 + \begin{pmatrix} 46.1 & 36.3 & -23.9 \\ 36.3 & 46.1 & -23.9 \\ -23.9 & -23.9 & 33.6 \end{pmatrix} *0.33*0.0156 = \begin{pmatrix} 0.48 & 0.38 & 0 \\ 0.38 & 0.48 & 0 \\ 0 & 0 & 0.35 \end{pmatrix}$$

[GPa]

*1E-9 [m^3]

*1E-9 [m^3]

Elimino coeff. 1E-6
[N*m]
[mN/mm]

5. Assemblaggio (modulo 6, da pag. 30 in poi):

$$A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3)$$

$$[\sigma]_k = [\bar{Q}]_k [\varepsilon]_k$$

$$\begin{bmatrix} N \\ \hline M \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{bmatrix} \varepsilon^0 \\ \hline K \end{bmatrix}$$

$$\left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] = \left(\begin{array}{ccc|ccc} 23.1 & 18.1 & 0 & 0 & 0 & -1.5 \\ 18.1 & 23.1 & 0 & 0 & 0 & -1.5 \\ 0 & 0 & 16.8 & -1.5 & -1.5 & 0 \\ \hline 0 & 0 & -1.5 & 0.48 & 0.38 & 0 \\ 0 & 0 & -1.5 & 0.38 & 0.48 & 0 \\ -1.5 & -1.5 & 0 & 0 & 0 & 0.35 \end{array}\right)$$

Opzionale: invertire l’equazione costitutiva del laminato. Considerando un laminato di dimensioni pari a 200x200 mm^2, applicare una forza pari a 20 N in direzione x, distribuendola sulla larghezza del laminato.

Calcolare le deformazioni del laminato.

Scilab:

```
ABD=[
23.1e-3, 18.1e-3, 0e-3, 0e-6, 0e-6, -1.5e-6;
18.1e-3, 23.1e-3, 0e-3, 0e-6, 0e-6, -1.5e-6;
0e-3, 0e-3, 16.8e-3, -1.5e-6, -1.5e-6, 0e-6;
0e-6, 0e-6, -1.5e-6, 0.48e-9, 0.38e-9, 0e-9;
0e-6, 0e-6, -1.5e-6, 0.38e-9, 0.48e-9, 0e-9;
-1.5e-6, -1.5e-6, 0e-6, 0e-9, 0e-9, 0.35e-9
];

_ABD = inv(ABD);

(la matrice va eventualmente scalata)

Vettore forze: [20/0.2,0,0,0,0,0] = [100,0,0,0,0,0]

F = [100E-6;0;0;0;0;0];

DEF = _ABD*F

 DEF  =

    0.0117641
  - 0.0082359
    0.
    0.
    0.
    15.120968
```

Queste sono le ε (0.0117641, - 0.0082359)

TORSIONE (15.120968)