16 Novembre

$$\lim_{x\to 0^+} \frac{\sqrt{1+2x^a}-1-x^2}{\log(1+x+x^2)-x} \qquad a\in\mathbb{R}_+$$

$$\boxed{\log(1+x+x^2)-x = \frac{x^2}{2}+o(x^2)}$$

$$\lim_{x\to 0}\frac{o(x^2)}{x^2}=0 \implies \frac{o(x^2)}{x^2}=o(1) \implies o(x^2)=x^2 o(1)$$

$$\boxed{\log(1+y)= y-\frac{y^2}{2}+o(y^2)} = y+o(y)$$

$$\log(1+y)= y+o(y) \qquad y=x+x^2$$

$$\log(1+x+x^2)= x+x^2+o\underbrace{(x+x^2)}_{o(x)} = x+x^2+o(x)$$

$$= x+o(x)$$

$$\log(1+x+x^2)=x+\frac{x^2}{2}+o(x^2)$$

$$\log(1+x+x^2)-x = \cancel{x}+o(x)-\cancel{x} = o(x)$$

$$\lim_{x\to 0^+} \frac{\sqrt{1+2x^a}-1-x^2}{\log(1+x+x^2)-x} \qquad a\in\mathbb{R}_+$$

$$\boxed{\log(1+x+x^2)-x = \frac{x^2}{2}+o(x^2)}$$

$$\lim_{x\to 0} \frac{o(x^2)}{x^2} = 0 \implies \frac{o(x^2)}{x^2} = o(1) \implies o(x^2)=x^2 o(1)$$

$$\frac{\sqrt{1+2x^a}-1-x^2}{\frac{x^2}{2}} \qquad \frac{\frac{x^2}{2}}{\frac{x^2}{2}+\cancel{2}\frac{x^2}{2}o(1)} = (1+o(1))$$

$$\lim_{x\to 0} \frac{\cancel{\frac{x^2}{2}}}{\cancel{\frac{x^2}{2}}\left(1+2o(1)\right)} = \lim_{x\to 0} \frac{1}{1+2o(1)} = 1$$

$$2o(1) = o(1) \implies \frac{1}{1+2o(1)} = \frac{1}{1+o(1)}$$

$$\lim_{x\to 0} \frac{1}{1+o(1)} = 1 \implies \frac{1}{1+o(1)} = 1+o(1)$$

$$\frac{\sqrt{1+2x^a}-1-x^2}{\log(1+x+x^2)-x} = 2\,\frac{\sqrt{1+2x^a}-1-x^2}{x^2}\,(1+o(1))$$

$$\lim_{x\to 0} 2\,\frac{\sqrt{1+2x^a}-1-x^2}{x^2}$$

$$2 \quad \lim_{x\to 0} \frac{\sqrt{1+2x^a}-1-x^2}{x^2}$$

$$\text{Num} = \sqrt{1+2x^a} - 1 - x^2$$

$$(1+y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2 + o(y^2) \quad , \; y = 2x^a$$

$$(1+2x^a)^{\frac{1}{2}} = 1 + \frac{1}{\cancel{2}}\cancel{2}x^a - \frac{1}{\cancel{8}_{2}}\cancel{4}x^{2a} + o(4x^{2a})$$

$$= 1 + x^a - \frac{x^{2a}}{2} + o(x^{2a})$$

$$\lim_{x\to 0} \frac{o(4x^m)}{x^m} = \lim_{x\to 0} \frac{o(4x^m)}{4x^m} \, 4 =$$

$$= 4 \lim_{x\to 0} \frac{o(4x^m)}{4x^m} = 4 \cdot 0 = 0$$

$$o(4x^m) = o(x^m)$$

$$(1+2x^a)^{\frac{1}{2}} = 1 + x^a - \frac{x^{2a}}{2} + o(x^{2a})$$

$$\text{Num} = (1+2x^a)^{\frac{1}{2}} - 1 - x^2 =$$

$$\text{Num} = \cancel{1} + x^a - \frac{x^{2a}}{2} + o(x^{2a}) \cancel{-1} - x^2$$

$$\text{Num} = x^a - \frac{x^{2a}}{2} - x^2 + o(x^{2a})$$

$$\text{Num} = x^a - \frac{x^{2a}}{2} - x^2 + o(x^{2a})$$

Dobbiamo scegliere il termine più grande, cioè quello dominante.

Quale sia il termine dominante dipende dalla relazione che c'è tra a e 2.

se $0<a<2$ allora

$$x^2 = o(x^a) \quad \text{perché} \quad \lim_{x\to 0^+} \frac{x^2}{x^a} =$$

$$= \lim_{x\to 0^+} x^{\overbrace{2-a}^{>0}} = 0$$

$$\text{Numeratore} = x^a + o(x^a) \quad \text{quando} \quad 0<a<2$$

$$\text{Num} = x^a - \frac{x^{2a}}{2} - x^2 + o(x^{2a})$$

se $0<a<2$

$$2\lim_{x\to 0^+} \frac{\sqrt{1+2x^a}-1-x^2}{x^2} = 2\lim_{x\to 0^+} \frac{x^a + \overbrace{o(x^a)}^{x^a o(1)}}{x^2}$$

$$= 2\lim_{x\to 0^+} \frac{x^a}{x^2}\cancel{(1+o(1))} = 2\lim_{x\to 0^+} x^{\overbrace{a-2}^{<0}} = +\infty$$

Così $a>2$ e $a=2$

$$\text{Num} = x^a - \frac{x^{2a}}{2} - x^2 + o(x^{2a})$$

Ora consideriamo il caso $a > 2$ allora

$$x^a = o(x^2)$$

$$\lim_{x\to 0^+} \frac{x^a}{x^2} = \lim_{x\to 0^+} x^{\overbrace{a-2}^{>0}} = 0$$

$$\text{Num} = -x^2 + o(x^2) = -x^2\,(1+o(1))$$

$$2 \lim_{x\to 0^+} \frac{\sqrt{1+2x^a} - 1 - x^2}{x^2} = 2 \lim_{x\to 0^+} \frac{-x^2\cancel{(1+o(1))}}{x^2}$$

$$= 2 \lim_{x\to 0^+} (-1) = -2$$

Caso $a = 2$

$$\text{Num} = x^a - \frac{x^{2a}}{2} - x^2 + o(x^{2a}) =$$

$$= \cancel{x^2} - \frac{x^4}{2} \cancel{-x^2} + o(x^4)$$

$$\text{Num} = -\frac{x^4}{2} + o(x^4) = -\frac{x^4}{2}\,(1+o(1))$$

$$2 \lim_{x\to 0^+} \frac{\sqrt{1+2x^a} - 1 - x^2}{x^2} = 2 \lim_{x\to 0^+} \frac{-\frac{x^4}{2}\cancel{(1+o(1))}}{x^2} = 0$$

Es 4 28/6/21 $f(x) = \sin(x^2) + \cos(x^2)$

Calcolare polinomi di Mc Laurin.

$$\sin(y) = \sum_{j=0}^{n} (-1)^j \frac{y^{2j+1}}{(2j+1)!} + o(y^{2n+1})$$

$$\cos(y) = \sum_{j=0}^{n} (-1)^j \frac{y^{2j}}{(2j)!} + o(y^{2n})$$

$$\sin(x^2) = \sum_{j=0}^{n} (-1)^j \frac{x^{4j+2}}{(2j+1)!} + o(x^{4n+2})$$

$$\cos(x^2) = \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!} + \underbrace{o(x^{4n})}_{o(x^{4n+2})}$$

$$f(x) = \underbrace{\sum_{j=0}^{n} (-1)^j \frac{x^{4j+2}}{(2j+1)!} + \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!}}_{P_{4n+2}(x)} + \underbrace{o(x^{4n+2}) + \underbrace{o(x^{4n})}_{o(x^{4n+2})}}_{o(x^{4n+2})}$$

$$\cos(x^2) = \sum_{j=0}^{n+1} (-1)^j \frac{x^{4j}}{(2j)!} + o(x^{4n+4}) =$$

$$= \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!} + \underbrace{(-1)^{n+1} \frac{x^{4n+4}}{(2n+2)!} + o(x^{4n+4})}_{o(x^{4n+2})}$$

$$\lim_{x\to 0} \frac{o(x^{4n+4})}{x^{4n+2}} = \lim_{x\to 0} \frac{o(x^{4n+4})}{x^{4n+2}\, x^2}\, x^2 =$$

$$= \lim_{x\to 0} \frac{o(x^{4n+4})}{x^{4n+4}}\, x^2 = 0 \cdot 0 = 0$$

$$\sin(x^2) = \sum_{j=0}^{n} (-1)^j \frac{x^{4j+2}}{(2j+1)!} + o(x^{4n+2})$$

P_{4n}

$$\cos(x^2) = \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!} + o(x^{4n})$$

$$f(x) = \sum_{j=0}^{n} (-1)^j \frac{x^{4j+2}}{(2j+1)!} + \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!} + o(x^{4n+2}) + o(x^{4n})$$

$$= \sum_{j=0}^{n-1} (-1)^j \frac{x^{4j+2}}{(2j+1)!} + \sum_{j=0}^{n} (-1)^j \frac{x^{4j}}{(2j)!} +$$

P_{4n}

$$+ \underbrace{(-1)^n \frac{x^{4n+2}}{(2n+1)!}}_{o(x^{4n})} + \underbrace{o(x^{4n+2})}_{o(x^{4n})} + o(x^{4n})$$

$o(x^{4n})$