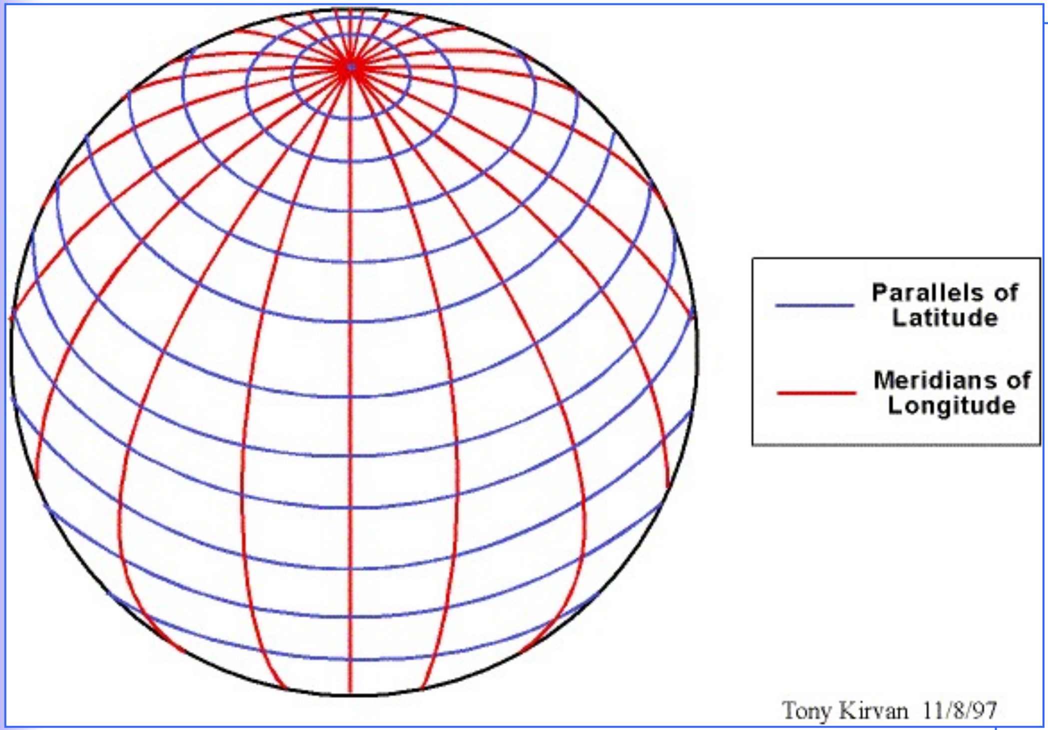
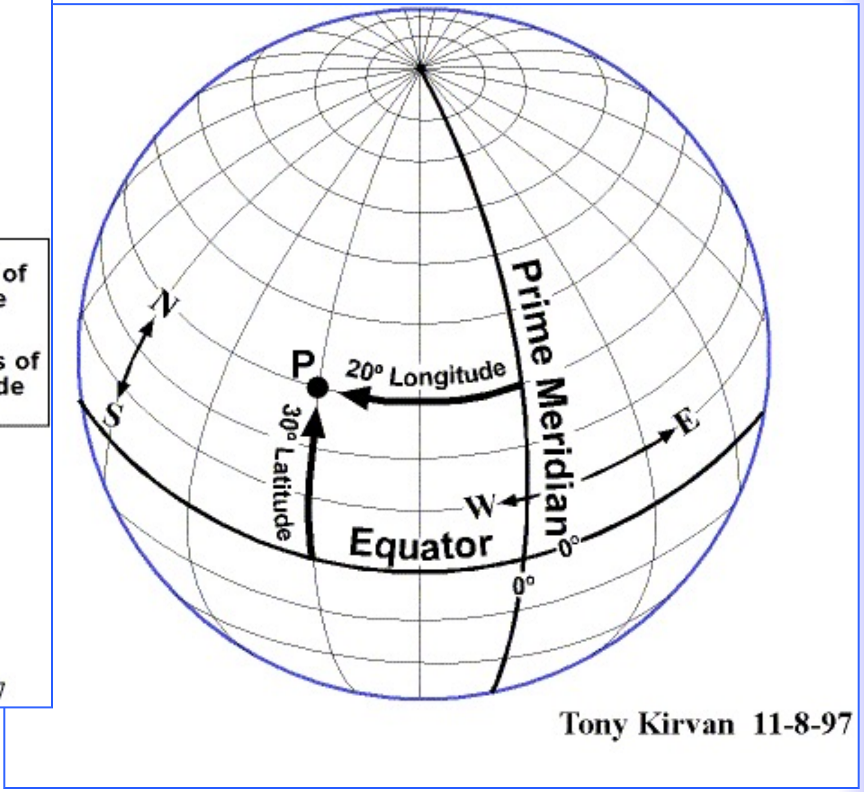
Geometria di un satellite

- Sfera Celeste
- Sistemi di Coordinate
- Studio Eclissi
- Geometria Terra / Satellite

SMAD Chapter 5
p. 95

Sfera Celeste 1/2





azimuth (longitudine ℓ)

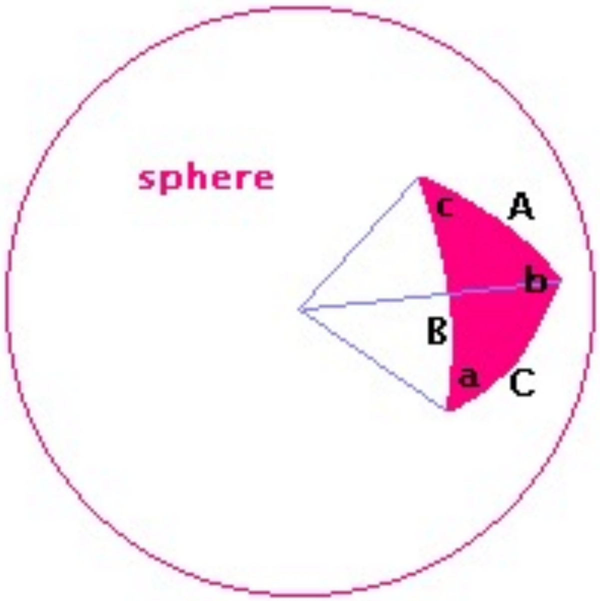
elevazione (latitudine λ)

$$x = \cos \ell \cos \lambda$$

$$y = \sin \ell \cos \lambda$$

$$z = \sin \lambda$$

Sfera Celeste 2/2



A spherical triangle consists of Great Circle Arcs, extending from the sphere's center, forming Great Circle Angles. Relations among arcs and angles are:

$$\cos(A) = \cos(B)\cos(C) + \sin(B)\sin(C)\cos(a)$$

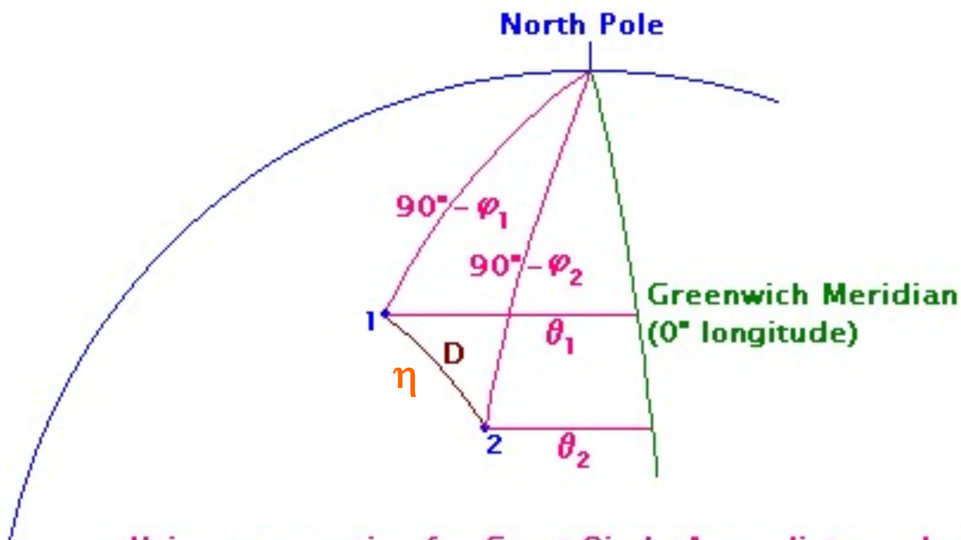
$$\cos(a) = -\cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

$$\sin(A)/\sin(a) = \sin(B)/\sin(b) = \sin(C)/\sin(c)$$

SMAD Appendix D
Table D-3 p. 907

φ_1, φ_2 elevazione

θ_1, θ_2 azimuth



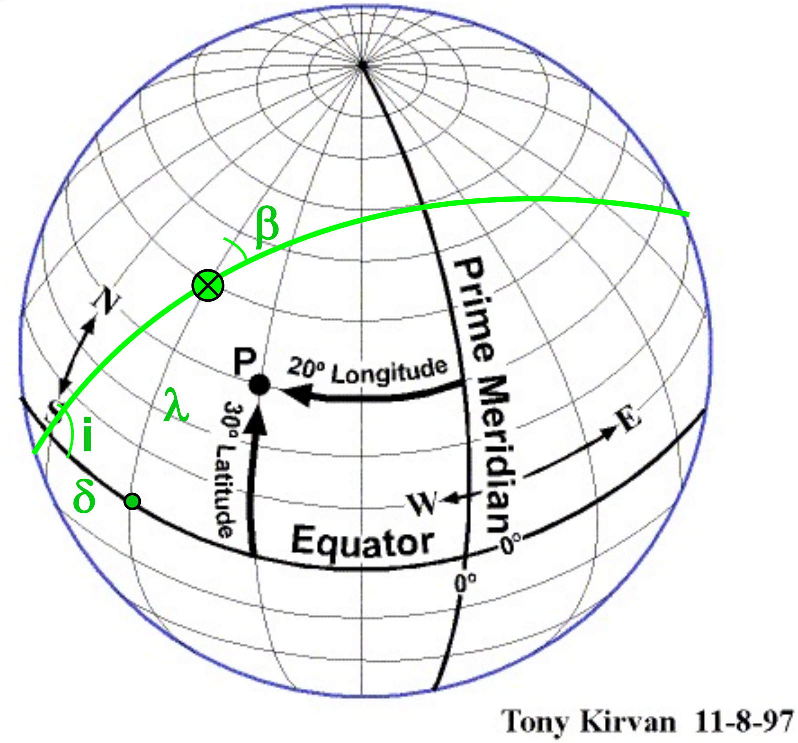
Using an equation for Great Circle Arcs, distance between 1 & 2 is estimated as:

$$\cos(\eta) = \cos(90^o - \varphi_1)\cos(90^o - \varphi_2) + \sin(90^o - \varphi_1)\sin(90^o - \varphi_2)\cos(\theta_1 - \theta_2)$$

$$D = 2\pi R_t/(2\pi)\ \mathrm{acos}(\sin(\varphi_1)\sin(\varphi_2) + \cos(\varphi_1)\cos(\varphi_2)\cos(\theta_1 - \theta_2))$$

Finestre di Lancio

P' (30° W, 40° N)



$\lambda > i$?

$\lambda = i$?

$\lambda < i$?

SMAD Appendix D
Table D-1 p. 905 riga 4 col. 3

SMAD Appendix D
Table D-1 p. 905 riga 5 col. 3

$$\sin \beta = \cos i / \cos \lambda$$

$$\cos \delta = \cos \beta / \sin i$$

$$LST = \Omega + \delta$$

$$LST = \Omega + 180^o - \delta$$

SMAD chapter 6.4
p. 153-155

$$v_{sud} = - v_o \cos \gamma \cos \beta_L$$

$$v_{est} = v_o \cos \gamma \sin \beta_L - v_\lambda$$

$$v_r = v_o \sin \gamma \quad (v_z)$$

$$v_\lambda = 464.5 \cos \lambda \text{ m/s}$$

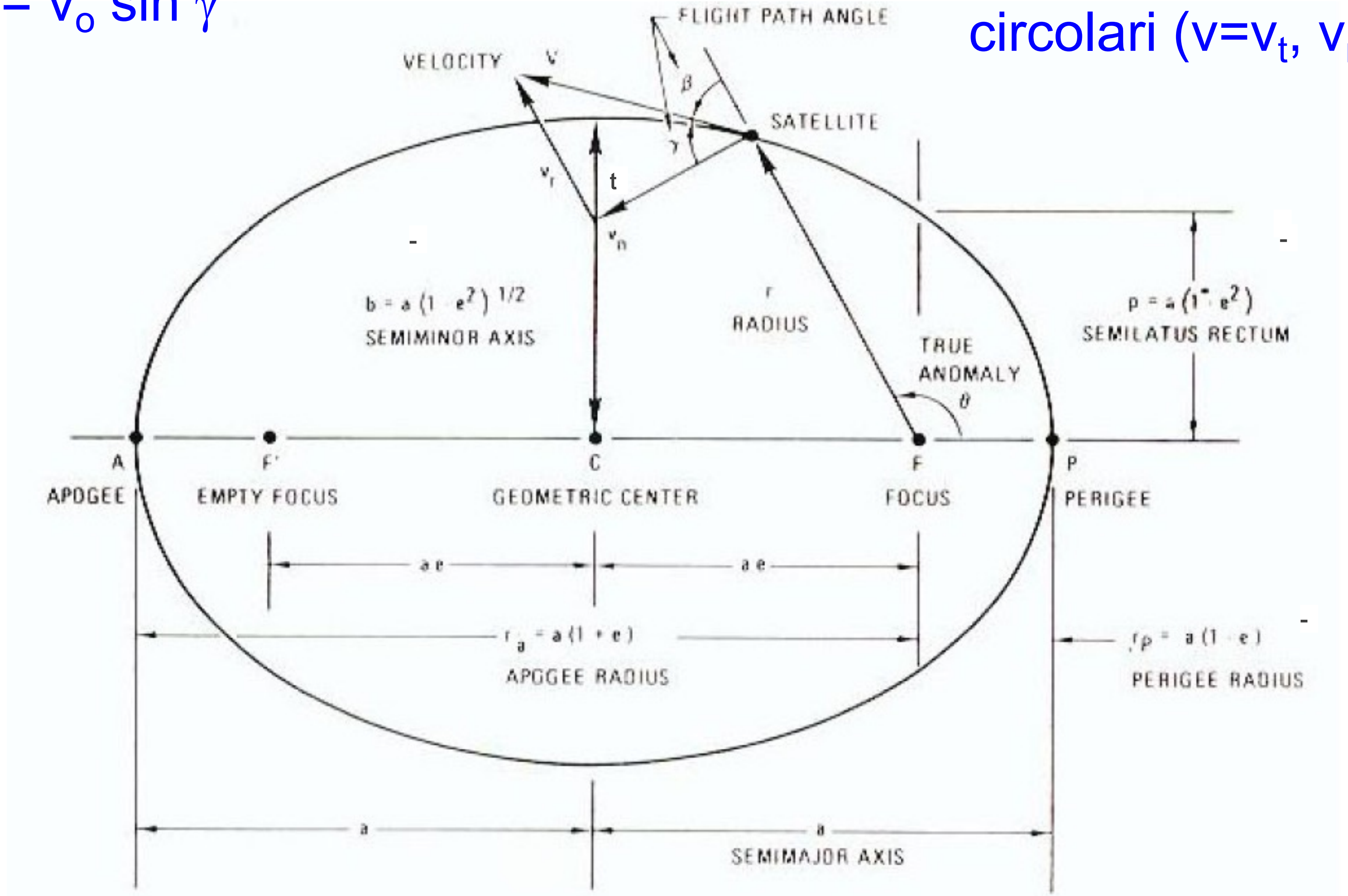
β azimuth di lancio
γ angolo traiettoria volo al *burn-out*
(vedi ultima trasparenza su orbite)

Parametri Ellisse

$v_t = v_o \cos \gamma$ (*)

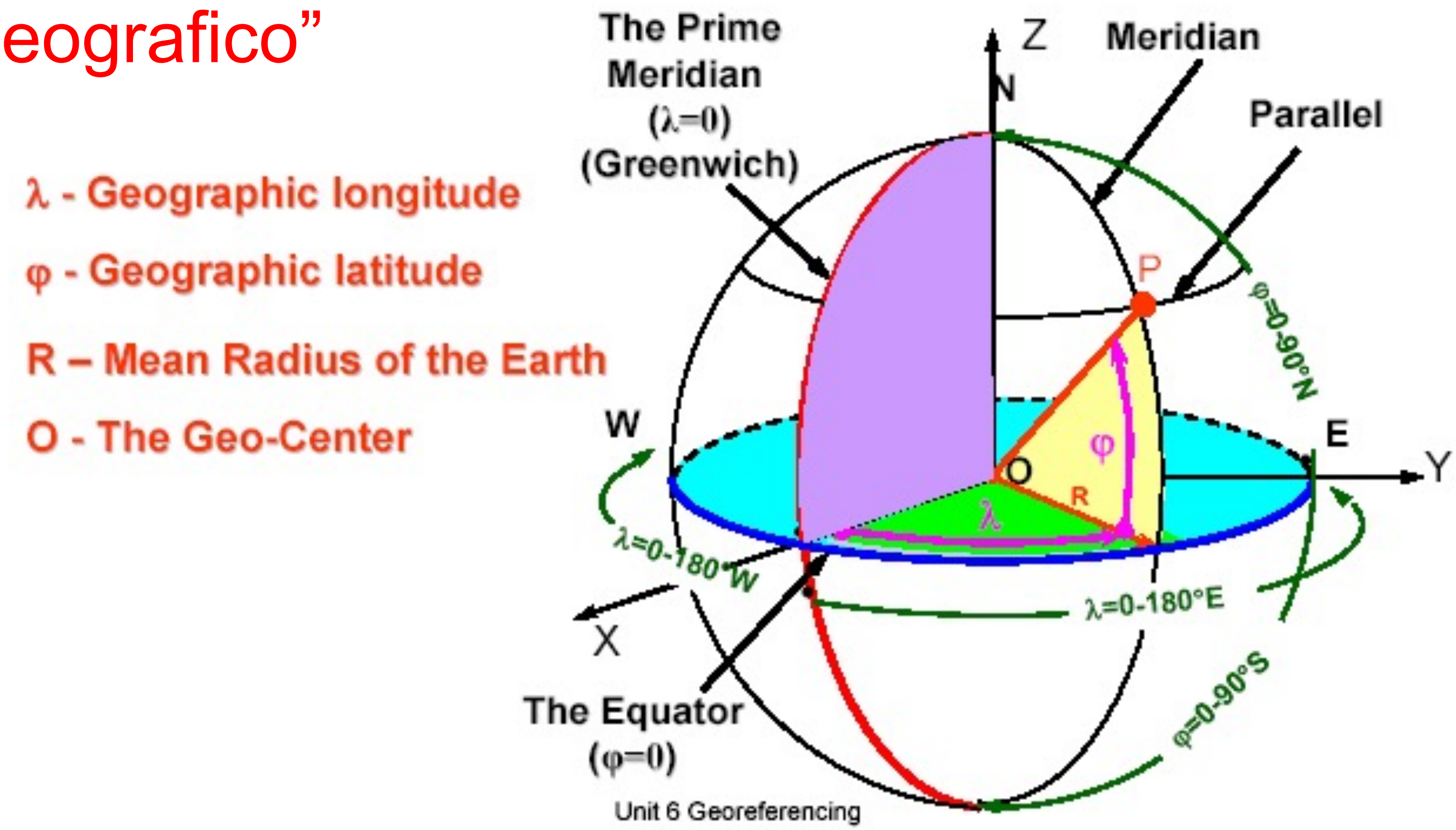
$v_r = v_o \sin \gamma$

$\gamma=0$ per orbite circolari ($v=v_t$, $v_r=0$)



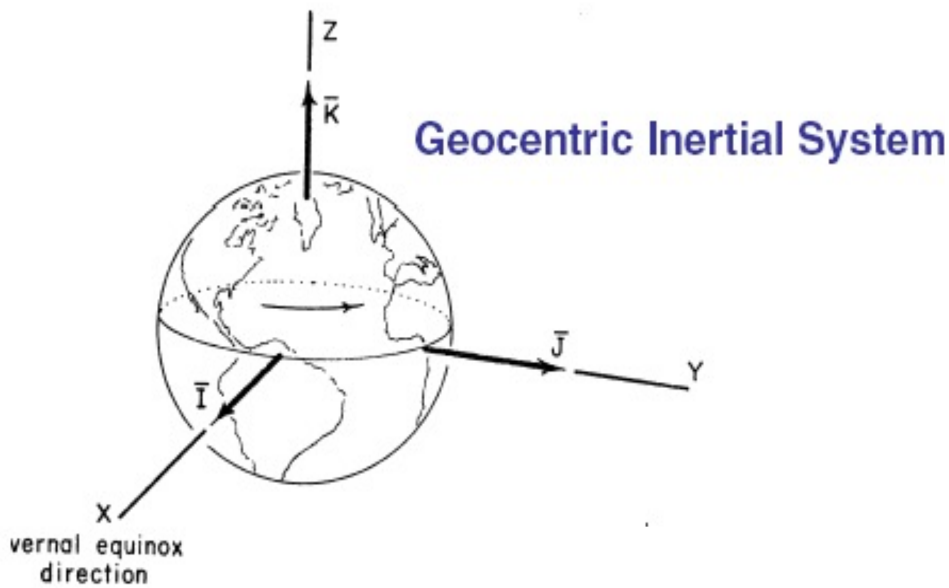
Sistemi di Coordinate 1/3

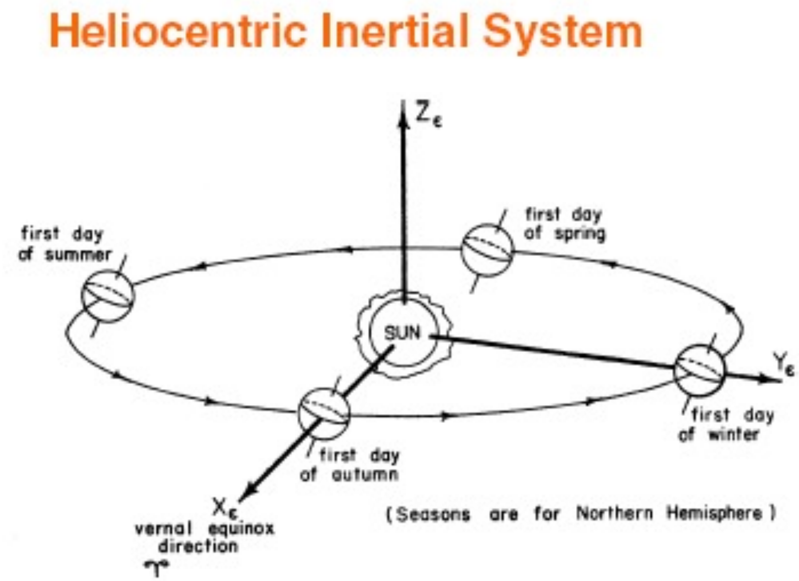
Sistema Geocentrico "Geografico"



Attenzione all'indicazione lat/long !

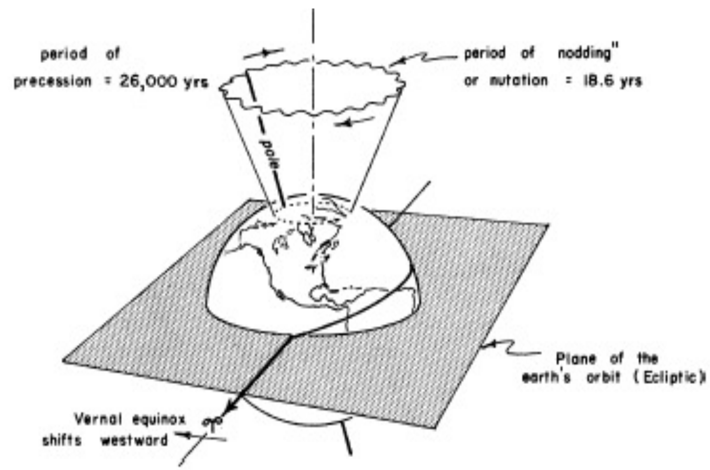
Sistemi di Coordinate 2/3





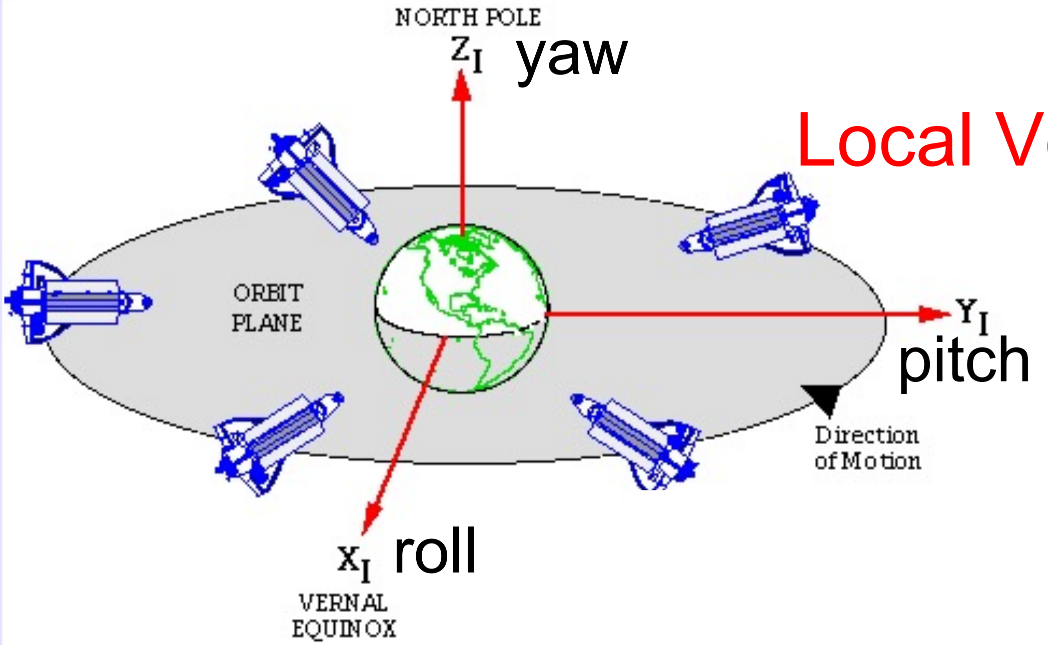
azimuth = ascensione retta α

elevazione = declinazione δ



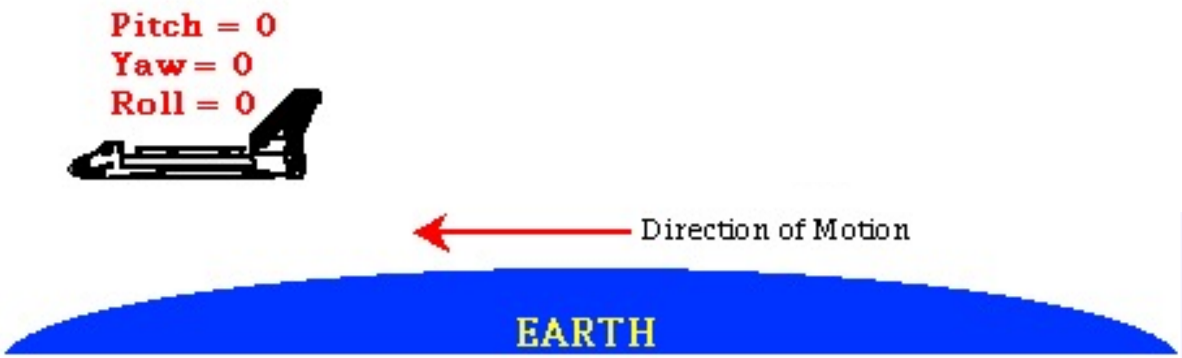
50.2786” westerly drift of the Vernal Equinox per year

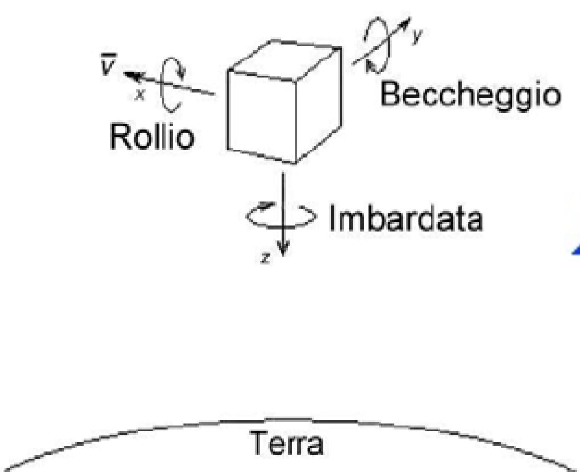
Sistemi di Coordinate 3/3



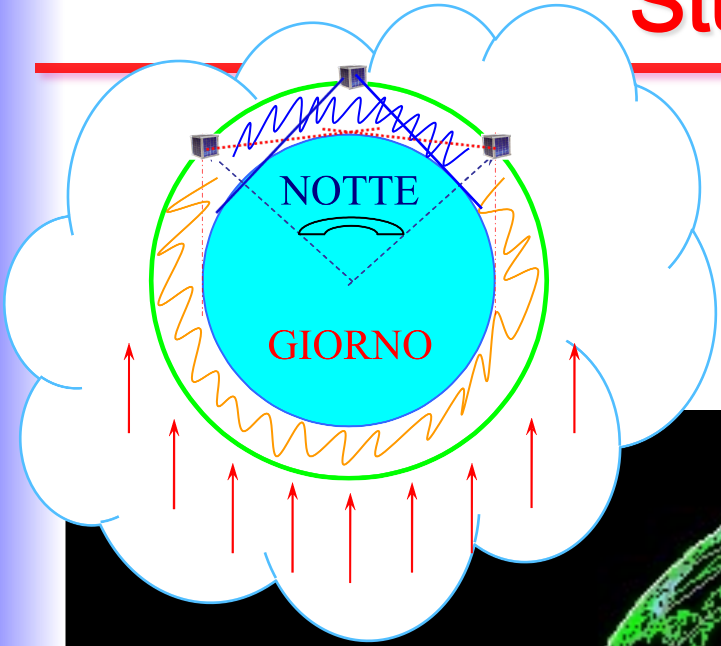
LVLH

Local Vertical – Local Horizontal

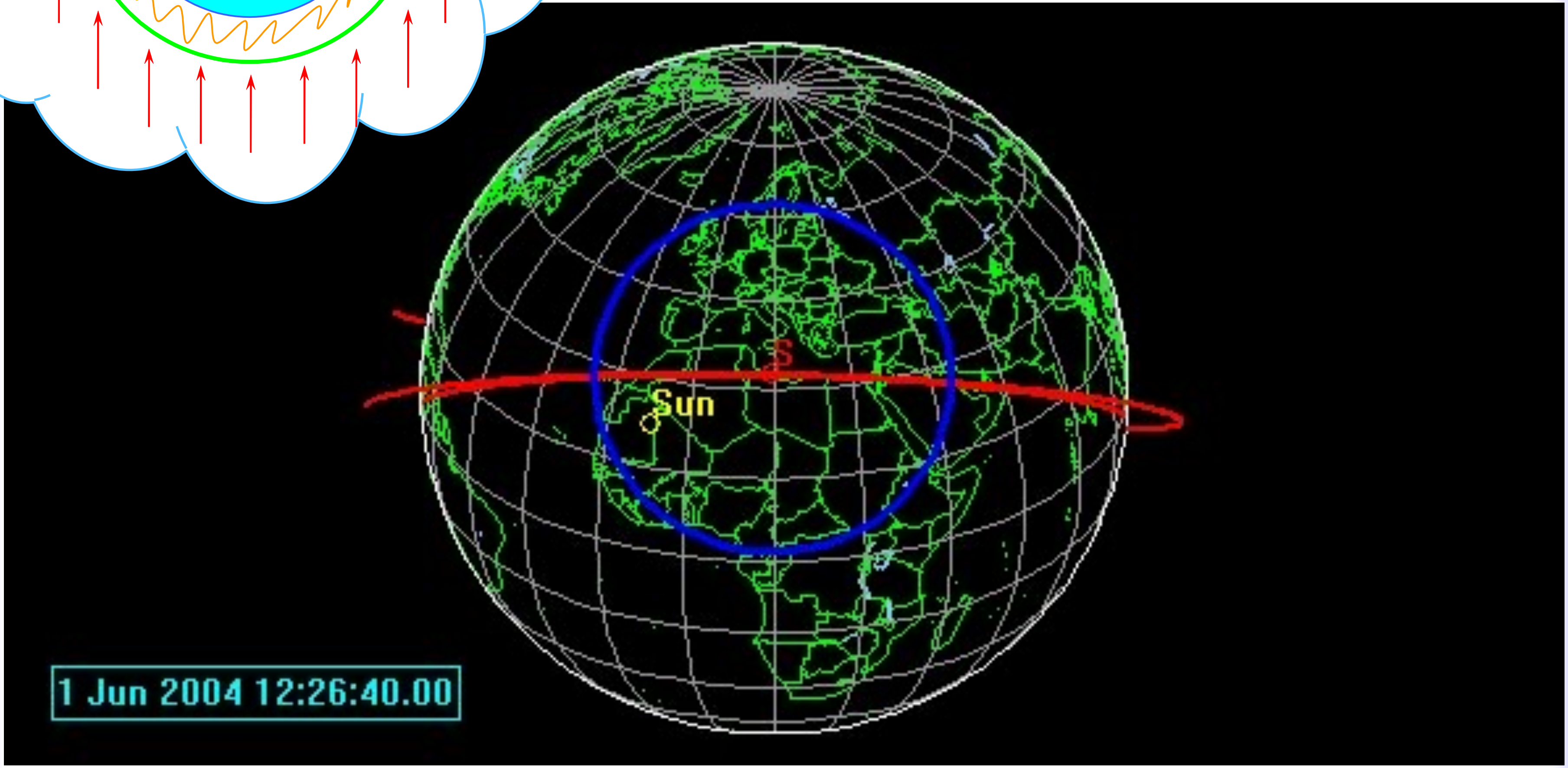




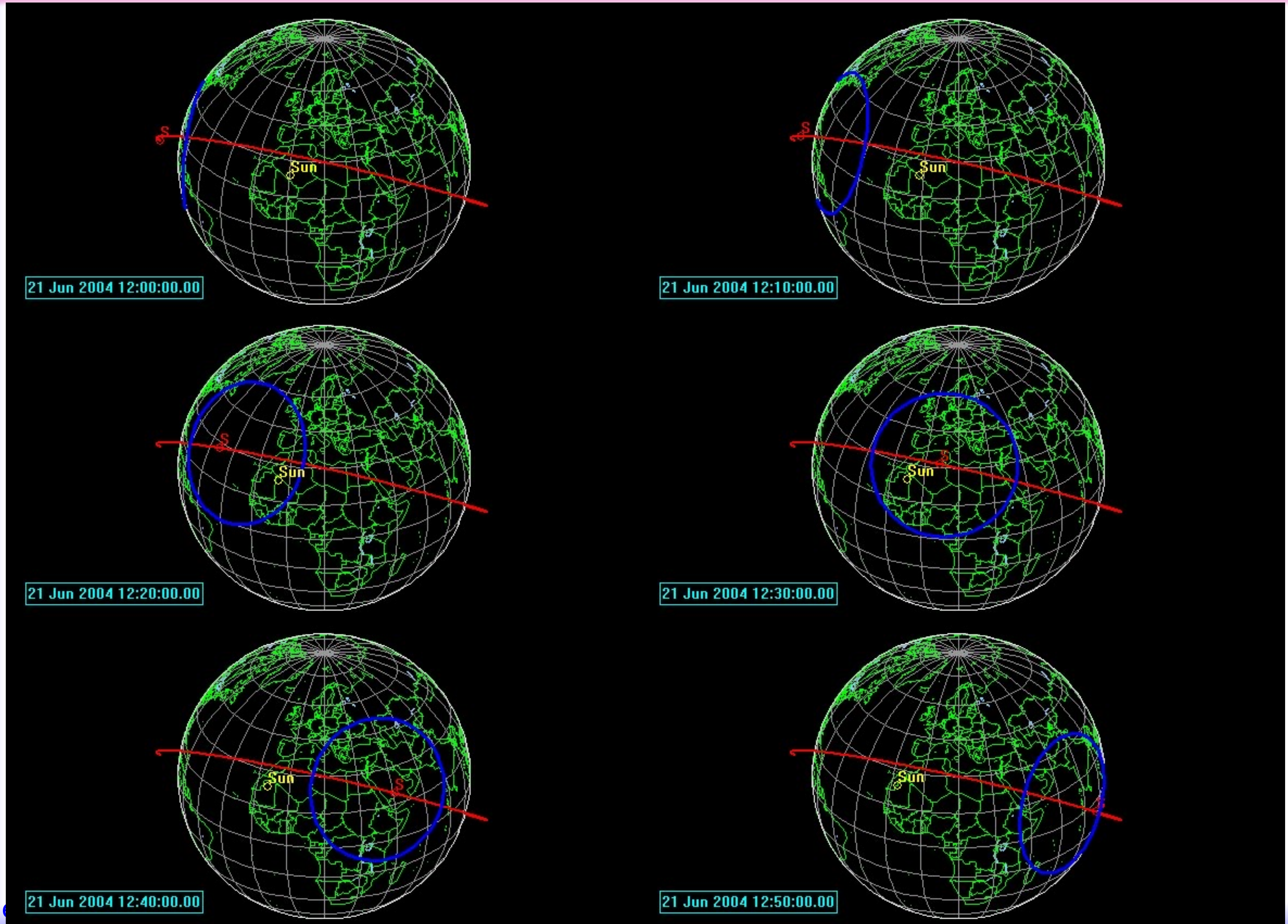
Studio Eclissi 1/3



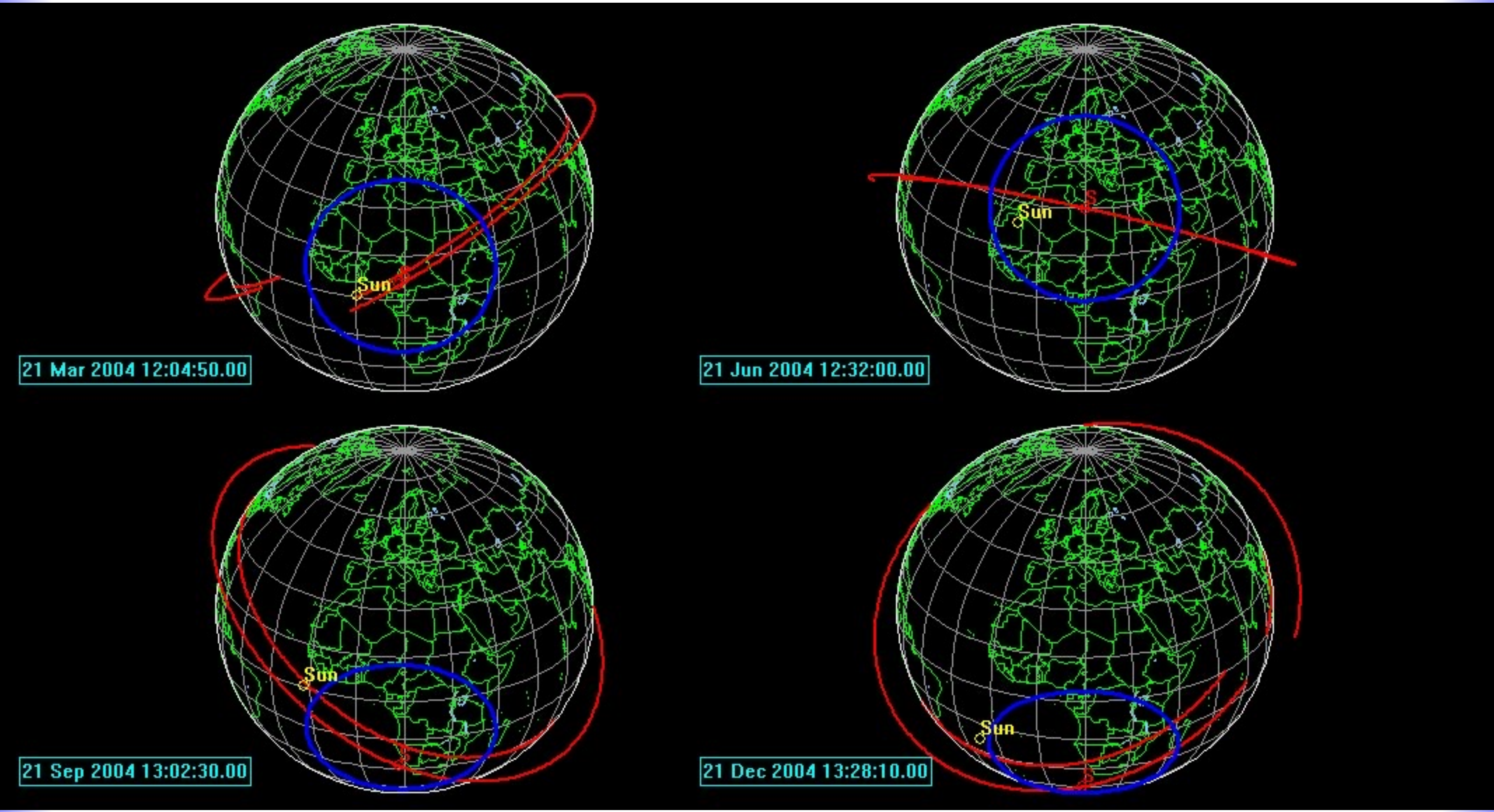
h = 1000 km, i = 32°



Studio Eclissi 2/3

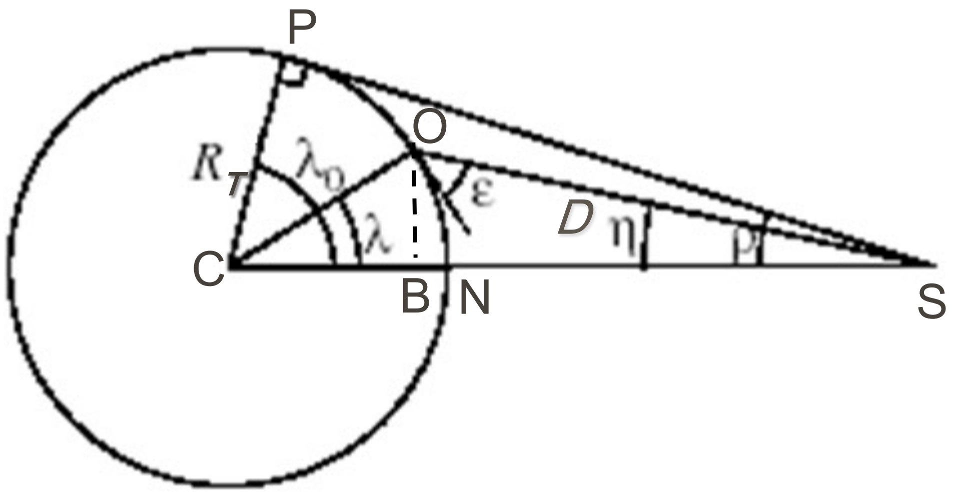


Studio Eclissi 3/3



SMAD chapter 5.1
Example 1, 2 e 3
p. 105-110

Geometria Terra / Satellite 1/3



ρ raggio angolare Terra

η angolo di nadir

ε elevazione

λ angolo centrale Terra (swath width)

SMAD chapter 5.2
fig 5-13 p. 110-113

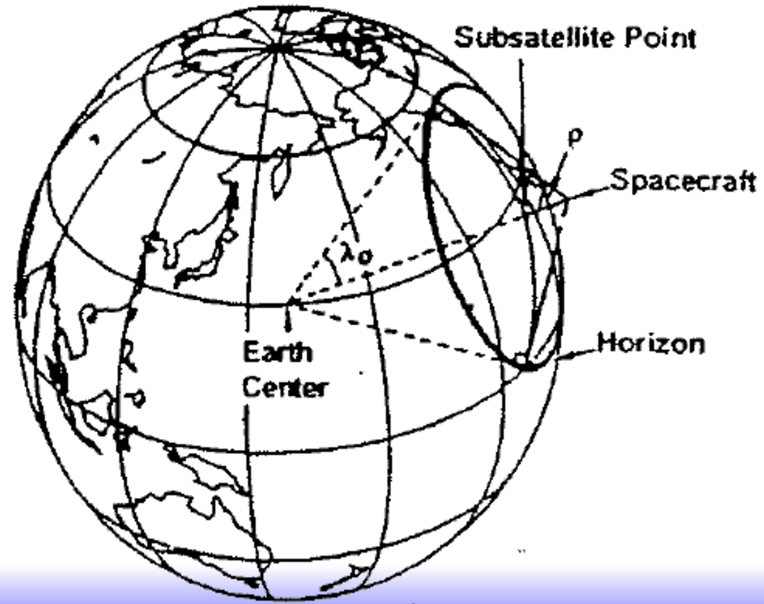
$\sin \rho = \cos \lambda_0 = R_T/R = R_T / (R_T+h)$

$\sin \eta = \cos \varepsilon \sin \rho$

$\lambda = \pi/2 - \eta - \varepsilon$

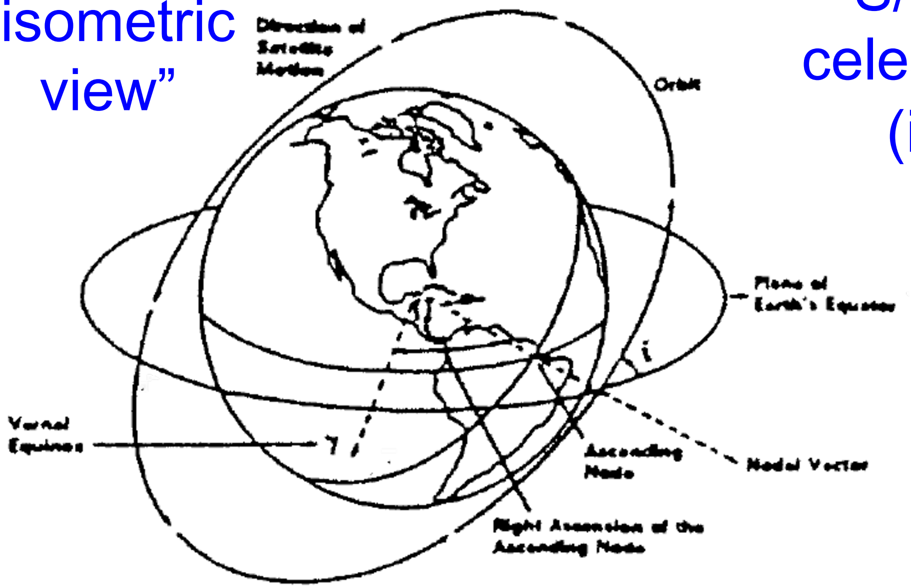
$\mathrm{tg}\, \eta = \sin \rho \sin \lambda / (1 - \sin \rho \cos \lambda)$

$D = R_T \sin \lambda / \sin \eta$

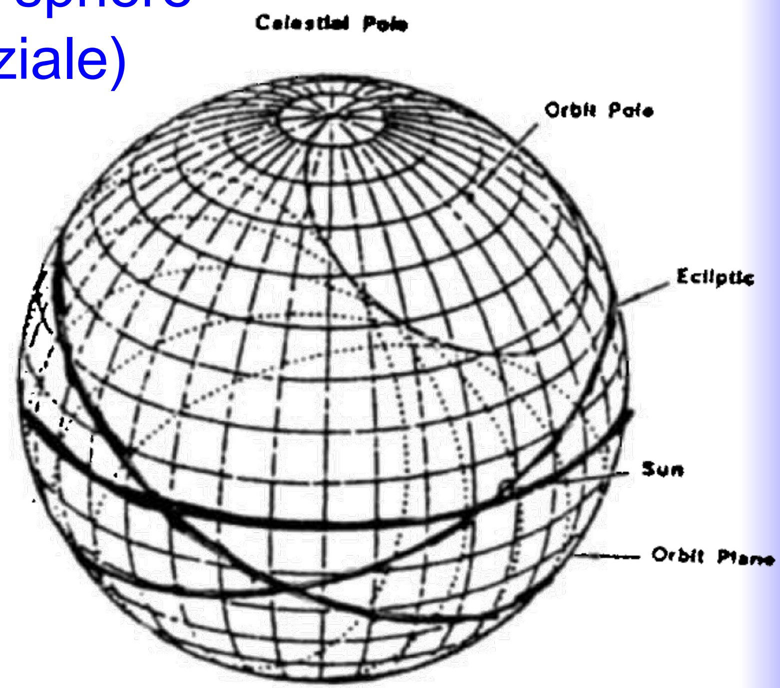


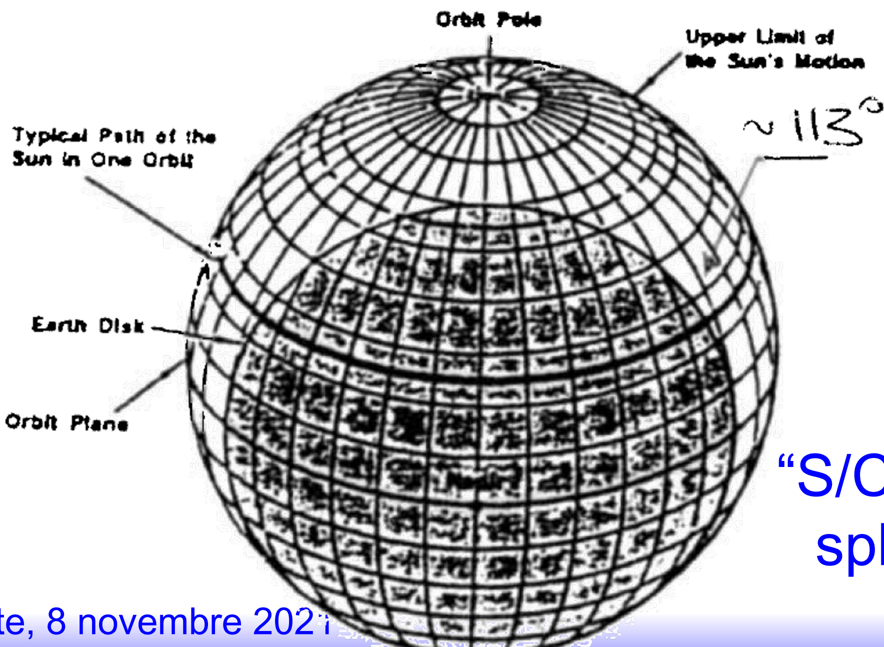
Geometria di un satellite 2/3

“isometric view”



“S/C centered celestial sphere” (inerziale)





$h = 1000$ km, $i = 32^\circ$ $\Rightarrow$

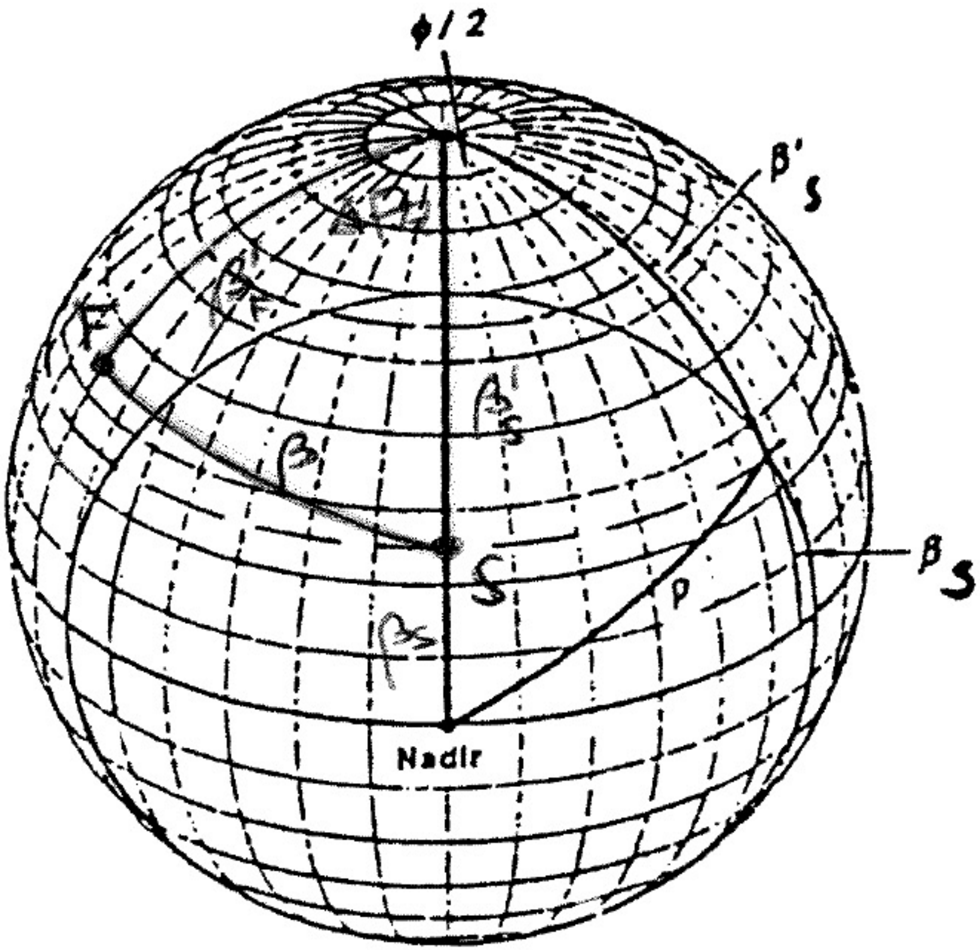
$\tau = 105$ min, $\rho = 60^\circ$

“S/C centered celestial sphere” (riferimento terrestre)

Geometria di un satellite 3/3

"S/C centered celestial sphere"

$\cos \Phi/2 = \cos \rho / \cos \beta_s$



$\beta_s = 25°$ $\Rightarrow$ $\Phi/2 = 56.5°$

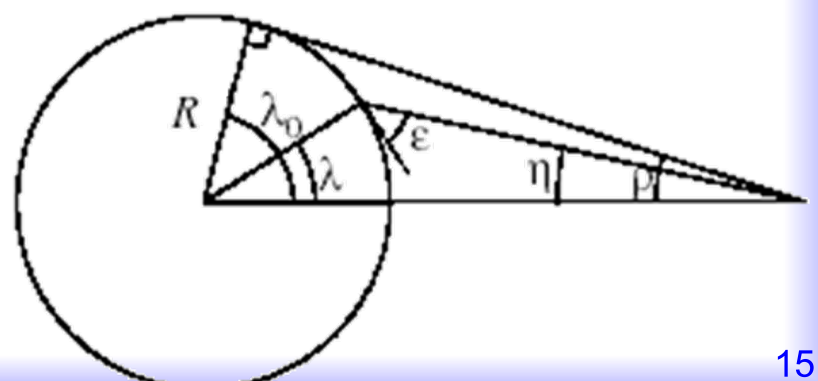
Durata fase notturna (eclisse): max e min

β_s : max e min β

$\beta_F = 35°$, $Az_o = 70°$

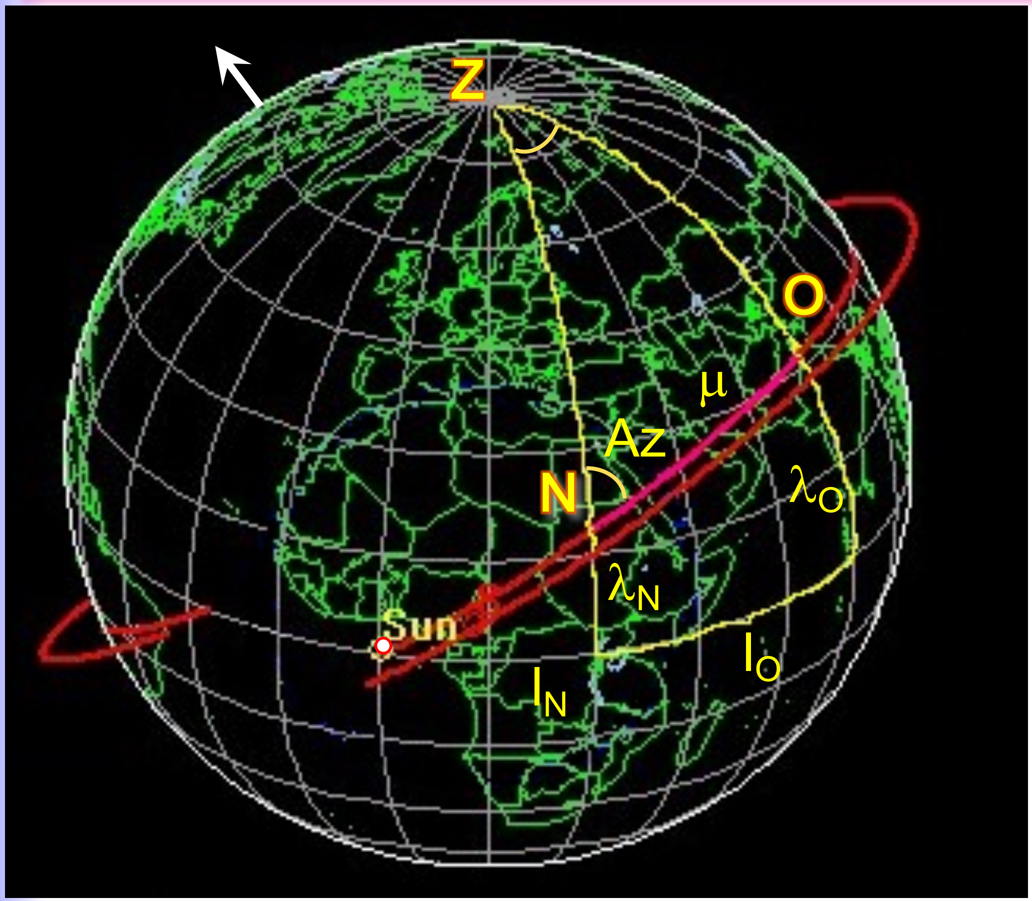
$A = 0.5\ m^2$ $\Rightarrow$

- "eclisse": $Az = Az_o \pm \Phi/2$
- "dietro": $\beta = \pm \pi/2$

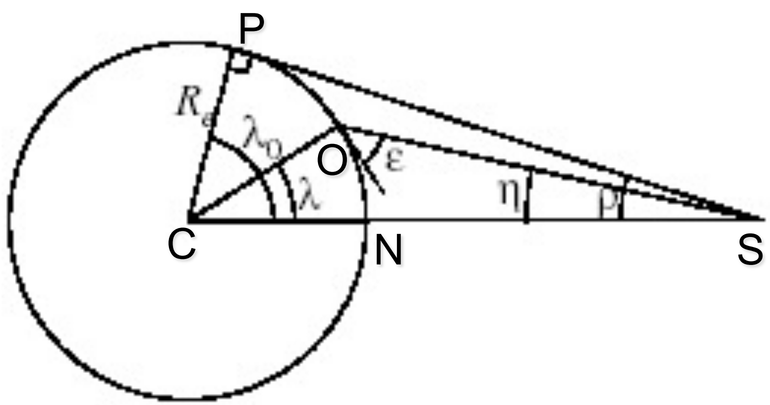


Analisi dell'eclissi da una LEO

Passaggio sopra la stazione 1/8



"Earth centered celestial sphere"



Meglio: geometria vista dal satellite!
Nota: quella segnata non è un' orbita del satellite ma un cerchio max che passa per O e N

N = Sub Satellite Point (l_N, λ_N)

O = Punto qls Terra (l_O, λ_O) !!!

$$\cos\mu = \sin\lambda_N \sin\lambda_O + \cos\lambda_N \cos\lambda_O \cos(l_O - l_N) \quad \Rightarrow \eta$$

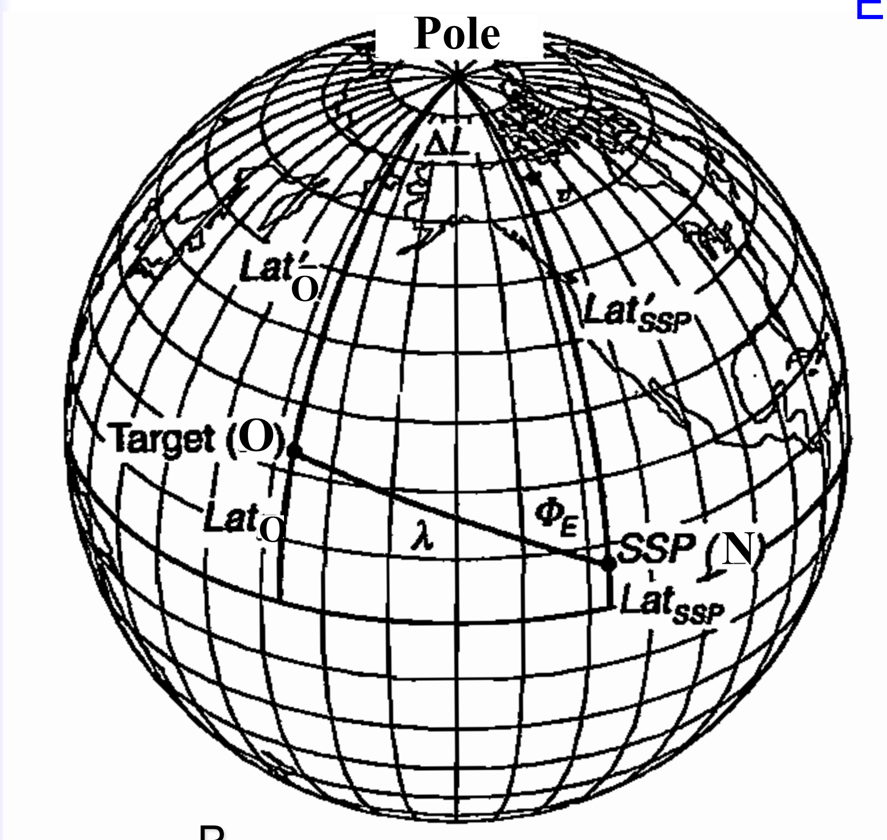
$$\sin\lambda_O = \sin\lambda_N \cos\mu + \cos\lambda_N \sin\mu \cos Az \qquad (\mu \equiv \lambda)$$

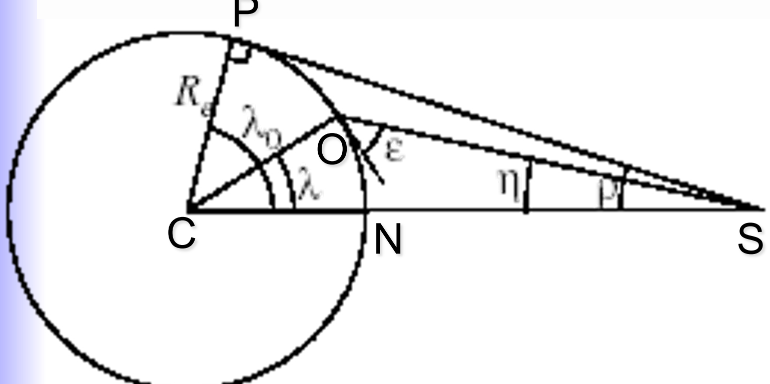
$$\cos \mathbf{Az} = (\sin\lambda_O - \sin\lambda_N \cos\mu) / \cos\lambda_N \sin\mu$$

SMAD chapter 5.2
fig 5-12 p. 112

Risultato: angoli da satellite

Passaggio sopra la stazione 2/8



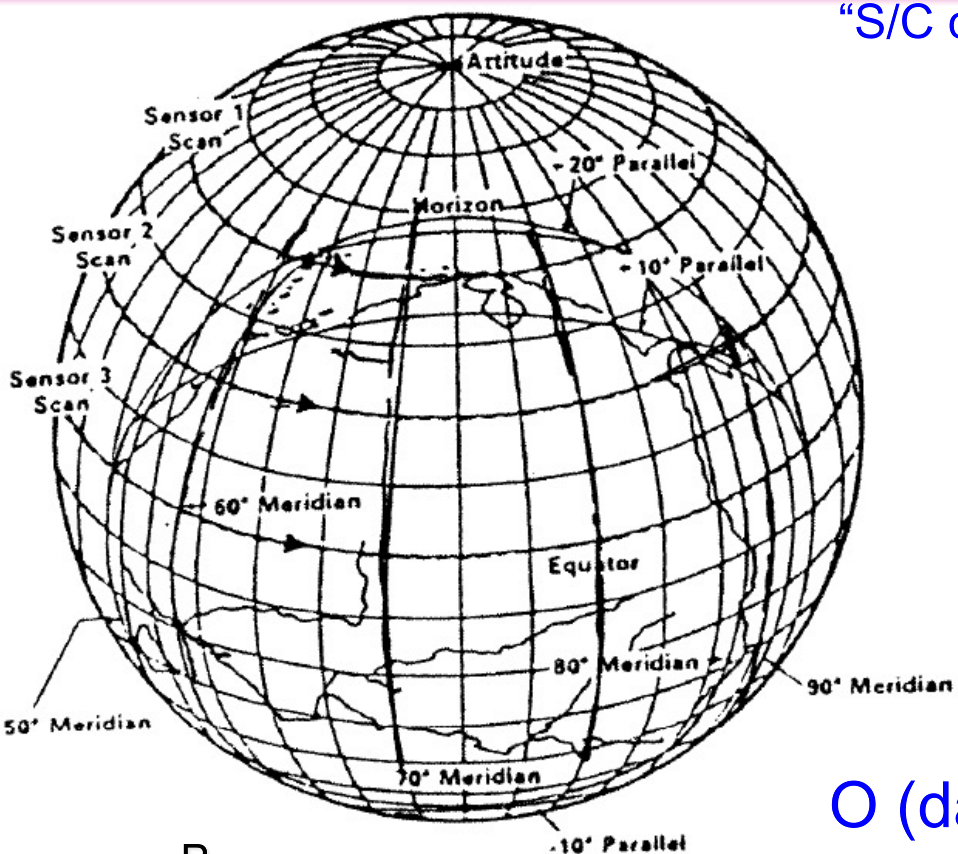


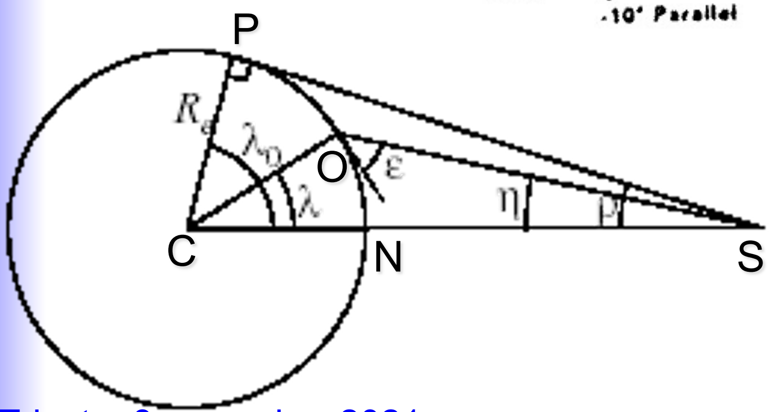
"Earth centered celestial sphere"

$l_o = 200^o$, $\lambda_o = 22^o$ (hawaii)

$l_N = 185^o$, $\lambda_N = 10^o$

Passaggio sopra la stazione 3/8





"S/C centered celestial sphere"

$l_o = 200^o$, $\lambda_o = 22^o$ (hawaii)

$l_N = 185^o$, $\lambda_N = 10^o$

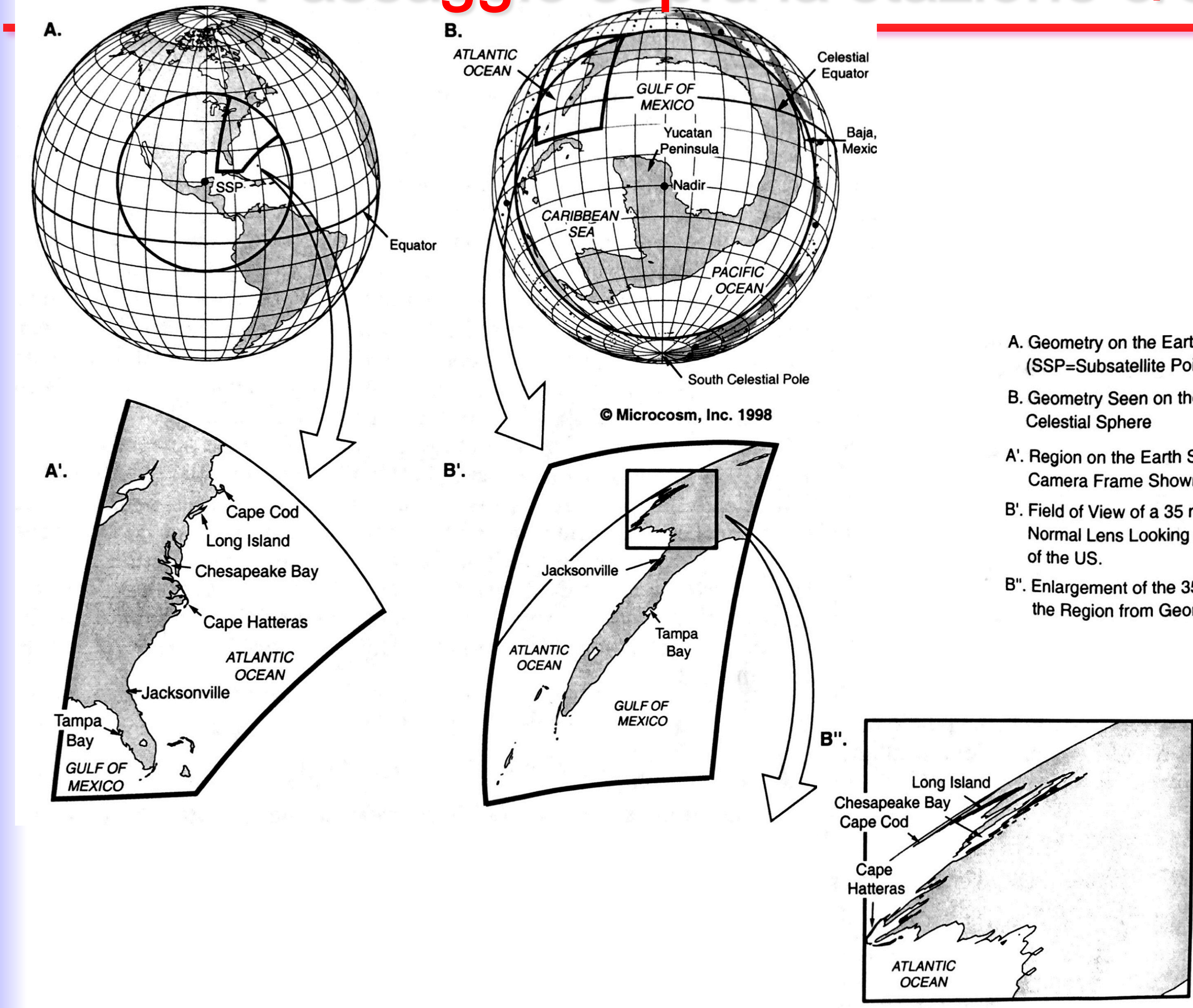
$\Rightarrow$

$\rho = 59.8^o$, $\lambda_0 = 30.2^o$,
$D_{max} = 3709$ km

O (da N)
- $\lambda = 18.7^o$ (swath width)
- Az = 48.3^o

O (da satellite)
- $\eta = 56.8^o$ ($\varepsilon=14.5^o$)
- D = 2444 km

Passaggio sopra la stazione 5/8



A. Geometry on the Earth's Surface (SSP=Subsatellite Point)

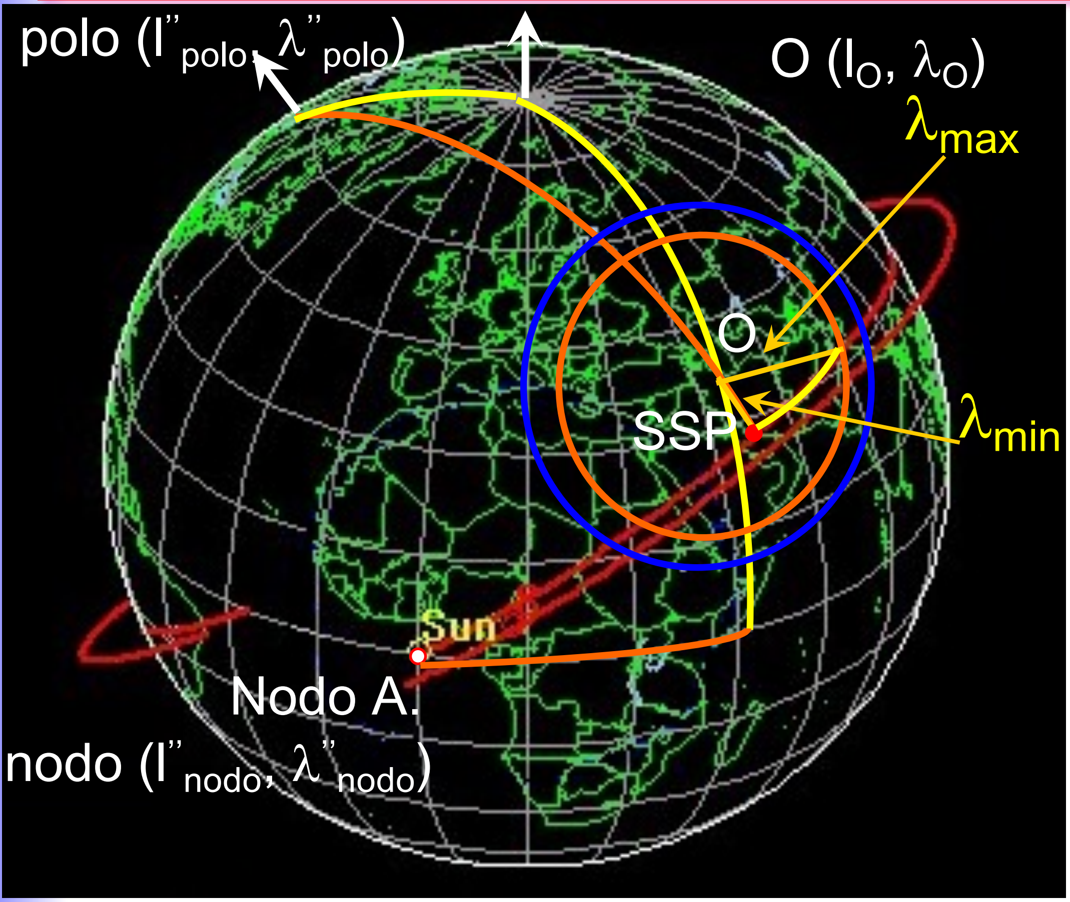
B. Geometry Seen on the Spacecraft Centered Celestial Sphere

A'. Region on the Earth Seen by the 35 mm Camera Frame Shown in (B')

B'. Field of View of a 35 mm Camera with a Normal Lens Looking Along the East Coast of the US.

B". Enlargement of the 35 mm Frame Showing the Region from Georgia to Massachusetts.

Passaggio sopra la stazione 6/8



"Earth centered celestial sphere"

$\sin \eta_{max} = \cos \varepsilon_{min} \sin \rho$

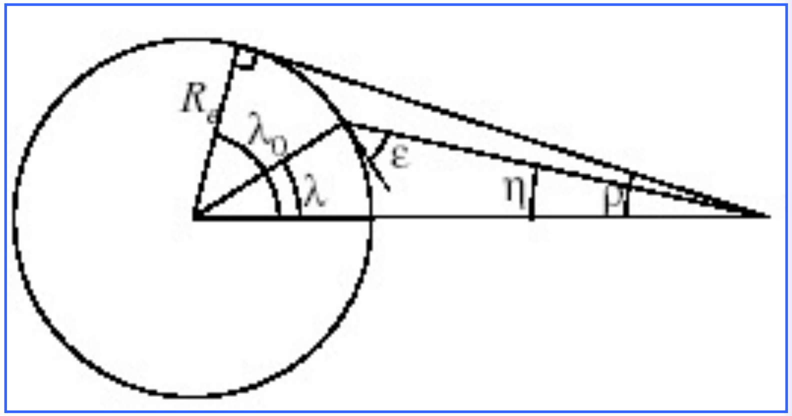
$\lambda_{max} = \pi/2 - \eta_{max} - \varepsilon_{min}$

$D_{max} = R_T \sin \lambda_{max} / \sin \eta_{max}$

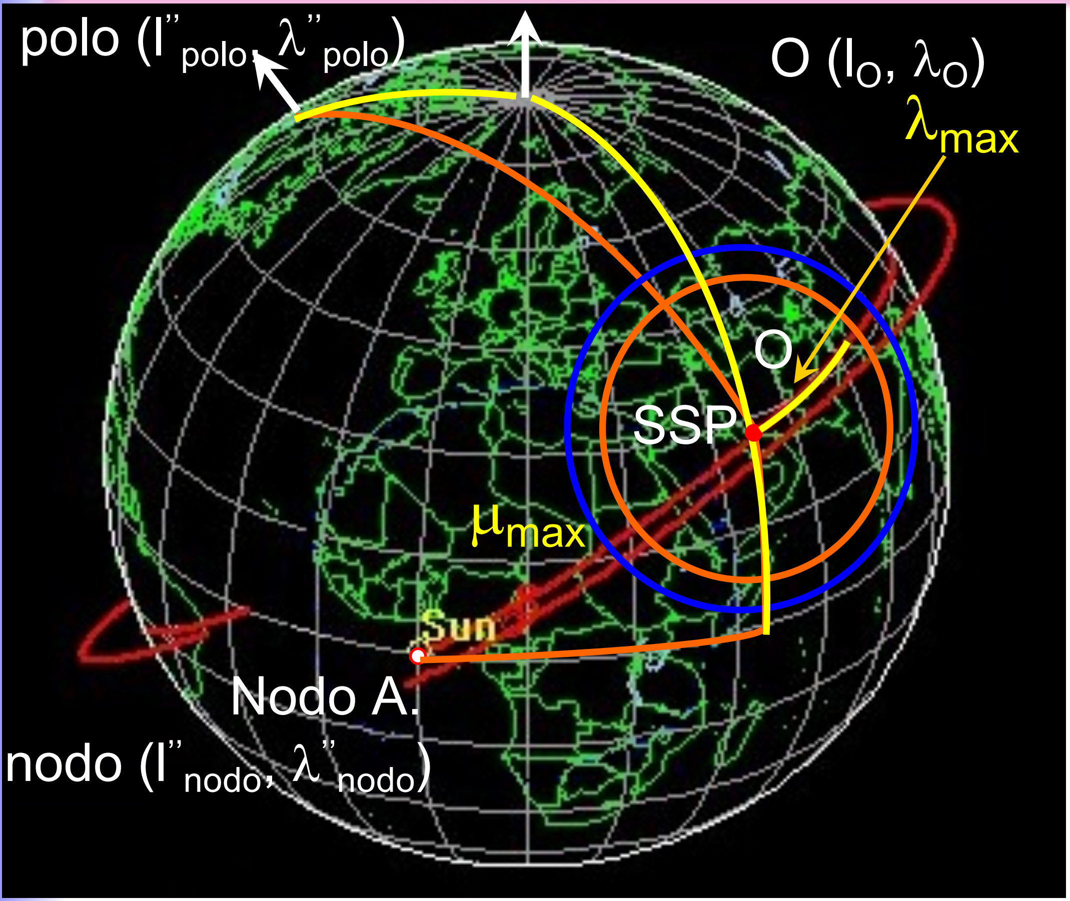
$\lambda''_{polo} = \pi/2 - i$ (lat)

$l''_{polo} = l''_{nodo} - \pi/2$ (long)

SMAD chapter 5.3.1
fig 5-17 p. 118-121



Passaggio sopra la stazione 7/8



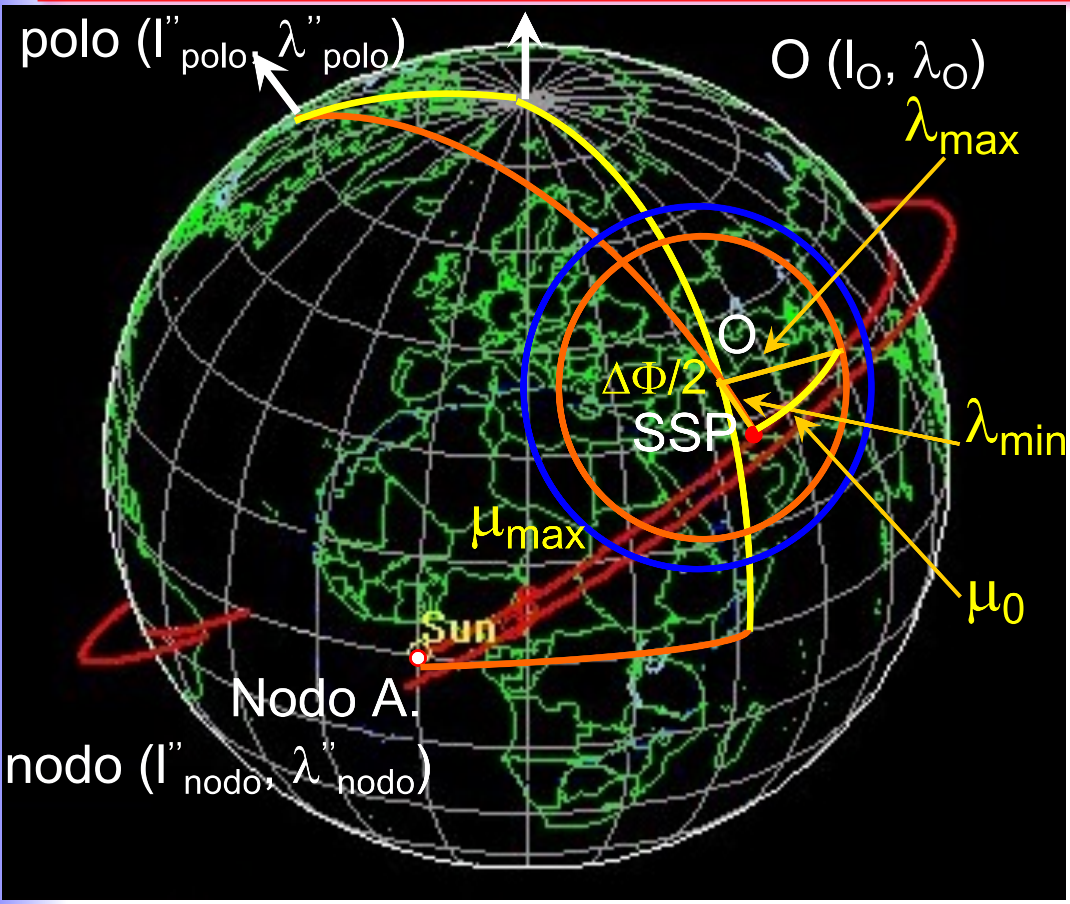
"Earth centered celestial sphere"

$\sin(l_O - l''_{nodo}) = \tan \lambda_O / \tan i$

$\sin \mu_{max} = \sin \lambda_O / \sin i$

O≡SSP

Passaggio sopra la stazione 8/8



"Earth centered celestial sphere"

$\sin(l_O - l''_{nodo}) = \tan \lambda_O / \tan i$

$\sin \mu_{max} = \sin \lambda_O / \sin i$

O≡SSP

$\sin \lambda_{min} = \sin \lambda''_{polo} \sin \lambda_O +$

$+ \cos \lambda''_{polo} \cos \lambda_O \cos(l_O - l''_{polo})$

$\tan \eta_{min} = \sin \rho \sin \lambda_{min} / (1 - \sin \rho \cos \lambda_{min})$

$\varepsilon_{max} = \pi/2 - \eta_{min} - \lambda_{min}$

$R_T \sin \lambda_{min} = D_{min} \sin \eta_{min}$

$\omega_{max} = \dot{\theta}_{max} = v_{sat} / D_{min}$

$\cos \Delta\Phi/2 = \tan \lambda_{min} / \tan \lambda_{max}$

$T = \tau/180^o \, \mathrm{acos}(\cos \lambda_{max} / \cos \lambda_{min})$