

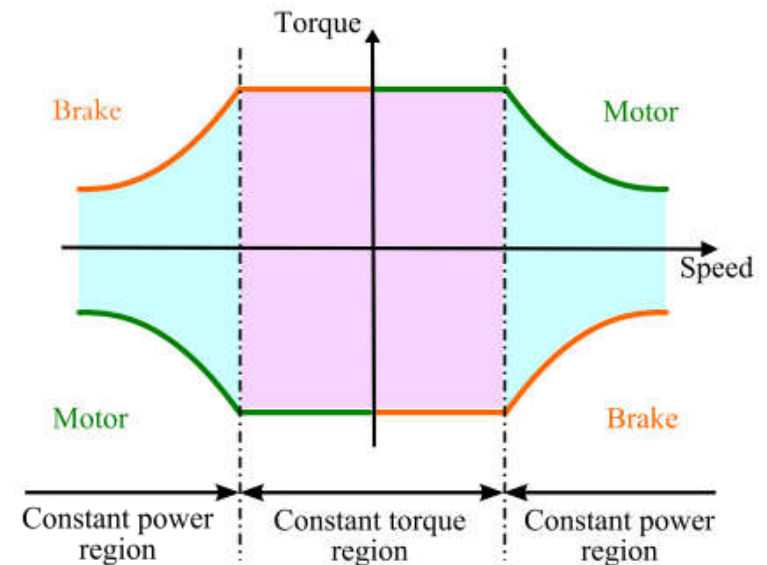
Macchine sincrone a  
magneti permanenti

# Electrical drive operating limits

They are of the mechanical type, torque and rotor speed, which are associated with their electrical limitations, current and voltage.

These limitations have physical nature related to the maximum electrical, thermal or mechanical stresses that the drive components are able to withstand.

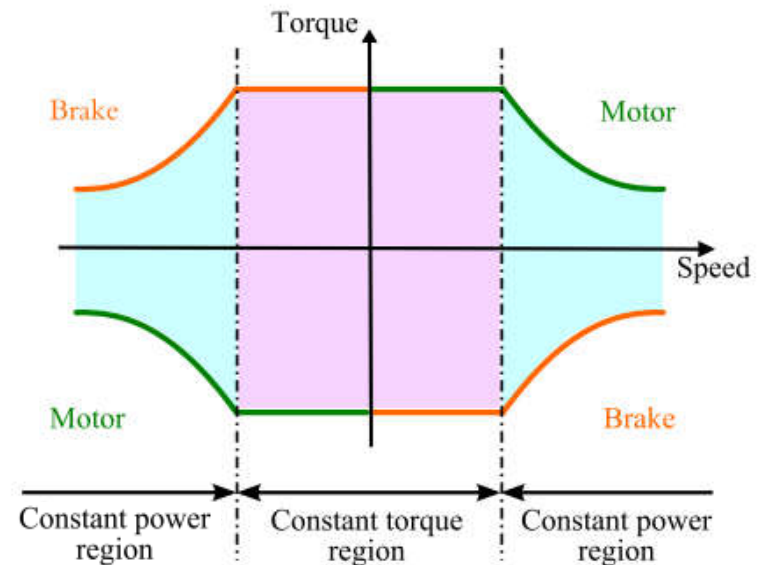
These limitations are generally reported in the rotor speed vs torque plan as shown in Fig.



Electrical drive operating limits in all quadrants.

- **Constant torque region:** this region is limited by constant current locus at its maximum value while the voltage is lower or equal to its maximum value. In this area, the electrical drive can deliver its maximum torque at any speed lower than the base speed which is reached when the voltage achieves its maximum value.

- **Constant power region:** this region is limited by constant current and voltage loci both equal to their maximum value. In this area, the electrical drive can deliver its maximum power at any speed higher than the base speed but with increasing the speed the torque decreases.



Electrical drive operating limits in all quadrants.

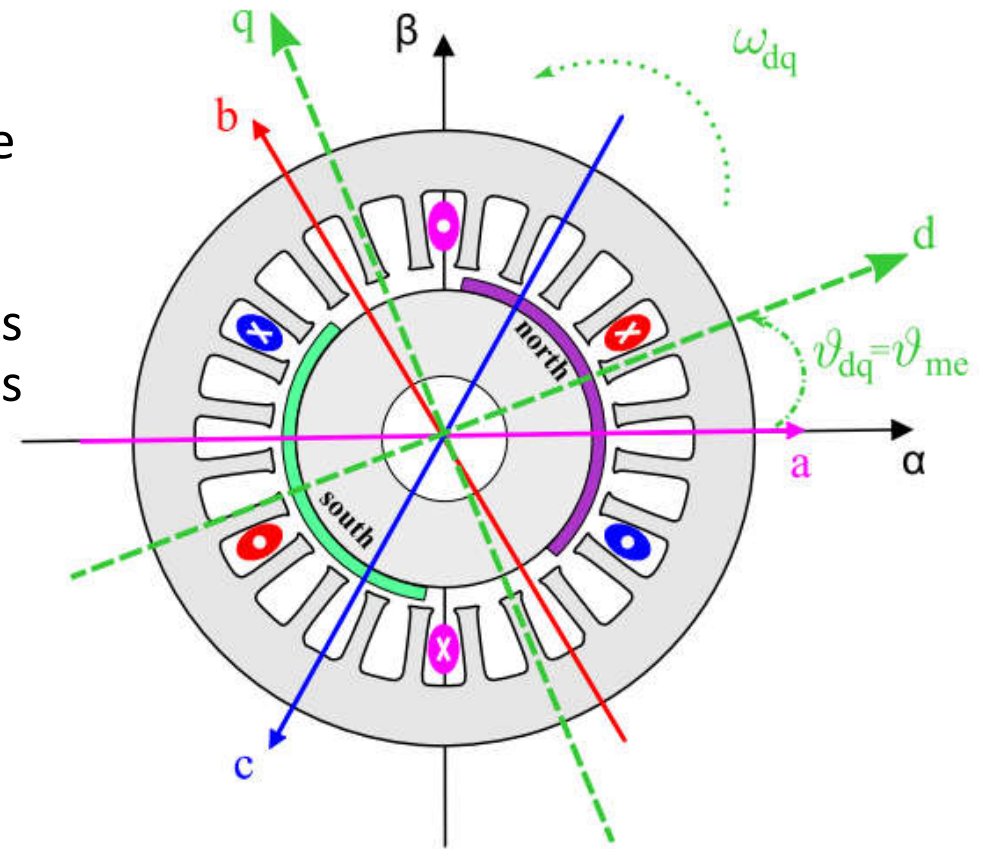
These definitions of operating limits have been done considering continuous operation of the drive. The overload of the drive for a limited period is generally allowed but limited by the thermal time constant of each electrical drive part.

Hereinafter the principal reasons that limit the electrical machine and the power converter have been analyzed:

- **Electrical machine:** the power capability of an electrical machine is limited by the maximum allowable temperature of its windings. Generally it is possible to make a current overload (torque) up to about three times of the nominal value for a time of a few minutes.
- **Power converter:** the power capacity of a power converter is limited by the maximum allowable temperature of its switching device, it is rated to its maximum current, or maximum insulation of its device, it is rated to its maximum voltage. Generally it is possible to make a current or voltage overload (torque or speed) up to about a few percent of the nominal value for a time of a second or a few ones.

# Synchronous electrical machine fundamental equations

The stator reference frame ( $\alpha; \beta$ ) is fixed in the direction of the  $a$ -phase-axis and in the direction perpendicular to this phase axis respectively. The rotor reference frame ( $d, q$ ) is fixed with the  $d$ -axis aligned to magnetic poles of the rotor, and the  $q$ -axis in the direction perpendicular to the former. The electromechanical angle  $\theta_{me}$  is highlighted between the rotor pole  $d$ -axis and the  $a$ -phase-axis.



At first an isotropic machine is considered in order to introduce the main relationships that characterize a PM synchronous machine.

Considering the three phases winding distributed along the stator with an electrical phase displacement of  $2\pi/3$  rad, the fundamental voltage equations for three-phase system,  $u_a(t)$ ,  $u_b(t)$ ,  $u_c(t)$ , are:

$$\begin{aligned}u_a(t) &= Ri_a(t) + \frac{d\lambda_a(t)}{dt} \\u_b(t) &= Ri_b(t) + \frac{d\lambda_b(t)}{dt} \\u_c(t) &= Ri_c(t) + \frac{d\lambda_c(t)}{dt}\end{aligned}$$

where the stator phase resistance  $R$  is assumed equal for all phases;

$i_a(t)$ ,  $i_b(t)$ ,  $i_c(t)$  are the phase currents,  $\lambda_a(t)$ ,  $\lambda_b(t)$ ,  $\lambda_c(t)$  are the stator phase flux linkages.

Assuming to neglect the iron saturation and eddy currents the stator flux linkages can be expressed as the sum of two components due to PM,  $\lambda_{mg}(t)$ , and phase winding currents,  $\lambda_i(t)$ :

$$\lambda_a(t) = \lambda_{a,mg}(t) + \lambda_{a,i}(t)$$

$$\lambda_b(t) = \lambda_{b,mg}(t) + \lambda_{b,i}(t)$$

$$\lambda_c(t) = \lambda_{c,mg}(t) + \lambda_{c,i}(t)$$

It is assumed that the flux linkage due to the magnets,  $\lambda_{mg}(t)$  is sinusoidal with the rotor electrical position and independent of stator currents; therefore the magnet flux equations become:

$$\lambda_{a,mg}(t) = \Lambda_{mg} \cos[\vartheta_{me}(t)]$$

$$\lambda_{b,mg}(t) = \Lambda_{mg} \cos[\vartheta_{me}(t) - 2\pi/3]$$

$$\lambda_{c,mg}(t) = \Lambda_{mg} \cos[\vartheta_{me}(t) - 4\pi/3]$$

where  $\Lambda_{mg}$  represents the PM flux linkage peak value, that is considered constant in time and position, and  $\vartheta_{me} = p\vartheta_m$

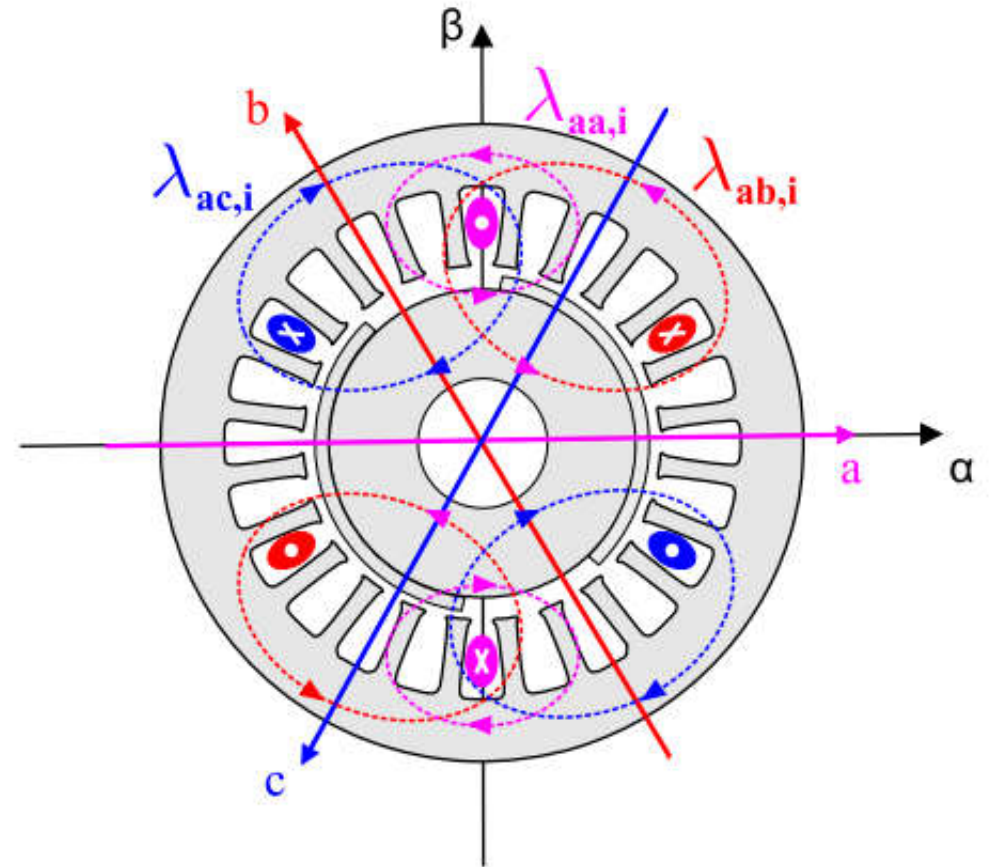
Now it is assumed that the permanent magnets are de-energized so the flux linkage due to the stator current can be expressed as:

$$\lambda_{a,i}(t) = \lambda_{aa,i}(t) + \lambda_{ab,i}(t) + \lambda_{ac,i}(t)$$

$$\lambda_{b,i}(t) = \lambda_{bb,i}(t) + \lambda_{ba,i}(t) + \lambda_{bc,i}(t)$$

$$\lambda_{c,i}(t) = \lambda_{cc,i}(t) + \lambda_{ca,i}(t) + \lambda_{cb,i}(t)$$

where  $\lambda_{aa,i}(t)$  is the flux linkage with a-phase winding due to a-phase current.  $\lambda_{ab,i}(t)$  and  $\lambda_{ac,i}(t)$  are the flux linkages with a-phase winding due to b-phase and c-phase current respectively.





The inductance components due to these fluxes can be expressed as:

$$\begin{aligned}L_{aa} &= \frac{\lambda_{aa,i}(t)}{i_a(t)} \\L_{M,ab} &= \frac{\lambda_{ab,i}(t)}{i_b(t)} = -|L_{M,ss}| < 0 \\L_{M,ac} &= \frac{\lambda_{ac,i}(t)}{i_c(t)} = -|L_{M,ss}| < 0\end{aligned}$$

Assuming, for geometric reasons, that the mutual-inductances  $L_{M,ss}$  between a-phase, b-phase, and c-phase are all equal to  $L_{M,ss}$  and reminding that the sum of three stator currents is null (i.e.  $i_a(t) + i_b(t) + i_c(t) = 0$ ) the synchronous inductance is defined as:

$$\begin{aligned}\lambda_{a,i}(t) &= \lambda_{aa,i}(t) + \lambda_{ab,i}(t) + \lambda_{ac,i}(t) \\&= L_{aa}i_a(t) - |L_{M,ss}| [i_b(t) + i_c(t)] \\&= (L_{aa} + |L_{M,ss}|) i_a(t) \\&= L_a i_a(t)\end{aligned}$$

Considering the isotropic machine all synchronous phase inductances are equal,  $L_a = L_b = L_c = L$  so the equations become:

$$\begin{aligned}\lambda_a(t) &= \Lambda_{mg} \cos[\vartheta_{me}(t)] + Li_a(t) \\ \lambda_b(t) &= \Lambda_{mg} \cos[\vartheta_{me}(t) - 2\pi/3] + Li_b(t) \\ \lambda_c(t) &= \Lambda_{mg} \cos[\vartheta_{me}(t) - 4\pi/3] + Li_c(t)\end{aligned}$$

Deriving the above equations with respect to time, they become:

$$\begin{aligned}\frac{d\lambda_a(t)}{dt} &= L \frac{di_a(t)}{dt} + e_a(t) \\ \frac{d\lambda_b(t)}{dt} &= L \frac{di_b(t)}{dt} + e_b(t) \\ \frac{d\lambda_c(t)}{dt} &= L \frac{di_c(t)}{dt} + e_c(t)\end{aligned}$$

where  $e_a(t)$ ,  $e_b(t)$ , and  $e_c(t)$  are the back electromotive forces (b.e.m.f) due to the PM flux linked by the phases:

$$\begin{aligned}e_a(t) &= \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2] \\e_b(t) &= \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2 - 2\pi/3] \\e_c(t) &= \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2 - 4\pi/3]\end{aligned}$$

where  $\omega_{me}(t)$  is the electrical speed.

By putting all together:

$$\begin{aligned}u_a(t) &= Ri_a(t) + L\frac{di_a(t)}{dt} + \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2] \\u_b(t) &= Ri_b(t) + L\frac{di_b(t)}{dt} + \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2 - 2\pi/3] \\u_c(t) &= Ri_c(t) + L\frac{di_c(t)}{dt} + \Lambda_{mg}\omega_{me}(t) \cos[\vartheta_{me}(t) + \pi/2 - 4\pi/3]\end{aligned}$$

It is possible to simplify the voltage equations by using the space vector notion:

$$\bar{u}^s(t) = R\bar{i}^s(t) + L\frac{d\bar{i}^s(t)}{dt} + j\omega_{me}(t)\bar{\lambda}_{mg}^s(t)$$

where the notation  $s$  indicate the stationary reference frame ( $\alpha$ ;  $\beta$ ). In particular the real component  $u_\alpha(t)$  and imaginary component  $u_\beta(t)$  of the voltage space vector are:

$$u_\alpha(t) = Ri_\alpha(t) + L\frac{di_\alpha(t)}{dt} - \omega_{me}(t)\lambda_{\beta,mg}(t)$$

$$u_\beta(t) = Ri_\beta(t) + \underbrace{L\frac{di_\beta(t)}{dt}}_{\text{Trasfomatoric EMF}} + \underbrace{\omega_{me}(t)\lambda_{\alpha,mg}(t)}_{\text{Motional EMF}}$$

Applying the transformation between the stationary reference frame, to the rotating reference frame the space vector of the PM flux  $\lambda_{r,mg}(t)$  results to have only real component being placed on the real axis of the rotating reference frame. Therefore the voltage space vector in the new reference frame becomes:

$$\bar{u}^r(t) = R\bar{i}^r(t) + L\frac{d\bar{i}^r(t)}{dt} + j\omega_{me}(t)L\bar{i}^r(t) + j\omega_{me}(t)\Lambda_{mg}$$

where the notation r indicate the rotating reference frame (d; q). The real component  $u_d(t)$  and imaginary component  $u_q(t)$  are:

$$u_d(t) = Ri_d(t) + L\frac{di_d(t)}{dt} - \omega_{me}(t)Li_q(t)$$

$$u_q(t) = Ri_q(t) + L\frac{di_q(t)}{dt} + \omega_{me}(t)Li_d(t) + \omega_{me}(t)\Lambda_{mg}$$

# Power balance

The power balance can be obtained multiplying the voltage equations for the respective currents and summing term by term; one can obtain

$$\underbrace{u_d(t)i_d(t) + u_q(t)i_q(t)}_{\frac{2}{3}P_e(t)} = \underbrace{R [i_d^2(t) + i_q^2(t)]}_{\frac{2}{3}P_{joule}(t)} + \underbrace{L \left[ i_d(t) \frac{di_d(t)}{dt} + i_q(t) \frac{di_q(t)}{dt} \right]}_{\frac{2}{3} \frac{dW_m(t)}{dt}} + \underbrace{\omega_{me}(t) \Lambda_{mg} i_q(t)}_{\frac{2}{3}P_{em}(t)}$$

where  $P_e(t)$  is the input electric power,  $P_{joule}(t)$  refers to dissipated Joule losses by the phase resistances,  $W_m(t)$  is the magnetic energy,  $P_{em}(t)$  is the electromechanical power. The ratio 2/3 in the equation allows to maintain the energy conservation during the transformation from the stationary reference frame to the d,q rotating reference frame.

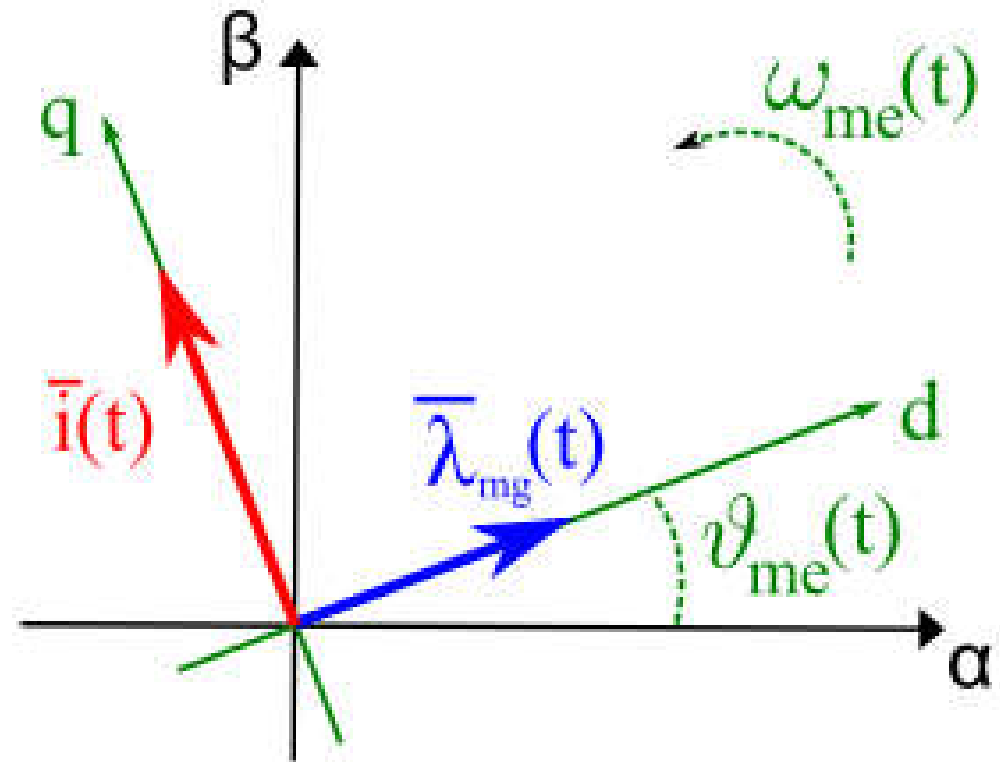
Taking into account that:  $P_{em}(t) = T(t)\omega_m(t)$

The electromagnetic torque is determined by the magnitude of the q-axis current and expressed as:

$$T(t) = \frac{3}{2}p\Lambda_{mg}i_q(t)$$

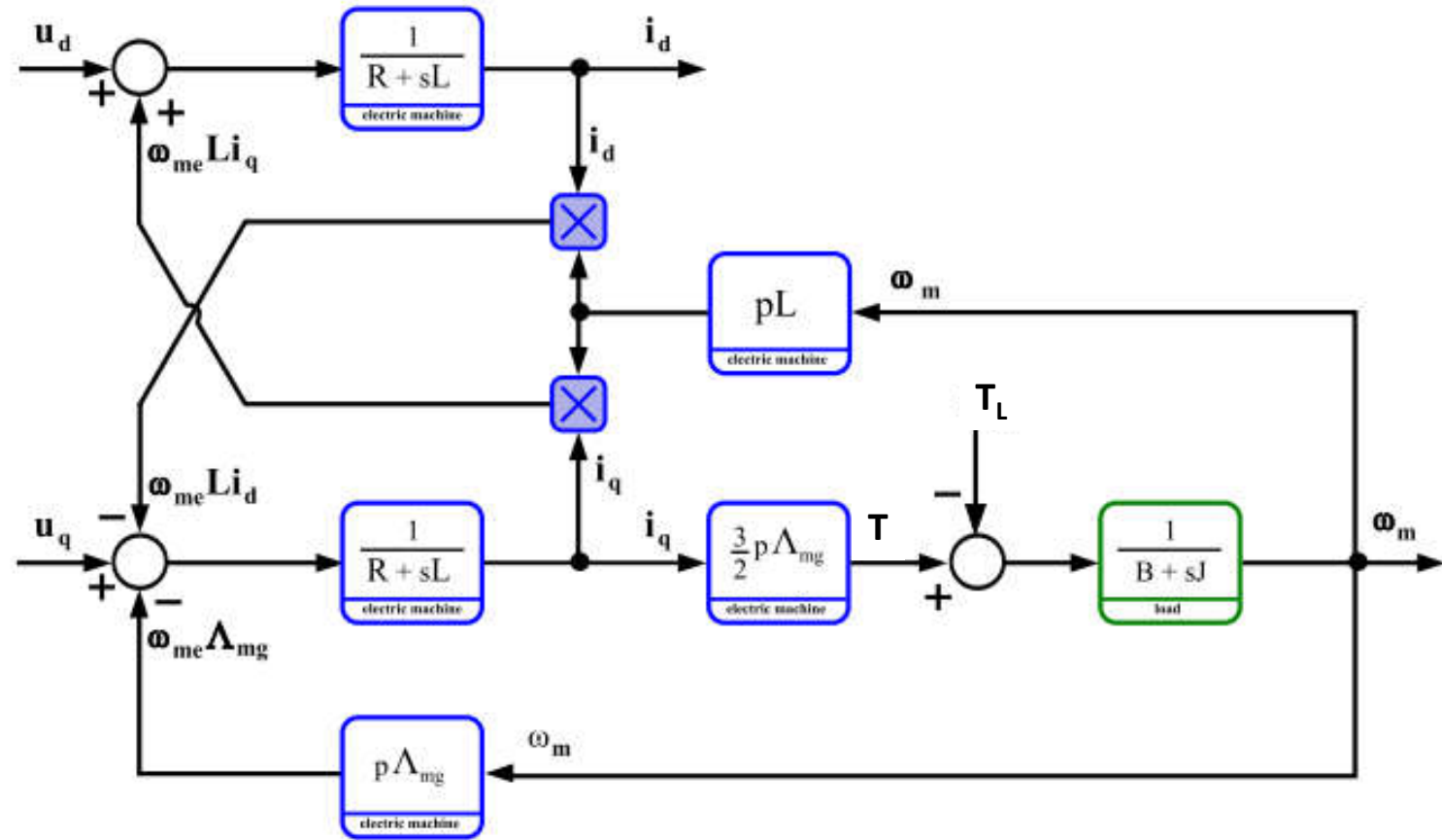
$$T(t) = \frac{3}{2} p \Lambda_{mg} i_q(t)$$

It is possible to highlighted that, in order to reduce the copper losses, the space vector of the current ( $i(t)$ ) must be aligned to  $q$ -axis because the PM flux space vector ( $\lambda_{mg}(t)$ ) is aligned in  $d$ -axis, for definition of rotating reference frame.



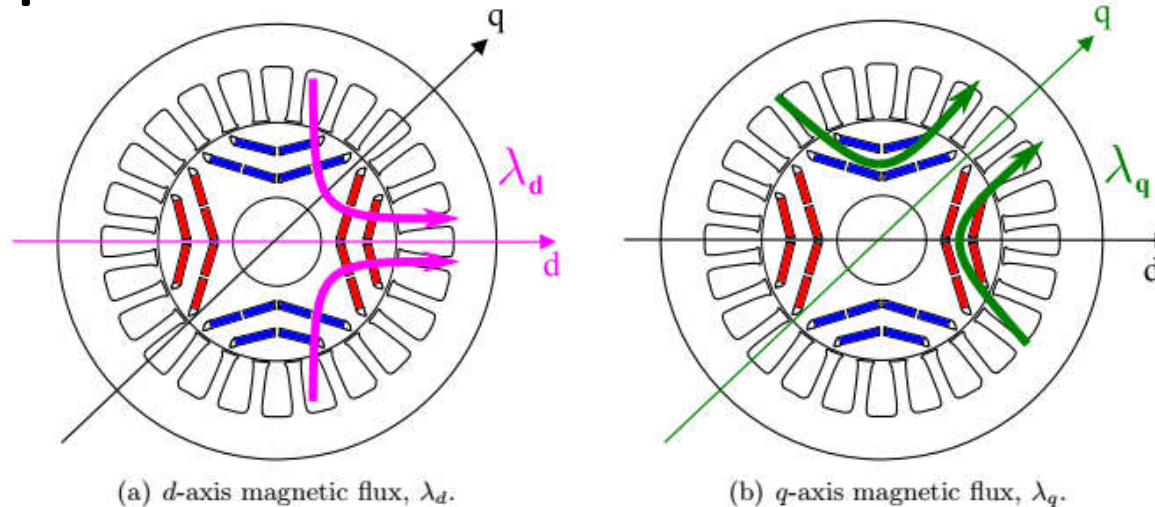
# Isotropic machine block diagram in $d; q$ reference frame

This block diagram contains nonlinear elements as multipliers; instead the linear blocks are represented by their transfer functions between their input and output. As it can be noted there is coupling between  $d$ -axis and  $q$ -axis, this contribution is expressed as  $+\omega_{me}Li_q$  that acts on the voltage  $u_d$  through the current  $i_q$  and vice versa  $-\omega_{me}Li_d$  that acts on the voltage  $u_q$  through the current  $i_d$ .





# Anisotropic machine considerations



The synchronous anisotropic PM machines are characterized by a rotor structure that yields a magnetic anisotropy, or rather a different magnetic behavior along the polar and inter-polar axes. Therefore with the same considerations that have been stated for the isotropic machine it can be assumed also in anisotropic machine that the conductors have a sinusoidal distribution and by a proper magnet shape it is possible to consider sinusoidal the flux linkages  $\lambda_{a,mg}(t)$ ,  $\lambda_{b,mg}(t)$ ,  $\lambda_{c,mg}(t)$ , so the equations are still valid. Instead it is not possible to characterize the machine with only one value of self-inductance and mutual-inductance because their values vary as a function of the rotor position.

In the rotating reference frame each axis is characterized by a proper inductance, thus the equations becomes:

$$u_d(t) = Ri_d(t) + L_d \frac{di_d(t)}{dt} - \omega_{me}(t)L_q i_q(t)$$

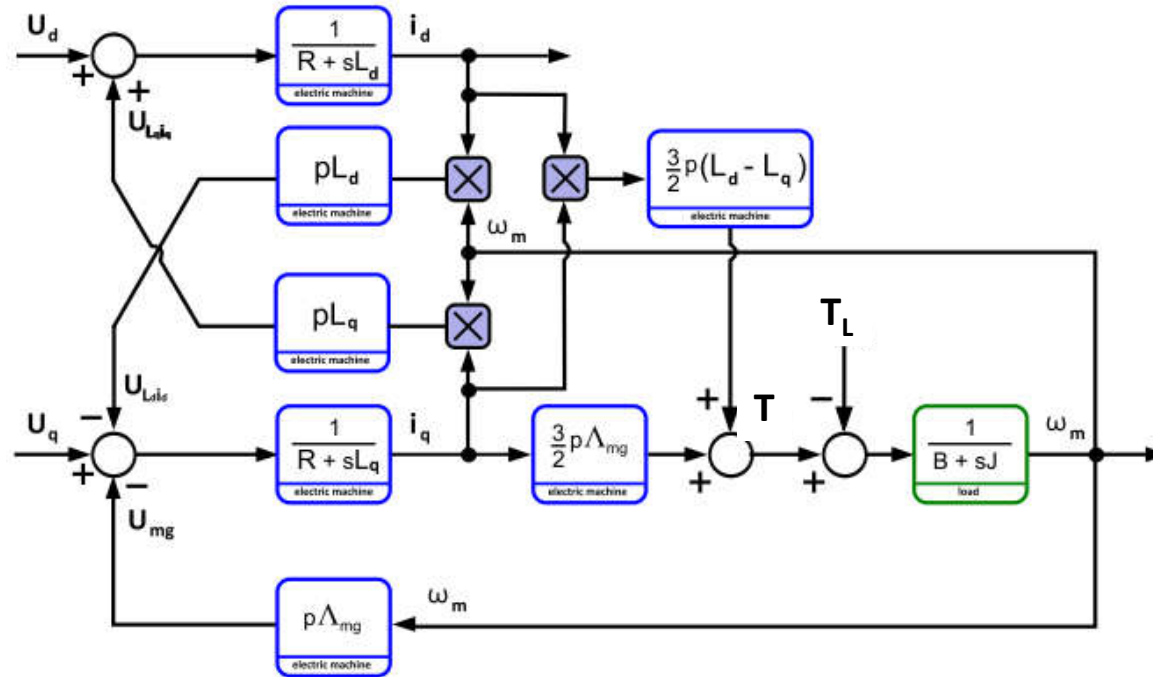
$$u_q(t) = Ri_q(t) + L_q \frac{di_q(t)}{dt} + \omega_{me}(t)L_d i_d(t) + \omega_{me}(t)\Lambda_{mg}$$

where  $L_d$  and  $L_q$  indicate the direct and quadrature inductances respectively. Generally in the IPM machine  $L_q$  is two/three times higher than  $L_d$ .

With these considerations the torque equation of an anisotropic machine becomes:

$$\begin{aligned} T(t) &= \frac{3}{2}p\Lambda_{mg}i_q(t) + \frac{3}{2}p(L_d - L_q)i_d(t)i_q(t) \\ &= \frac{3}{2}p[\Lambda_{mg} + (L_d - L_q)i_d(t)]i_q(t) \end{aligned}$$

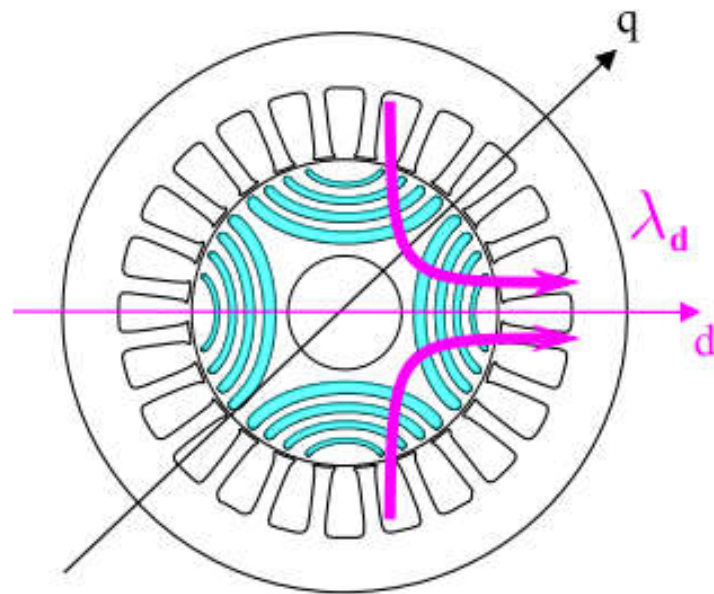
# IPM machine block diagram in $d; q$ reference frame



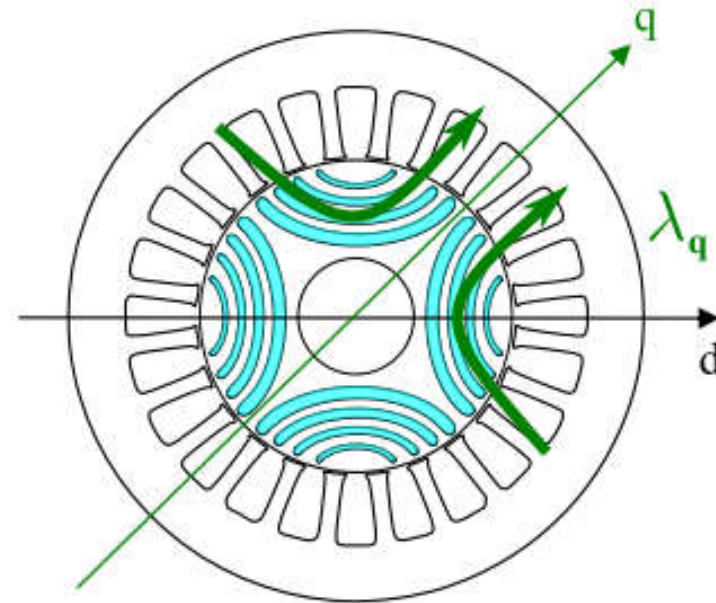
Likewise the block diagram of SPM machine, also the IPM machine contains nonlinear elements as multipliers. Instead the linear blocks are represented by their transfer functions between their input and their output. With this kind of rotor configuration there are two different types of coupling between  $d$ -axis and  $q$ -axis. The first is the same of SPM machine it is expressed as  $+\omega_{me}L_qi_q$  that acts on the voltage  $u_d$  through the current  $i_q$  and vice versa  $-\omega_{me}L_d i_d$  that acts on the voltage  $u_q$  through the current  $i_d$ . The second due to the reluctance torque component.

# Reluctance machine considerations

The reluctance machine is characterized by the absence of PMs; however the air barriers create a different magnetic behavior along the two rotating axes.



(a) *d*-axis magnetic flux,  $\lambda_d$ .



(b) *q*-axis magnetic flux,  $\lambda_q$ .

With the same assumptions done for IPM machine, also in reluctance machine it can be assumed that the conductors have a sinusoidal distribution the equations becomes:

$$u_d(t) = Ri_d(t) + L_d \frac{di_d(t)}{dt} - \omega_{me}(t) L_q i_q(t)$$

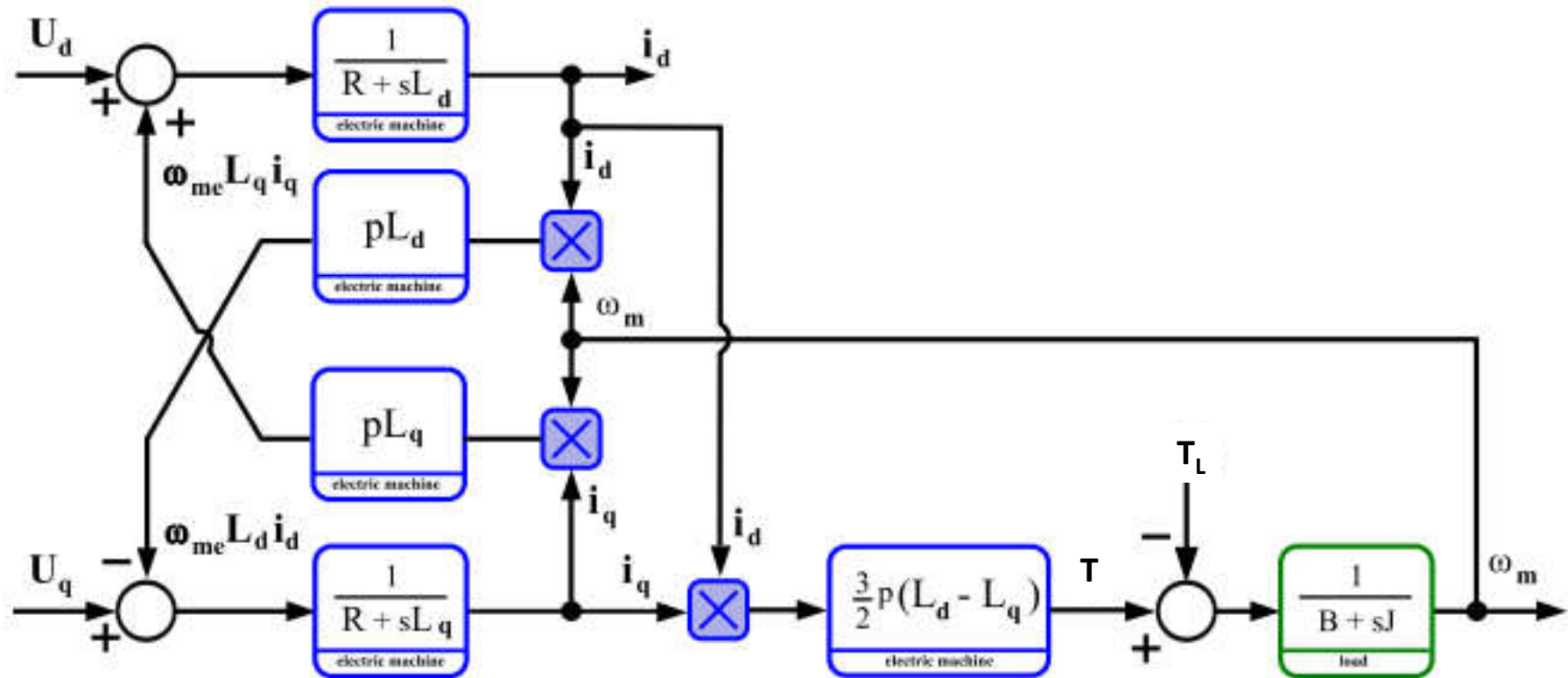
$$u_q(t) = Ri_q(t) + L_q \frac{di_q(t)}{dt} + \omega_{me}(t) L_d i_d(t)$$

where  $L_d$  and  $L_q$  indicate the direct and quadrature inductances respectively. Generally in the reluctance machine  $L_q$  is six/ten times higher than  $L_d$ .

With this considerations the torque equation of reluctance machine becomes:

$$T(t) = \frac{3}{2} p (L_d - L_q) i_d(t) i_q(t)$$

Reluctance machine block diagram in  $d; q$  reference frame.



# Electric machine operating regions

EM has its operating regions that are limited by the maximum available voltage and current that the drive is able to deliver. It is considered, with reasonable assumption, that the drive limits (i.e. maximum voltage and current of devices) for each operating point are the same of electric machine limits (i.e. thermal limits, insulation limits, and PMs demagnetization).

In order to investigate the EM operating working points some assumption have been taken into account. For example: steady state operations, sinusoidal currents and voltages (with constant amplitude and frequency), and constant electrical speed ( $\Omega_{me}$ ). Therefore in the synchronous rotating reference frame the voltage ( $U_d$  and  $U_q$ ) and current quantities ( $I_d$  and  $I_q$ ) have constant amplitude.

The nominal phase-to-phase voltage is indicated as  $U_{nom}$ , that is the maximum voltage value. The nominal phase current is  $I_{nom}$  and it complies with the thermal limit in steady state condition. Nevertheless it is possible to overload the EM with a current higher than the nominal one for a short period according to the thermal and demagnetizing current limits.

The current limit can be expressed directly by the  $d$ ;  $q$  components of the current ( $I_d$  and  $I_q$ ) as:

$$I_d^2 + I_q^2 \leq I_N^2$$

where  $I_N$  is the amplitude of the stator current space vector which is the peak value of the nominal phase current  $I_{\text{nom}}$  (i.e.  $I_N = \sqrt{2}I_{\text{nom}}$  with sinusoidal current). Similarly, the voltage limit is:

$$U_d^2 + U_q^2 \leq U_N^2$$

where  $U_N$  is the amplitude of the stator voltage space vector which is the peak value of the nominal phase voltage  $U_{\text{nom}}$  (i.e.  $U_N = \sqrt{\frac{2}{3}}U_{\text{nom}}$ ).



# SPM machine

Considering a synchronous SPM motor with isotropic rotor the voltage equations in rotating reference frame and in steady state condition are described with the following relationships:

$$U_d = RI_d - \Omega_{me}LI_q$$

$$U_q = RI_q + \Omega_{me}(LI_d + \Lambda_{mg})$$

The phase resistance R voltage drop can be neglected because it is typically a few percent of the nominal voltage. Therefore with this assumption it is possible to express the voltage limit as a function of the current as:

$$(\Omega_{me}LI_q)^2 + (\Omega_{me}LI_d + \Omega_{me}\Lambda_{mg})^2 \leq U_N^2$$

that can be written as:

$$\left(I_d + \frac{\Lambda_{mg}}{L}\right)^2 + I_q^2 \leq \left(\frac{U_N}{\Omega_{me}L}\right)^2$$

# Operating limits and working points of an isotropic SPM machine with high short circuit current.

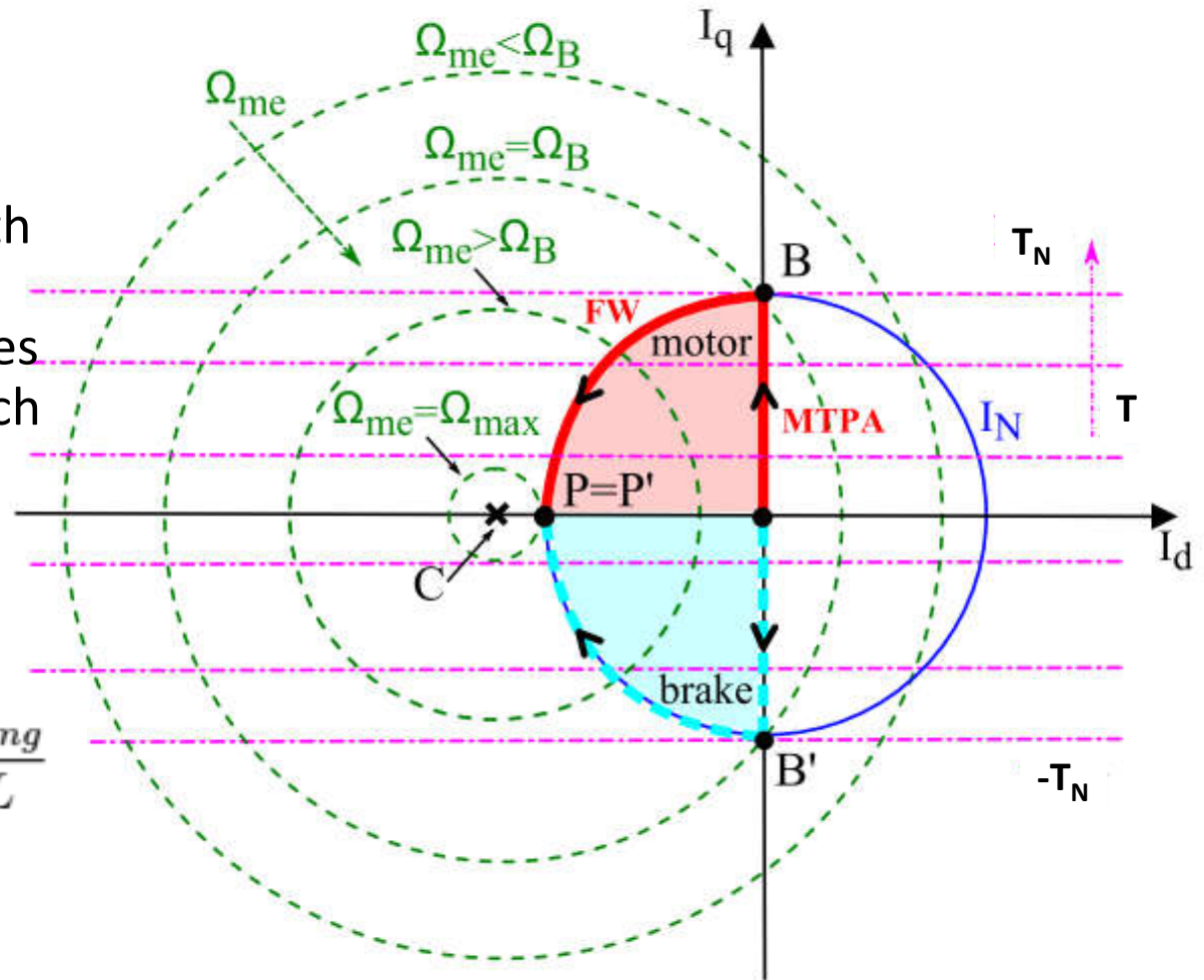
These operating limits can be easily reported in the  $d,q$  current plane as it is shown in Fig. The current limit is represented by a circle (blue solid line) with radius equal to  $I_N$  and the voltage limit is represented by a family of concentric circles (green dashed lines) the amplitude of which circle is inversely proportional to the rotor electrical speed.

The coordinates of the voltage limit center  $C$  is:

where these two coordinates correspond to the EM short circuit current components.

$$I_{Cd} = -\frac{\Lambda_{mg}}{L}$$

$$I_{Cq} = 0$$



These current components are obtained by neglecting the resistive voltage drops; in such case the short circuit current results independent of the speed.

Typically in an isotropic rotor the short circuit current is higher than the nominal one therefore the voltage limit circles center is placed outside the current limit circle.

In according with equation

$$T(t) = \frac{3}{2}p\Lambda_{mg}i_q(t)$$

in the isotropic EM the constant torque loci are parallel lines to  $Id$ -axis because the torque depends only on  $Iq$ .

An example of these lines are reported in magenta dashed-dotted lines. The trajectory  $BB'$  contains all the tangent point between the constant torque loci and the current limits. These points are characterized by the maximum ratio between the torque and the current, the  $BB'$  line is called Maximum Torque Per Ampere (MTPA) trajectory. Point  $B$  represents the positive value of the nominal torque  $MN$ . In this point the EM works as motor instead the point  $B'$  depicts the negative value of the nominal torque  $-MN$  and the EM operates as brake.

The MTPA trajectory is represented by:

$$I_{d,MTPA} = 0$$

$$|I_{q,MTPA}| \leq I_N$$

When the values of rotor speed and current vary the EM has to satisfy both of voltage and current limits. As an example at low speed the voltage circle radius is high, therefore the current limit is more restrictive, so the EM can be operated in the MTPA trajectory ( $BB'$ ) and it can provide the required torque up to the nominal one. This strategy can be adopted until the rotor speed reaches the base speed  $\Omega_B$ . At this speed both the voltage and current limits contain the points  $B$ , the base speed value is:

$$\Omega_B = \frac{U_N}{\sqrt{\Lambda_{mg}^2 + (LI_N)^2}}$$

At rotor speed higher than  $\Omega_B$ , the available limit for the operating points is in line  $BP$  (and  $B'P$ ) where the maximum value of the torque is lower than the nominal value even with the same current. This operating region limited by lines  $BP$  and  $B'P$  is called flux-weakening (FW) region. The maximum speed  $\Omega_{max}$  is defined when the operating point  $P$  is reached. In this point the torque becomes null and current  $I_d = -I_N$ . The maximum electrical speed is:

$$\Omega_{max} = \frac{U_N}{\Lambda_{mg} - LI_N}$$

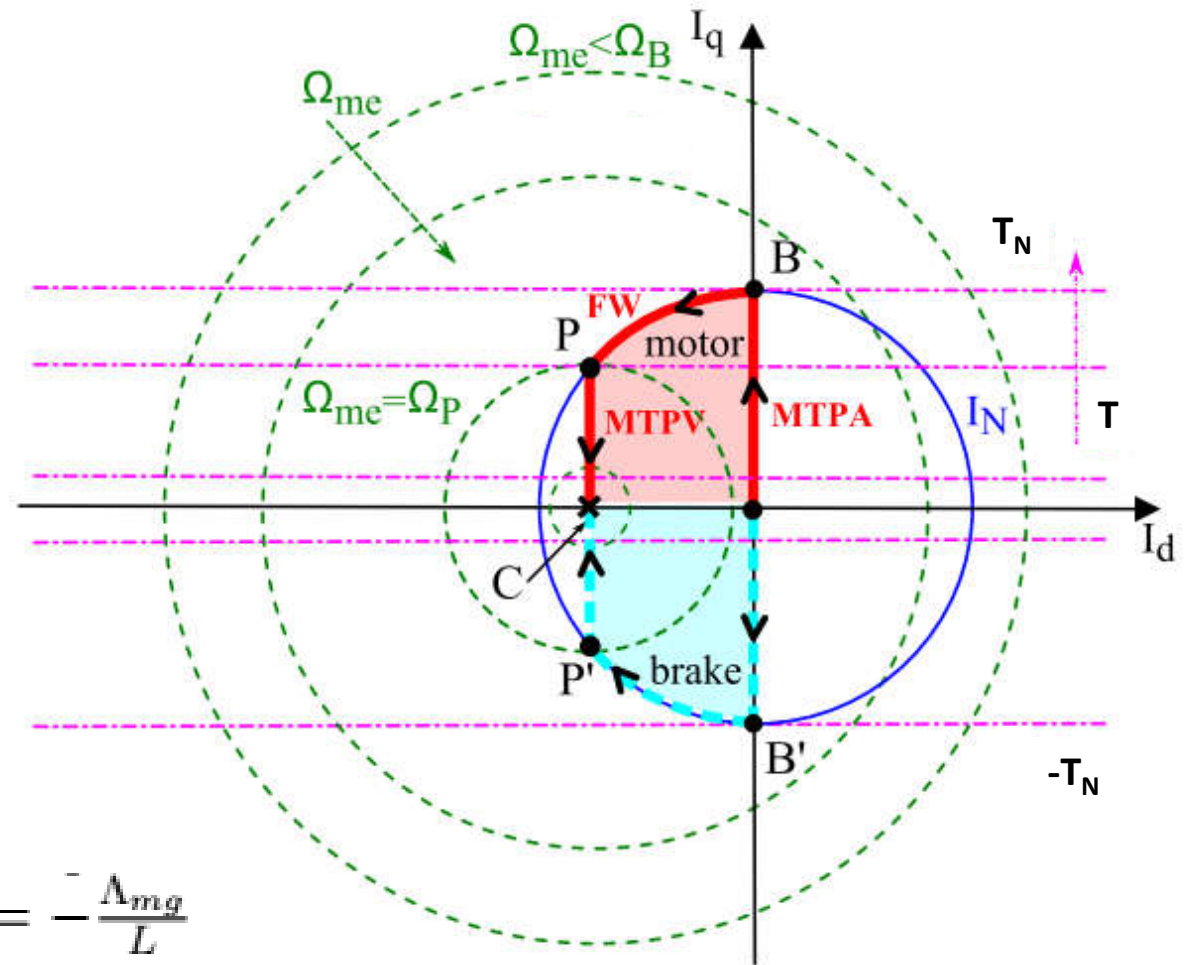
# Operating limits and working points of an isotropic SPM machine with low short circuit current.

Considering a short circuit current lower than the nominal one, the voltage limit center  $C$  is located inside the current limit circle and the operating strategy changes according to Fig. Let us note that the maximum speed can reach the infinite value if the short circuit current is less or equal to the nominal current.

$$\Omega_P = \frac{U_N}{\sqrt{(LI_N)^2 - \Lambda_{mg}^2}}$$

satisfying the conditions

$$I_q^2 = I_N^2 - I_d^2 \text{ and } I_d = -\frac{\Lambda_{mg}}{L}$$



For electrical speed higher than  $\Omega_p$ , the maximum available torque is achieved supplying the motor with current vectors along the segment  $PP'$ . Similarly as above for  $BB'$ , the segment  $PP'$  allows to reach the maximum ratio between the torque and the available voltage, i.e. the Maximum Torque Per Volts (MTPV) condition.

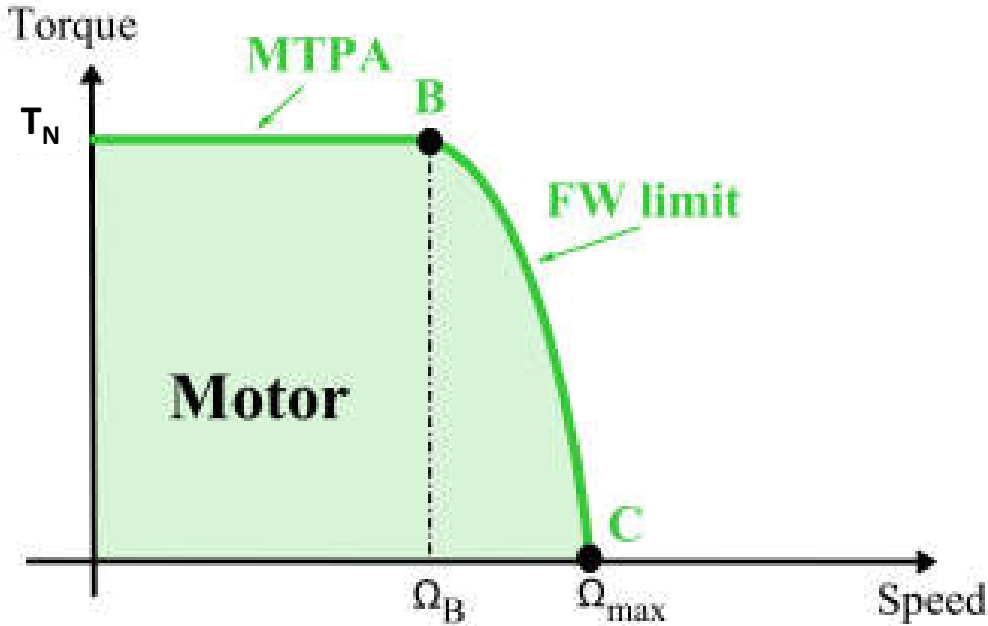
The equations that describe the MTPV trajectory are:

$$I_{d,MTPV} = -\frac{\Lambda_{mg}}{L}$$
$$|I_{q,MTPV}| \leq \sqrt{I_N^2 - I_{d,MTPV}^2}$$

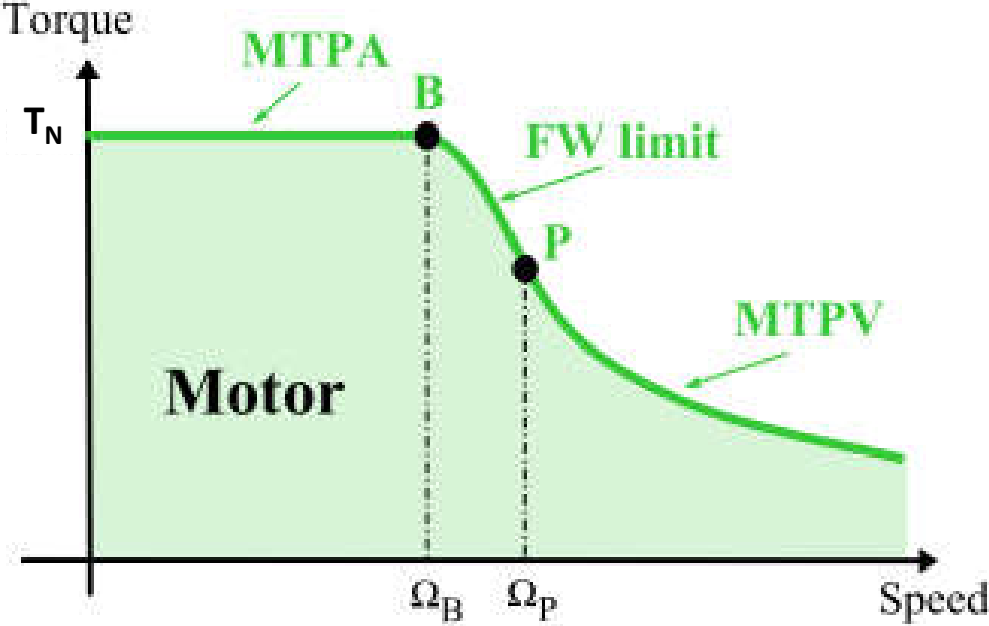
Adopting this strategy there is not a speed limit: the voltage circle center C is reached ideally at infinite speed.

NOTE: Typically the isotropic motors are characterized by a PM flux value ( $\Lambda_{mg}$ ) higher than  $LI_N$ , therefore the maximum speed becomes slightly higher than the base speed (typically about 20 ÷ 30% higher of  $\Omega_B$ ).

Torque vs speed region of an isotropic SPM machine with different values of short circuit current.



(a) High short circuit current.



(b) Low short circuit current.

# IPM machine

In the case of anisotropic rotor in steady state condition the voltage components, that are reported in equations became:

$$U_d = RI_d - \Omega_{me}L_qI_q$$

$$U_q = RI_q + \Omega_{me}(L_dI_d + \Lambda_{mg})$$

Neglecting the resistive the voltage limit can be expressed as:

$$(\Omega_{me}L_qI_q)^2 + (\Omega_{me}L_dI_d + \Omega_{me}\Lambda_{mg})^2 \leq U_N^2$$

Therefore:

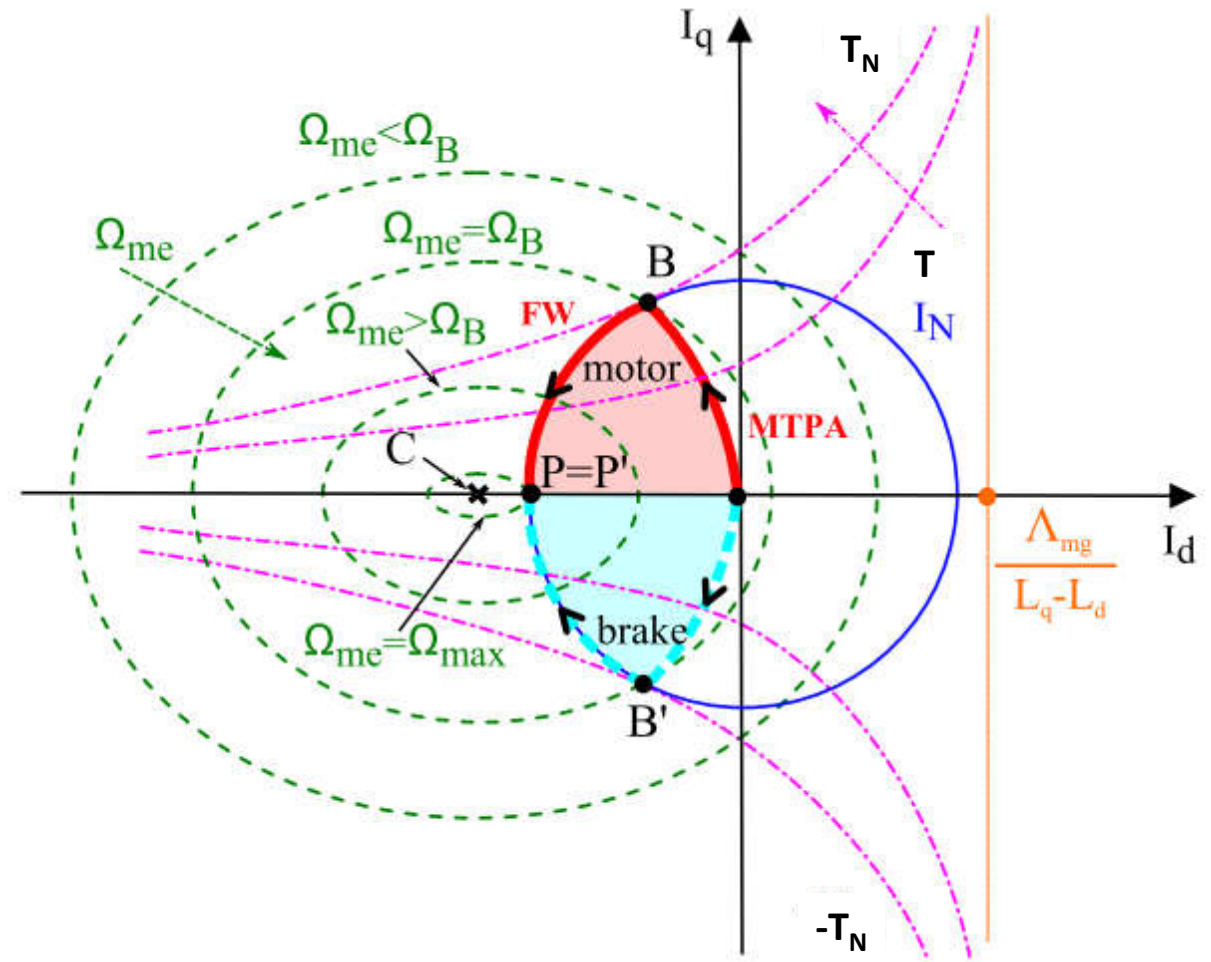
$$\left(I_d + \frac{\Lambda_{mg}}{L_d}\right)^2 + \left(\frac{L_q}{L_d}I_q\right)^2 \leq \frac{U_N^2}{(\Omega_{me}L_d)^2}$$

The voltage limit is represented by a family of concentric ellipses.



# Operating limits and working points of an anisotropic IPM machine with high short circuit current.

for anisotropic IPM machine the constant torque characteristics are a family of hyperboles that has as asymptotes the  $d$ -axis and the vertical line with coordinate  $I_d = \Lambda_{mg}/(L_q - L_d)$ , as reported in orange dashed-dotted line.



The ellipse center C coordinates are:

$$I_{Cd} = -\frac{\Lambda_m}{L_d}$$

$$I_{Cq} = 0$$

The curve  $BB'$  indicates the MTPA trajectory. This curve contains the tangent point between the constant torque hyperboles and the current circles at different current values. The expression of the MTPA trajectory is obtained by imposing the orthogonality condition of the tangent line ( $l_1$ ) of the constant torque line ( $M_x$ ) at a given point  $Q$  with the joining straight line ( $l_2$ ) from this point  $Q$  to the origin of the axes. The point  $Q$  is in current limit circle ( $I_x$ ).

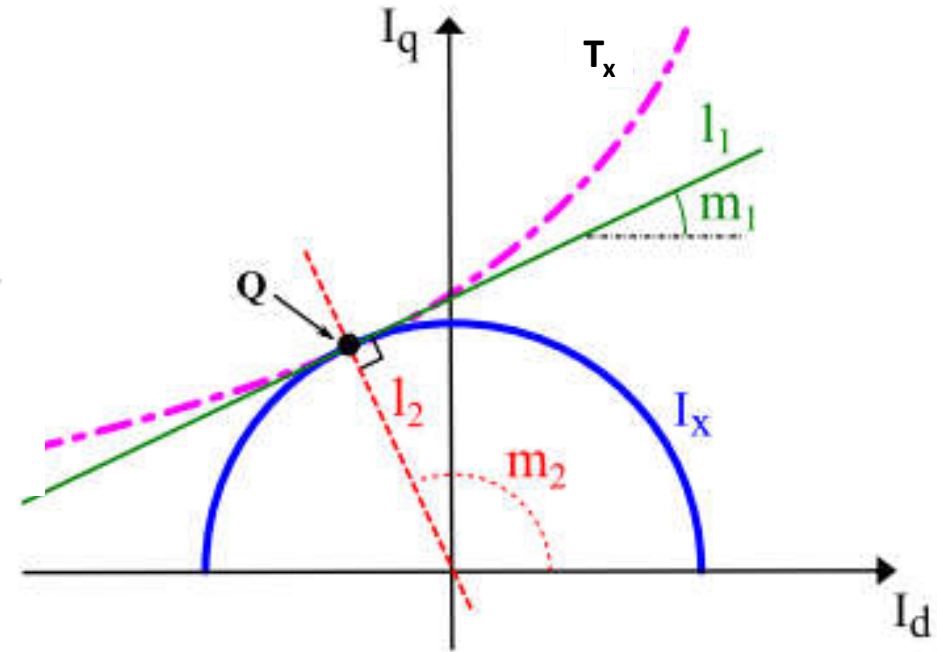
The angular coefficient  $m_1$  of the tangent line ( $l_1$ ) is computed as:

$$m_1 = \frac{dI_q}{dI_d} = \frac{2}{3} \frac{\tau_x}{p} \left\{ \frac{-1}{[\Lambda_{mg} + (L_d - L_q)I_d]^2} (L_d - L_q) \right\}$$

$$= \frac{(L_q - L_d)I_q}{\Lambda_{mg} + (L_d - L_q)I_d}$$

Where

$$I_q = \frac{2}{3} \frac{\tau_x}{p} \frac{1}{\Lambda_{mg} + (L_d - L_q)I_d}$$



$$m_2 = I_q / I_d.$$

The orthogonality condition of these two line is expressed by imposing the condition  $m_1 m_2 = -1$  so that the MTPA loci equation becomes:

$$I_{d,MTPA} < 0$$

$$I_{q,MTPA} = \pm \sqrt{\frac{I_{d,MTPA} [\Lambda_{mg} + (L_d - L_q) I_{d,MTPA}]}{L_d - L_q}}$$

Let us note that this strategy is applied until the curve BB' remains inside the voltage limit, i.e. the electrical speed is equal to the base speed ( $\Omega_{me} = \Omega_B$ ), up to the base speed it is always possible to achieve the nominal torque.

For rotor speed higher than the base one ( $\Omega_{me} > \Omega_B$ ) the available operating points are located in line BP where the maximum value of the torque is lower than the nominal value even with the same current as seen in isotropic case. This region is called fluxweakening (FW) region.

The value of the maximum speed is:

$$\Omega_{max,e} = \frac{U_N}{\Lambda_{mg} - L_d I_N}$$



To calculate expression of the MTPV loci it is possible to make a translation of voltage ellipses x-axis by quantities  $I_d = \frac{\Lambda_{mg}}{L_d}$  and a change of variable  $I'_q = \frac{L_q}{L_d} I_q$  so the voltage limit equations become a circle in a new origin instead the constant torque loci remain hyperbolas. Doing the same manipulation done for MTPA loci the MTPV loci equations are:

$$I_{d,MTPV} < -\frac{\Lambda_{mg}}{L_d}$$

$$I_{q,MTPV} = \pm \frac{L_d}{L_q} \sqrt{\frac{(I_{d,MTPV} + \frac{\Lambda_{mg}}{L_d}) [\Lambda_{mg} + (L_d - L_q) I_{d,MTPV}]}{L_d - L_q}}$$

# REL machine

Considering a reluctance machine the voltage equations in rotating reference frame and in steady state condition are described with the following relationships:

$$U_d = RI_d - \Omega_{me} L_q I_q$$

$$U_q = RI_q + \Omega_{me} L_d I_d$$

Neglecting the phase resistance R it is possible to express the voltage limit as a function of the current:

$$(\Omega_{me} L_q I_q)^2 + (\Omega_{me} L_d I_d)^2 \leq U_N^2$$

That can be rewritten as:

$$I_d^2 + \left( \frac{L_q}{L_d} I_q \right)^2 \leq \left( \frac{U_N}{\Omega_{me} L} \right)^2$$

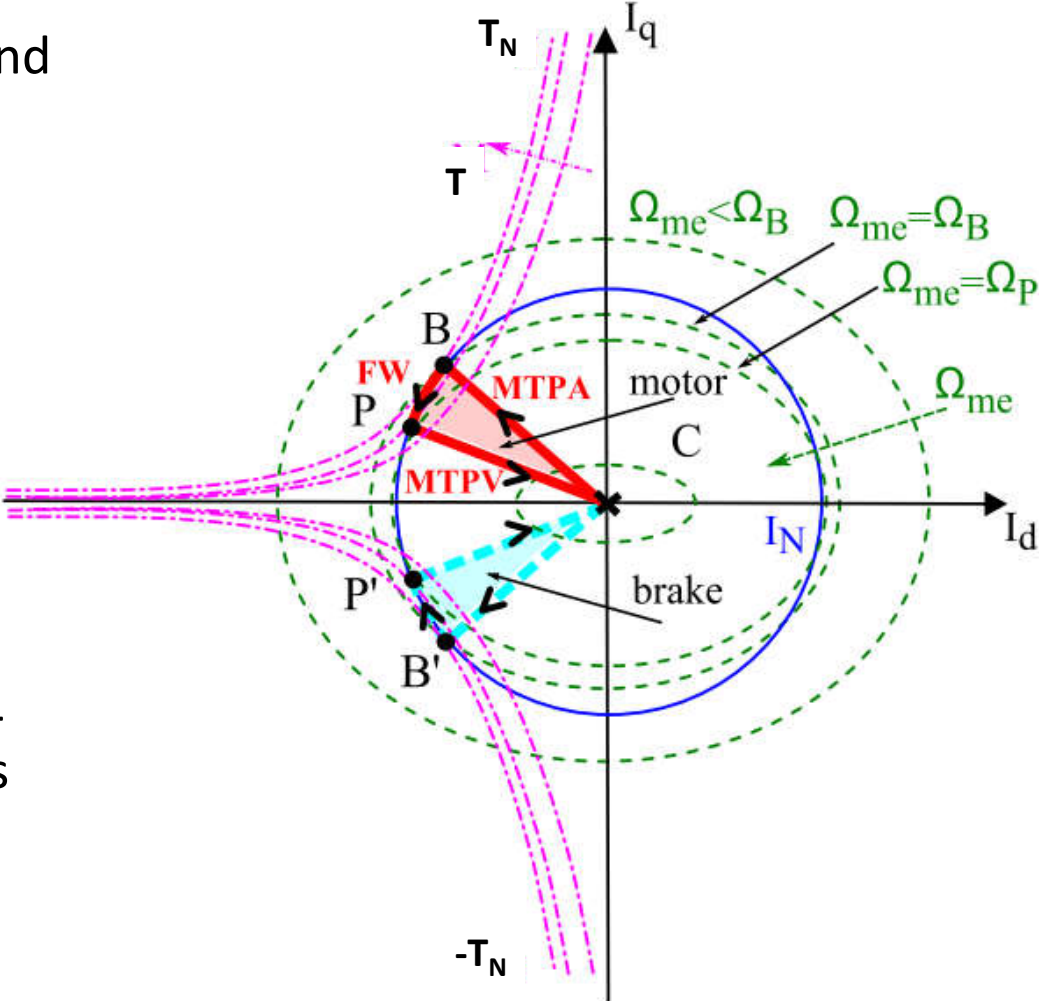
# Operating limits and working points of an anisotropic REL machine.

The current limit is represented by a circle with radius equal to  $I_N$ , shown with blue solid line, and the voltage limit is represented by a family of concentric ellipses. The ellipses center  $C$  of the voltage limit is located in the plain origin:

$$I_{Cd} = 0$$

$$I_{Cq} = 0$$

Because in this type of EM the short circuit current is null.  
 The constant torque loci for an anisotropic REL machine corresponds to a family of hyperboles with asymptotes locate in the d-axis and q-axis



The MTPA trajectory is shown as the line  $BB'$ . This curve contains the tangent point between the constant torque hyperboles and the current circles at different current values. With the same considerations that have been done for IPM machine the MTPA trajectory is expressed by the following equations:

$$I_{d,MTPA} < 0$$

$$I_{q,MTPA} = \pm I_{d,MTPA}$$

Up to the base speed ( $\Omega_B$ ) it is possible to provide the nominal torque satisfying both the voltage and current limits the value of this speed is:

$$\Omega_B = \frac{\sqrt{2}U_N}{I_N \sqrt{L_d^2 + L_q^2}}$$

At higher rotor speed ( $\Omega_{me} > \Omega_B$ ), the available operating points are limited by line BP called flux-weakening (FW) trajectory. In this case the control strategy of the motor follows the same criteria adopted in the Section above, up to the speed  $\Omega_p$ . At this point the intersection between the voltage ellipse and the current circle shows also the constant torque hyperboles tangent to the voltage ellipses.



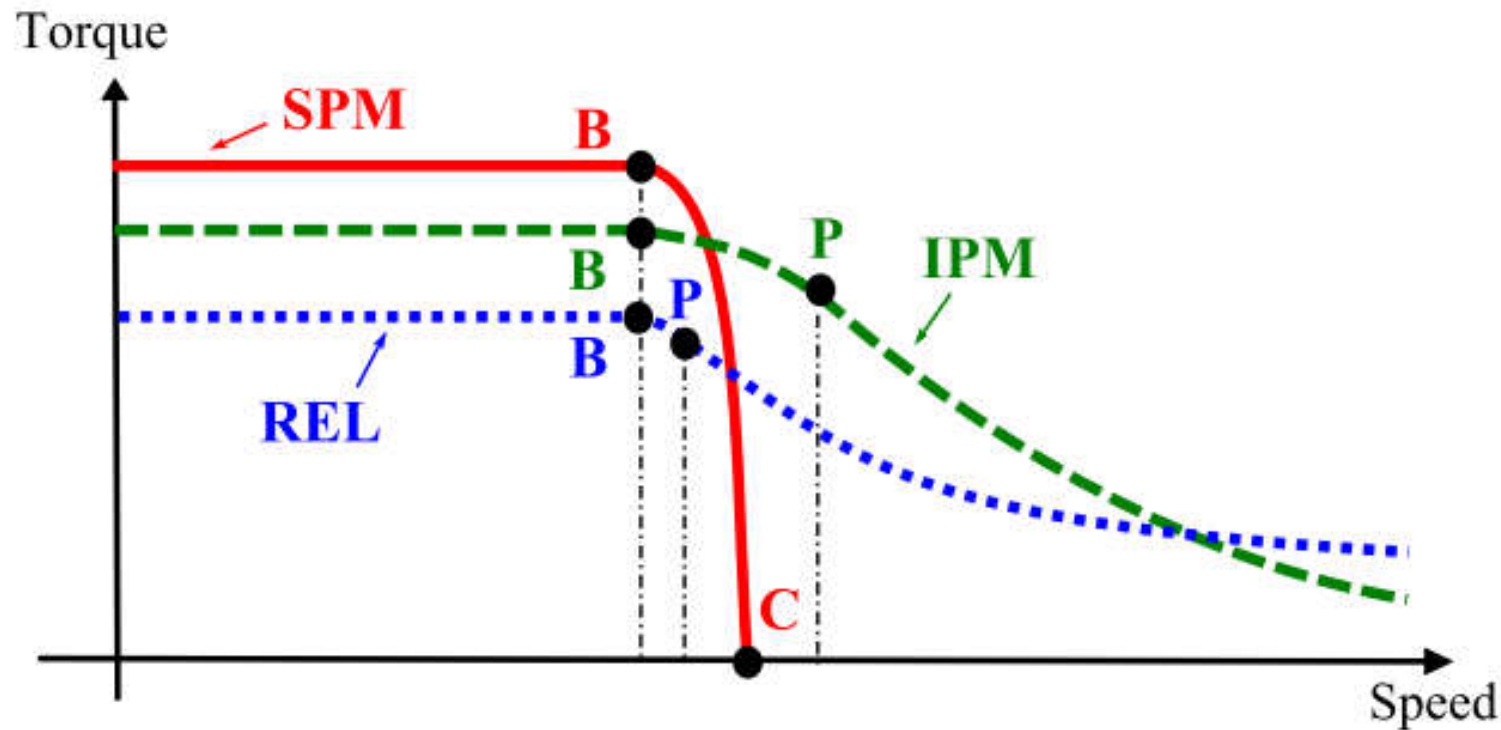
At speed higher than  $\Omega_p$  the maximum available torque is obtained operating the machine along the  $PP'$  loci. Analogously of the curve  $BB'$ , the curve  $PP'$  allow to achieve the Maximum Torque Per Volts (MTPV) condition. Also in this case there is no limit to the maximum speed because it tends to infinite.

By using the same consideration adopted for computed the MTPV trajectory with IPM machine in this case the equations are:

$$I_{d,MTPV} < 0$$

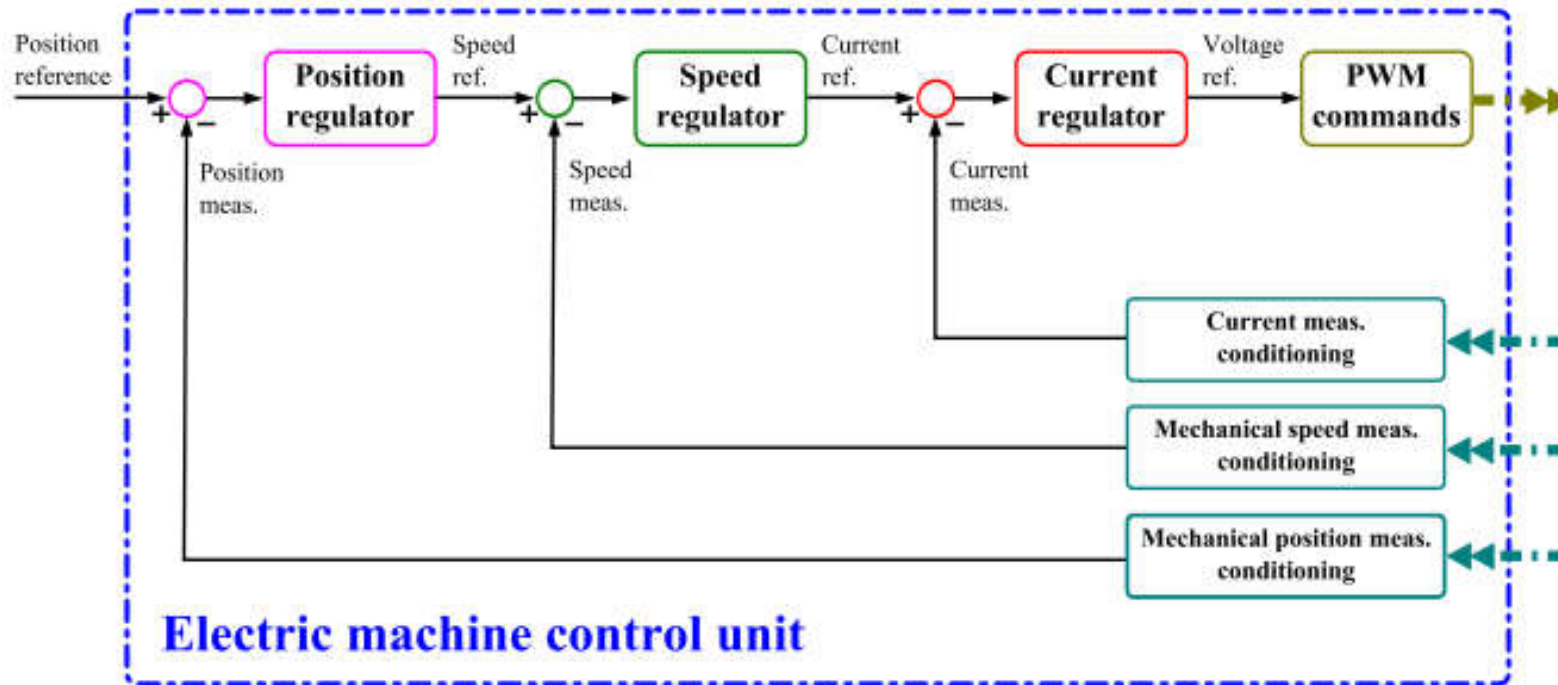
$$I_{q,MTPV} = \pm \frac{L_d}{L_q} I_{d,MTPV}$$

# Synchronous EM performance comparison



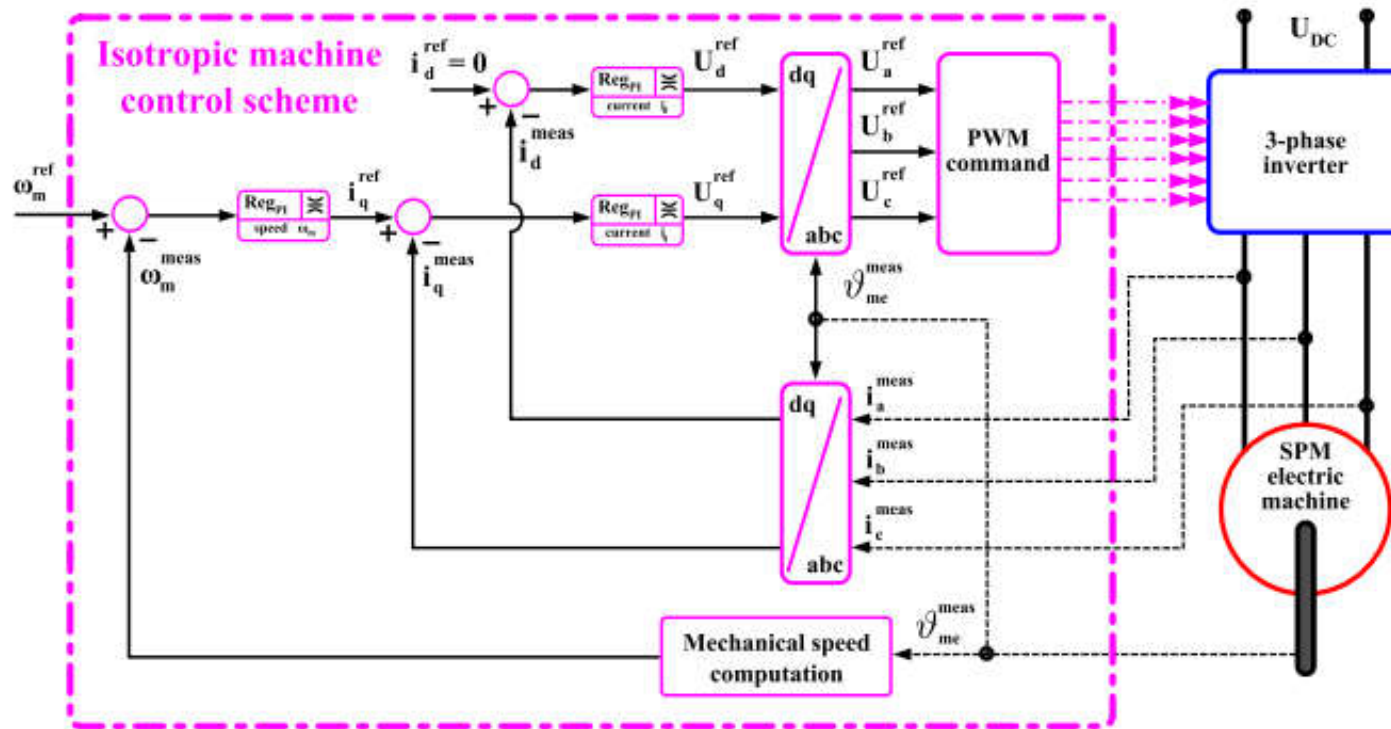
Usually the SPM machine exhibits higher torque but lower maximum speed at the contrary the REL machine has ideally an infinite rotor speed but lower torque because it is without PMs. Instead the IPM machine shows a good compromise between high torque and extended FW region.

# Electrical drive control with PM machines



Usually, the controlled electrical variables are current, voltage, and power. The mechanical variables under control are position (angle), speed, acceleration, and torque. Through the regulation of these electric and mechanical variables the electric drive can exhibit the desired performances. In the control of electric drive, typically only the current of EM is measured as electric variable. In addition, as mechanical variable, the mechanical position is measured or estimated from other measured variables.

# Isotropic machine control scheme



A speed control block scheme of a synchronous isotropic electric machine is shown. It highlights that the EM control is generally done in the rotating reference frame of EM. Referring to operating regions of this EM the stator current vector has to be synchronized with the rotor polar axis ( $d$ -axis), therefore the position measurement by position sensor as encoder or resolver is needed. Furthermore in order to maximize the EM performance the MTPA trajectory is normally implemented.

As it is highlighted in Fig. the EM control scheme is composed by an outer speed loop and by two inner current loops ( $d$ - and  $q$ -axis). In according to the MTPA control the  $d$ -axis current is maintained to zero instead the  $q$ -axis current reference is delivered by the speed loop regulator. For the electric drive control the controllers are generally simple PI regulators with their output limited at the maximum values of current (speed regulator) and voltage (current regulators). Referring to Fig. the input of EM control is the reference speed and the its output is the reference voltage that is provided to PWM command of 3-phase inverter. All loop are closed through the measurement of quantities as currents and speed instead the position measurement is used to change the reference frame from stationary to rotating. In the description that will be made below the inverter and sampler have been considered as first order delay and their transfer functions are given by the equation

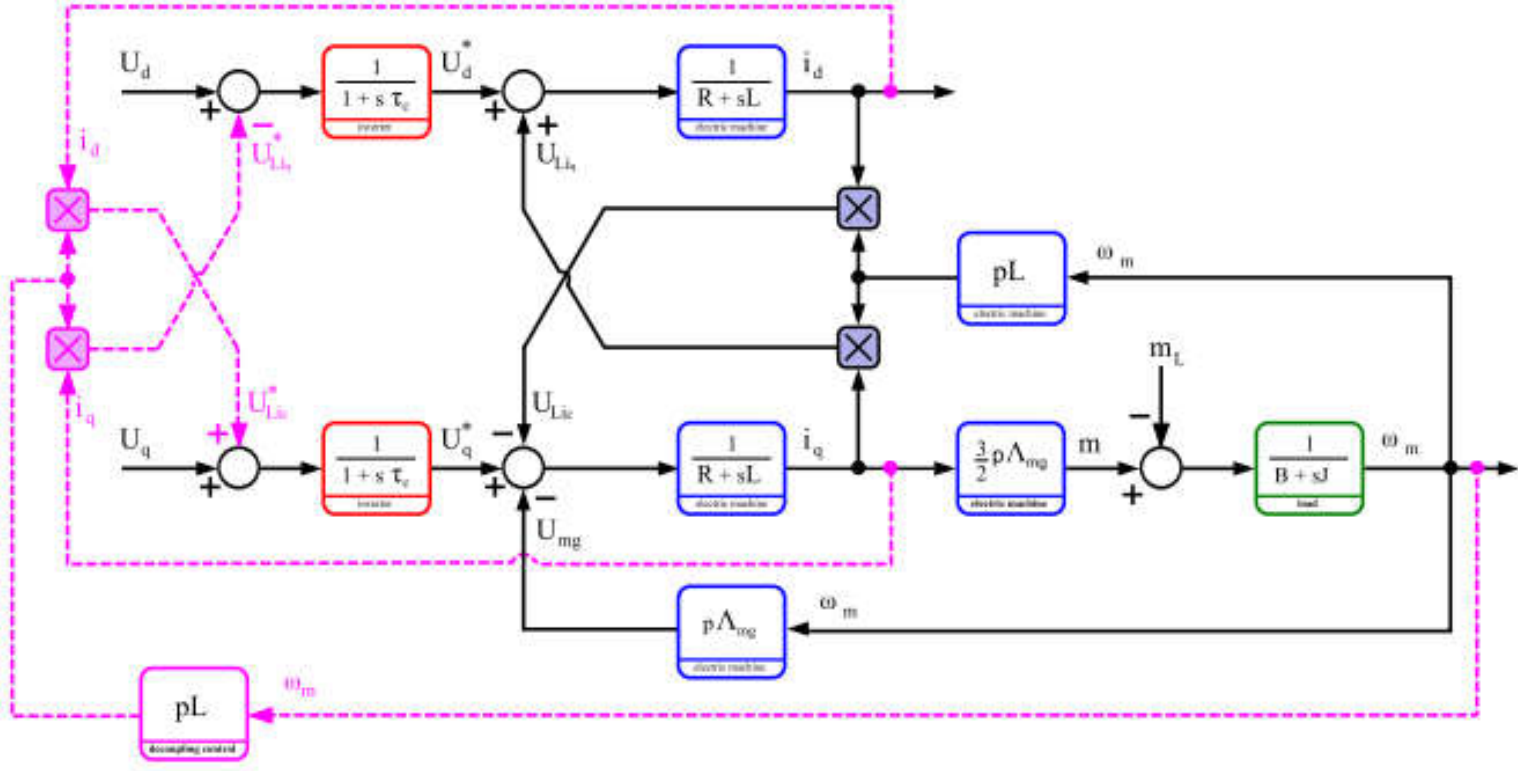
$$G_C(s) = \frac{U'(s)}{U(s)} \simeq \frac{1}{1 + s\frac{T_{SW}}{2}} = \frac{1}{1 + s\tau_c}$$

In more electric drive the sampling time and switching time have the same value, i.e  $T_S = T_{SW}$  . Furthermore considering a long computation time  $T_C$  the reference value can be processed into the next switching time step so the converter time constant becomes  $\tau_c = 3/2 T_{SW}$  .

Instead the dynamic of measurements (current, position, and speed) is neglected.

The current regulators design is complicated because the two current loops are not independent but they influence each other due to mutual coupling between the  $d$ - and  $q$ -axis of the electric machine.

In certain conditions it is possible to overcome this problem of mutual coupling between two current axes by doing the current axes decoupling inside the EM control software. Generally it can be considered the time constant of the converter ( $\tau_c$ ) is much smaller than the other time constants of the system, therefore it is possible to add at the respective voltage references of the inverter a signal opposite to the mutual coupling quantities as shown



The advantage of this software compensation is that after the current axes decoupling it is possible to analyze the two current axes separately, instead the principal problems of this technique are the existence of mutual cross saturation between the two axes and that the knowledge of exact value of the inductance is necessary.

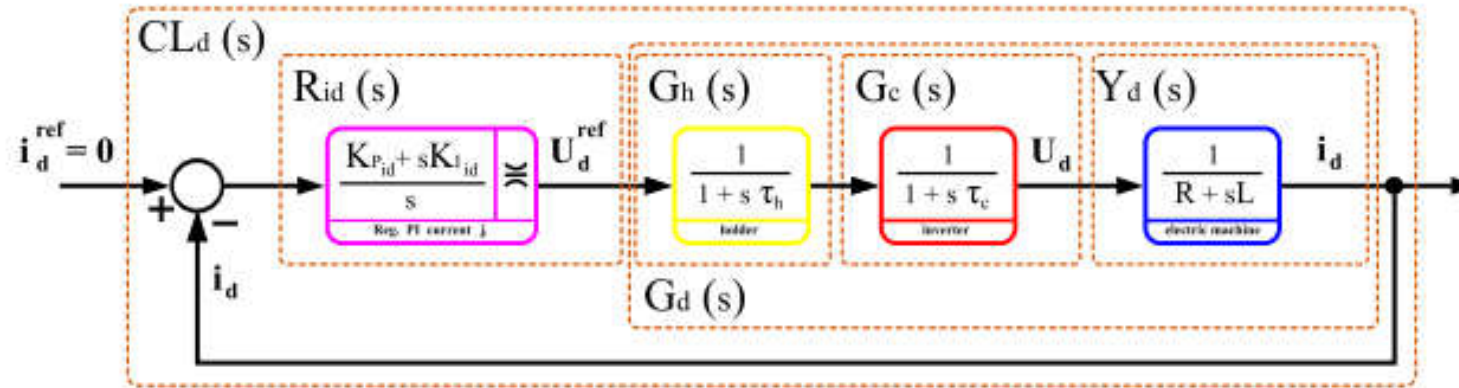
In order to draw the transfer functions of the control loops some useful time constants are defined below:

$$\begin{aligned} \tau_e &= \frac{L}{R} && \Rightarrow \text{electric time constant} \\ \tau_m &= \frac{J}{B} && \Rightarrow \text{mechanical time constant} \\ \tau_{m1} &= \frac{JR}{\frac{3}{2}(p\Lambda_{mg})^2} && \Rightarrow \text{electromechanical time constant} \end{aligned}$$

$$\begin{aligned} \tau_e &\ll \tau_m \\ \tau_{m1} &\ll \tau_m \end{aligned}$$

$R \Rightarrow$  stator phase resistance  
 $L \Rightarrow$  synchronous phase inductance  
 $J \Rightarrow$  system inertia  
 $B \Rightarrow$  viscous friction  
 $p \Rightarrow$  pole pair  
 $\Lambda_{mg} \Rightarrow$  PM flux linkage peak value

# $d$ -axis current loop after axes decoupling



After the current axes decoupling the EM block scheme, can be simplified and by adding the block schemes of power converter, sampler/holder, and PI regulator the block scheme of  $d$ -axis current loop becomes such as that reported.

$$Y_d(s) = \frac{I_d(s)}{U_d(s)} = \frac{1}{R + sL} = \frac{R}{1 + s\tau_e}$$

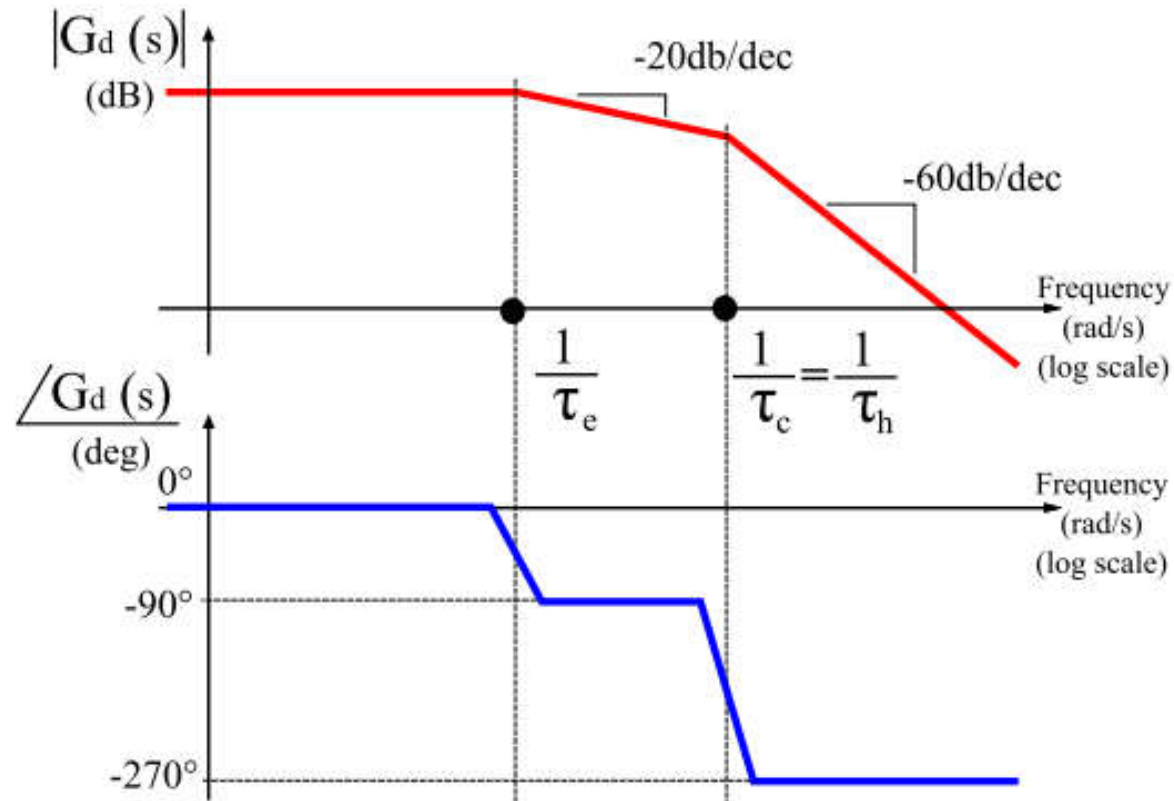
$$G_d(s) = \frac{I_d(s)}{U_d^{ref}(s)} = G_h(s)G_c(s)Y_d(s)$$

$$CL_d(s) = \frac{I_d(s)}{I_d^{ref}(s)} = \frac{OL_d(s)}{1 + OL_d(s)} = \frac{R_{id}(s)G_d(s)}{1 + R_{id}(s)G_d(s)}$$

where the notation for the transfer function are  $CL_d(s)$  for  $d$ -axis current close loop and  $OL_d(s)$  for  $d$ -axis current open loop.



A qualitative frequency-response of the process,  $G_d(s)$ . It highlights that at control frequencies the phase margin is typically high ( $70^\circ \div 90^\circ \text{ deg}$ ) for this kind of transfer function, this means that it is always easy to regulate this loop by a simple PI controller.



# $q$ -axis current loop after axes decoupling

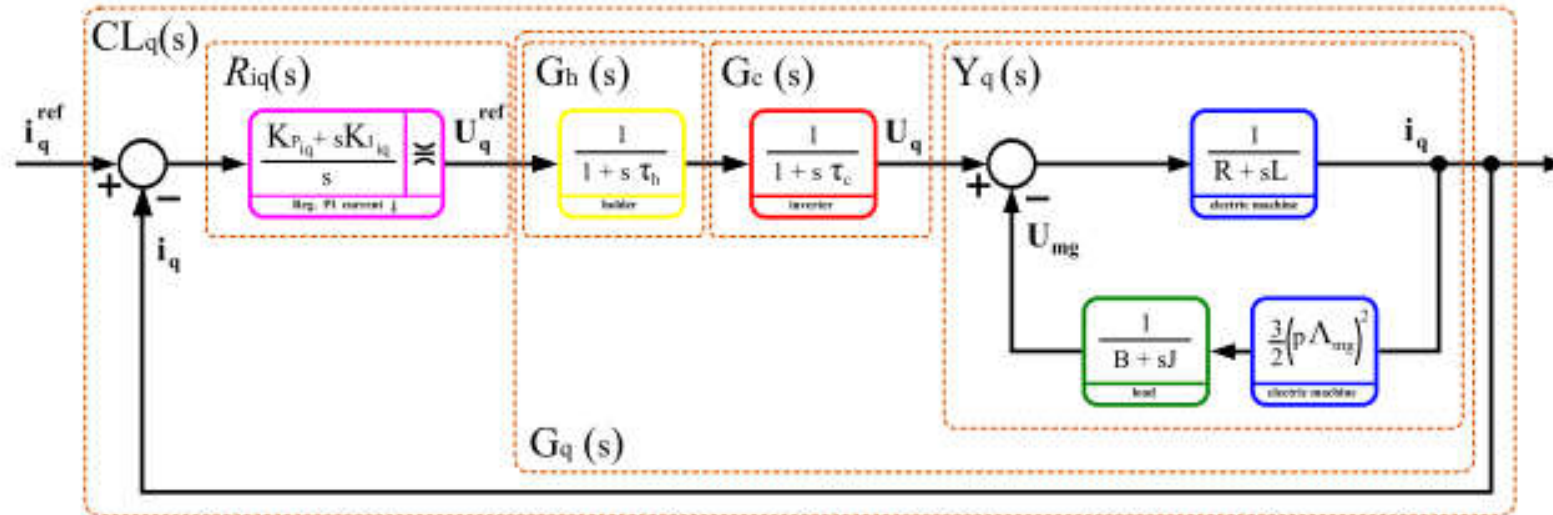


Fig. shows the block scheme of  $q$ -axis current loop after the current axes decoupling. It highlights that the  $q$ -axis loop unlike the  $d$ -axis one it also contains the load equations, so its transfer functions are:

$$Y_q(s) = \frac{I_q(s)}{U_q(s)} = \frac{1}{\frac{3}{2}(p\Lambda_{mg})^2} \frac{B + sJ}{D(s)} = \frac{\frac{2}{3}}{(p\Lambda_{mg})^2} \frac{1 + s\tau_m}{BD(s)}$$

$$G_q(s) = \frac{I_q(s)}{U_q^{ref}(s)} = G_h(s)G_c(s)Y_q(s)$$

$$CL_q(s) = \frac{I_q(s)}{I_q^{ref}(s)} = \frac{OL_q(s)}{1 + OL_q(s)} = \frac{R_{iq}(s)G_q(s)}{1 + R_{iq}(s)G_q(s)}$$

where the transfer function  $D(s)$  is defined as:

$$D(s) = s^2 \tau_e \tau_{m1} + s \tau_{m1} \left(1 + \frac{\tau_e}{\tau_m}\right) + \left(1 + \frac{\tau_{m1}}{\tau_m}\right) = \left(1 + \frac{RB}{\frac{3}{2} (p \Lambda_{mg})^2}\right) \left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right)$$

and assuming  $B = 0$  the equation can be simplified:

$$D(s)|_{B=0} \simeq \left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right)$$

The values of poles  $p_1$  and  $p_2$  depend from the values of the time constants  $\tau_{m1}$  and  $\tau_e$ .

Therefore the three possible cases are:

1.  $\tau_{m1} = 4\tau_e \Rightarrow$  coincident real roots

$$p_{1,2} = +\frac{1}{2\tau_e}$$

2.  $\tau_{m1} > 4\tau_e \Rightarrow$  distinct real roots, if  $\tau_{m1} \gg 4\tau_e$

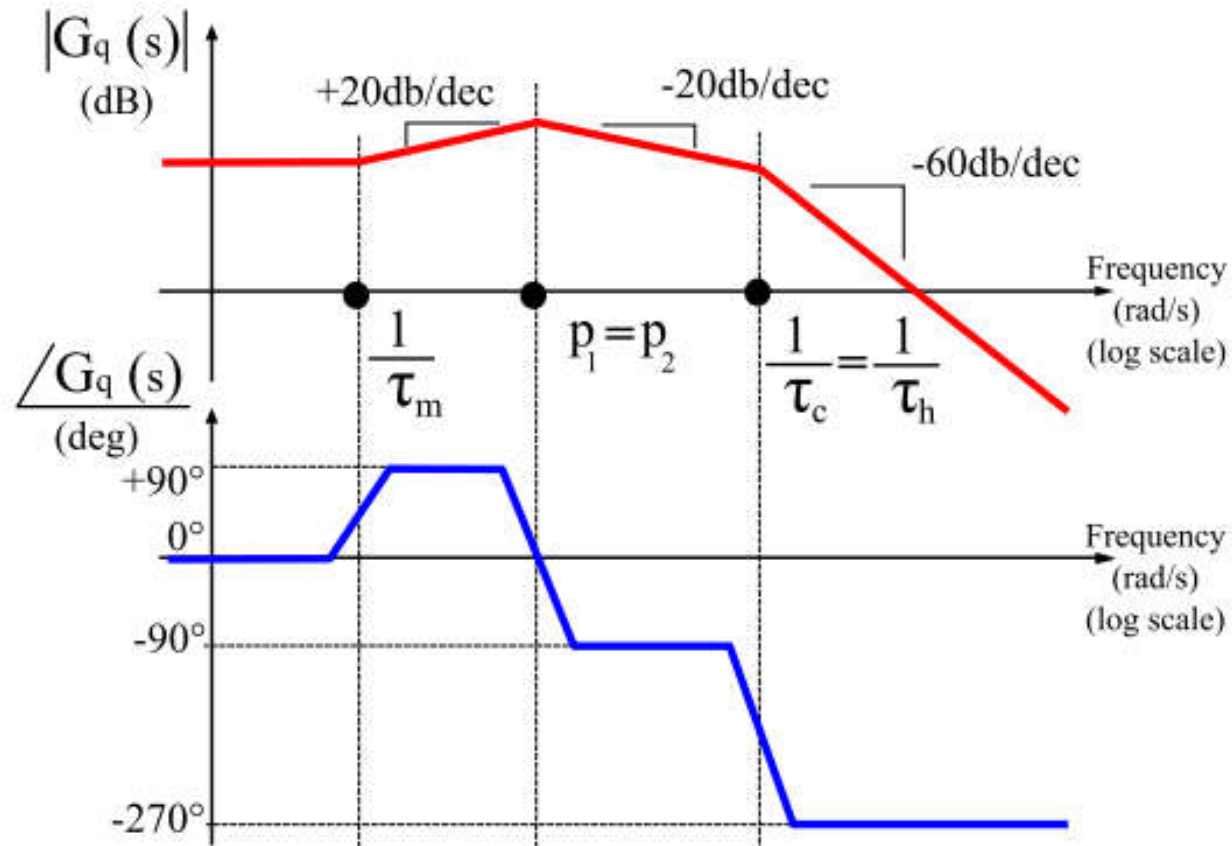
$$p_1 \simeq +\frac{1}{\tau_e} \quad \text{and} \quad p_2 \simeq +\frac{1}{\tau_{m1}}$$

3.  $\tau_{m1} < 4\tau_e \Rightarrow$  complex conjugates roots

$$p_{1,2} = +\frac{1}{2\tau_e} \left( 1 \pm j \sqrt{\frac{4\tau_e}{\tau_{m1}} - 1} \right)$$

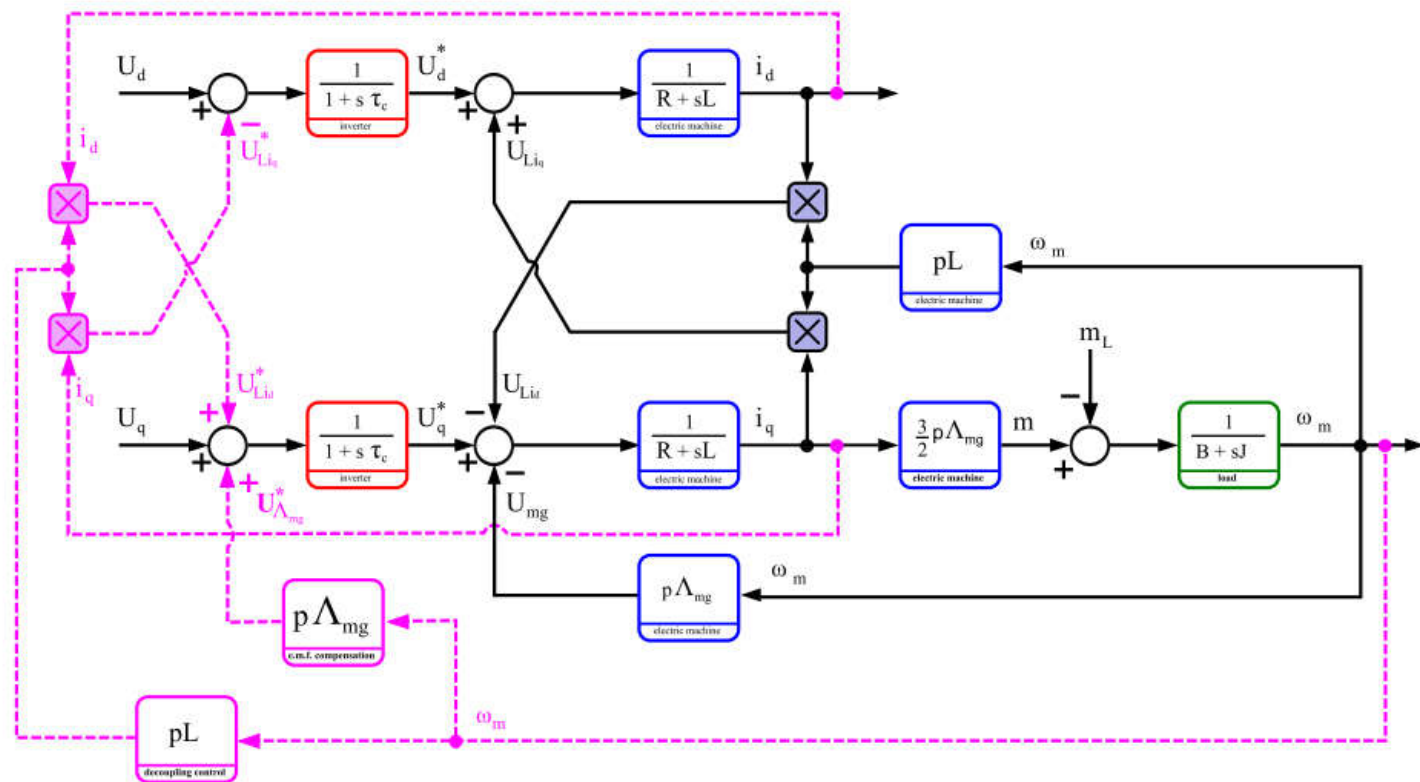
In electric drive with PM electric machine the condition that  $\tau_{m1} < 4\tau_e$  is more frequent, so the roots of the denominator are complex conjugate.

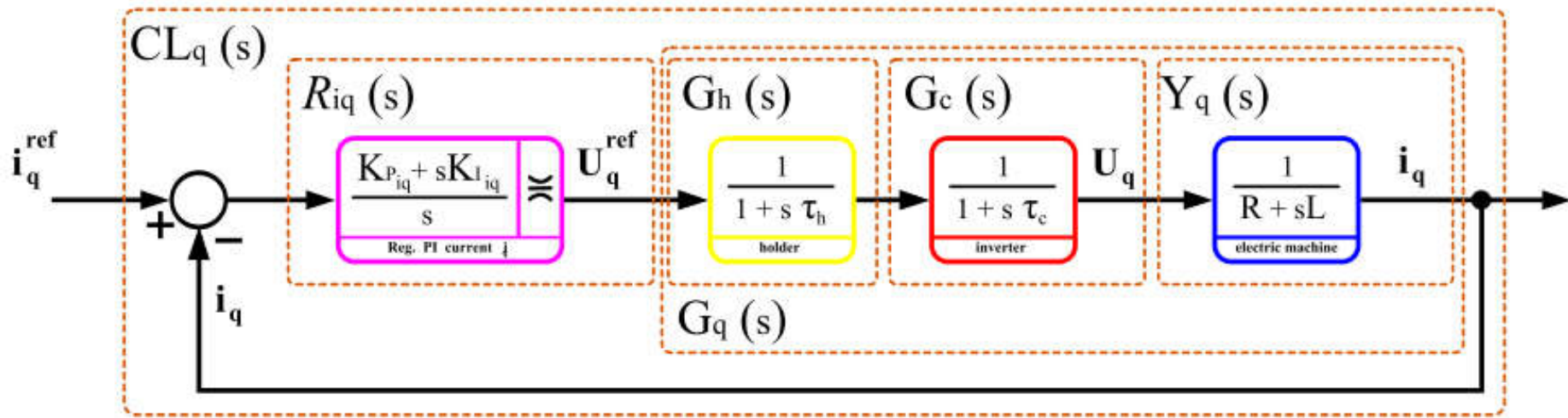
A qualitative frequency-response of the process,  $Gq(s)$ , is reported. It represents the distinct real roots case (i.e.  $\tau_{m1} > 4\tau_e$ ). It highlights, at control frequencies, that also in this case the phase margin of this transfer function is typically high ( $50^\circ \div 70^\circ \text{ deg}$ ) so by a simple PI controller this loop is easy to regulate.



# E.m.f. compensation

In the case of electric drive with synchronous PM electric machine often the dynamic performance request of current control is generally high. For this reason the electromotive force (e.m.f.) compensation is done by software. As shown in Fig. it is achieved by adding at the output of the  $q$ -axis current regulator a signal proportional to the electromotive force but with opposite sign.

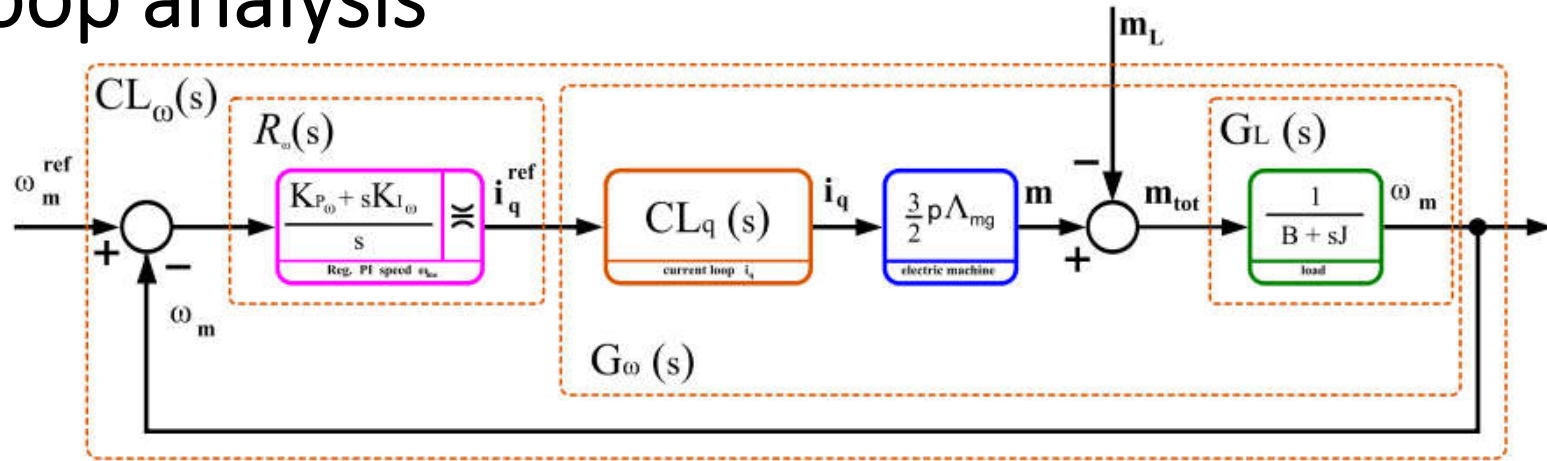




Therefore neglected the converter delay, same assumption that has been adopted for the axes decoupling, the block diagram of  $q$ -axis current loop after axes decoupling and e.m.f. compensation is reported.

The principal advantages are that also the  $q$ -axis current loop does not depend on load dynamic, as highlighted, and so  $q$ -axis current loop is easier to design as the  $d$ -axis current one. Instead the principal problem is the positive feedback, for compensating the e.m.f., because it can create instability for this reason it is better to underestimate the flux value  $\Lambda_{mg}$ .

# Speed loop analysis



The block scheme of speed loop after axes decoupling but without the e.m.f. compensation is reported. It highlights that inside the speed loop there is only  $CL_q(s)$ , i.e. the close loop transfer function of  $q$ -axis current, therefore in a isotropic machine only  $q$ -axis current produces EM torque. Fundamental transfer functions equations of speed loop are:

$$CL_q(s) \simeq \frac{1}{1 + \frac{s}{\nu_{Aiq}}} \frac{1}{1 + s\tau_c} \frac{1}{1 + s\tau_h}$$

$$G_L(s) = \frac{\Omega_m(s)}{M_{tot}(s)} = \frac{1}{B + sJ} = \frac{B}{1 + s\tau_m}$$

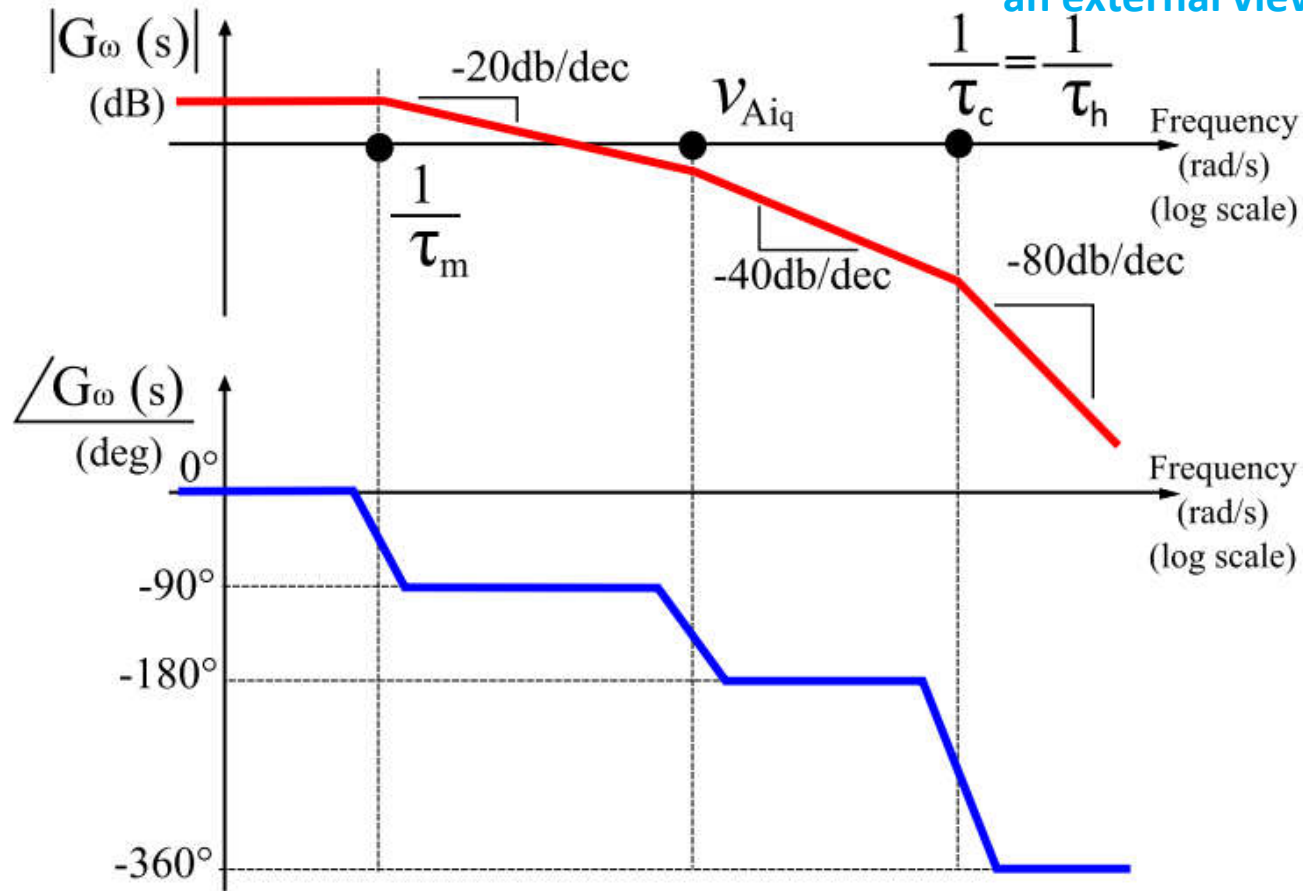
$$CL_\omega(s) = \frac{\Omega_m(s)}{\Omega_m^{ref}(s)} = \frac{OL_\omega(s)}{1 + OL_\omega(s)} = \frac{R_\omega(s)G_\omega(s)}{1 + R_\omega(s)G_\omega(s)}$$

where  $\nu_{Aiq}$  is the  $q$ -axis current bandwidth.



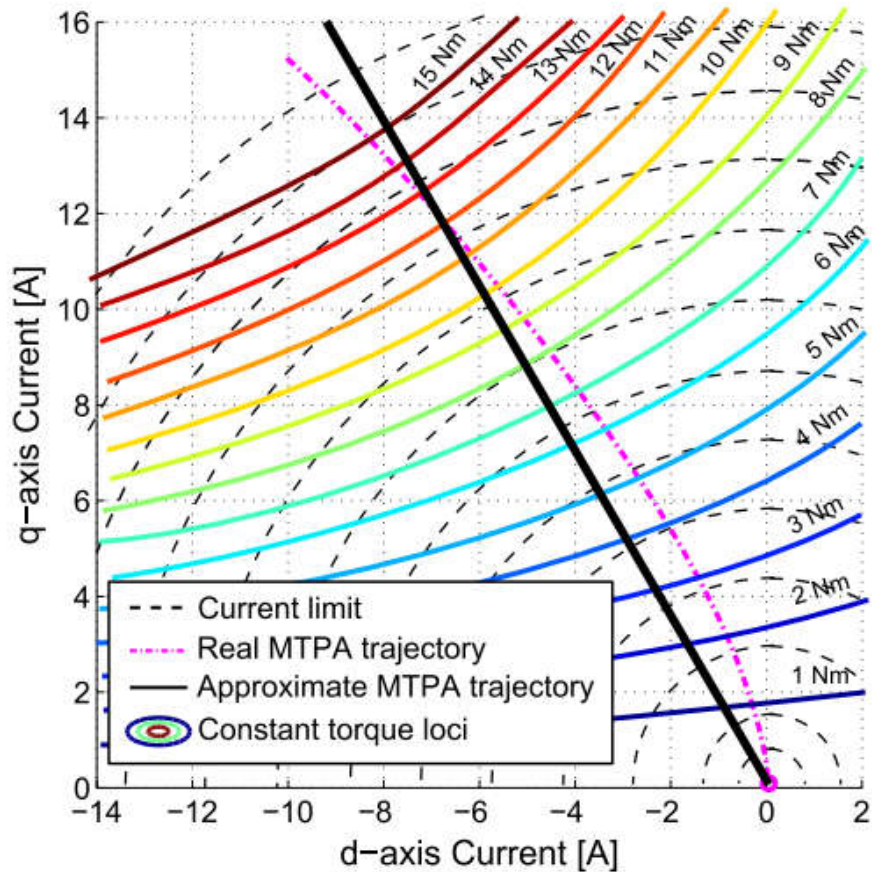
A qualitative frequency-response of  $G(s)$ , is reported in Fig. In order to have a good phase margin ( $40^\circ \div 60^\circ \text{ deg}$ ) the speed loop bandwidth of the system must be less than at least eight/ten times respect the  $q$ -axis current bandwidth,  $v_{Aiq}$ .

“To see the inner current loop as a unity block; to neglect it from an external viewpoint”



# Example of drive control design

As an example the considered IPM machine is a 10 poles and 12 slots, whose rotor is characterized by two flux-barriers per pole.



In order to control the IPM machine along the MTPA trajectory the constant torque loci of the EM maps have been computed by a finite element analysis.

# Electric drive fundamental parameters.

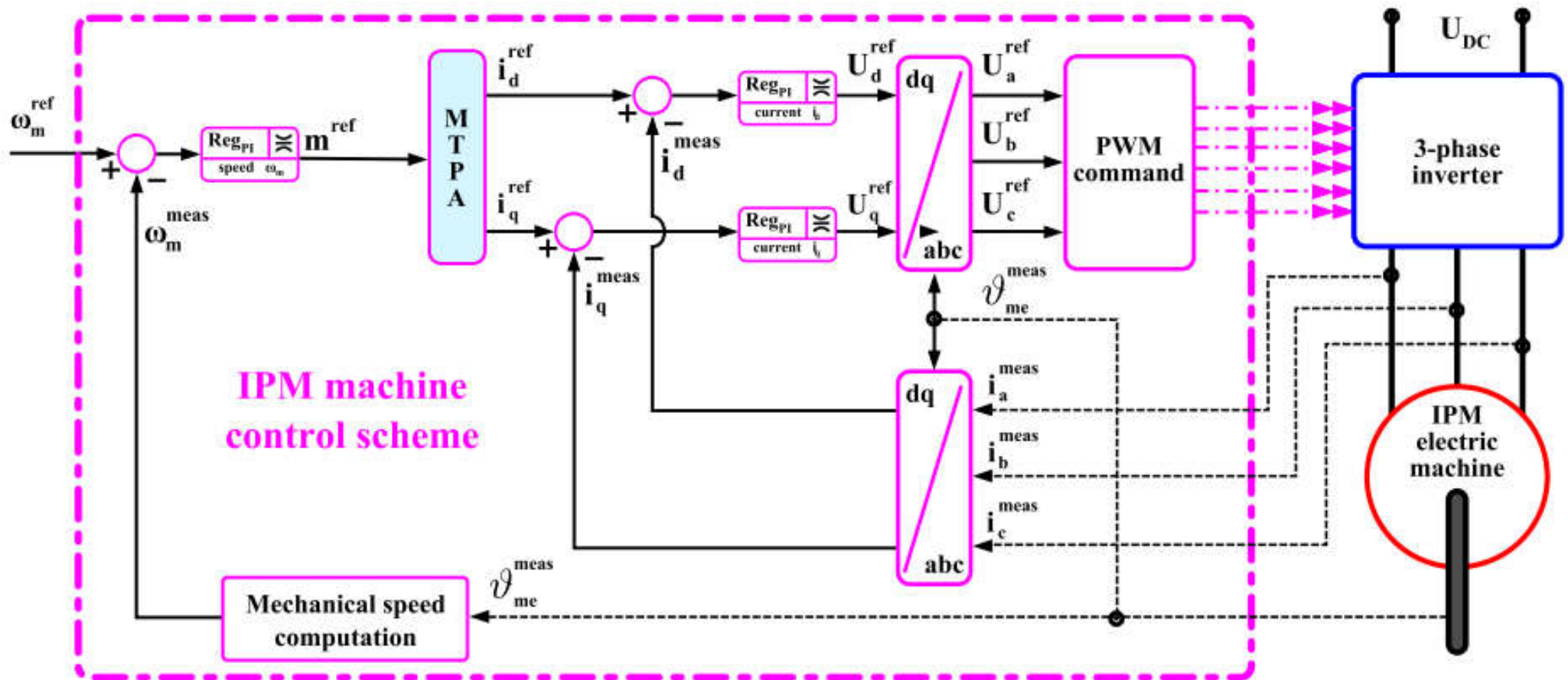
(a) IPM electric machine

Parameter	Value
Pole number	10
Slot number	12
Nominal torque	10 $Nm$
Nominal current	10 $A_{rms}$
Phase resistance	1.2 $\Omega$
Direct inductance	12 $mH$
Quadrature inductance	20 $mH$
PM flux	0.08 $Wb$
Inertia	1.3 $gm^2$

(b) Power converter

Parameter	Value
Number of phases	3
Type of switch	IGBT
DC bus voltage	550 $V$
Nominal current	15 $A_{rms}$
Sampling time	100 $\mu s$
Switching time	100 $\mu s$

# Speed control scheme of a IPM machine.



In order to design the speed and two current loop regulators the fundamental time constants of EM have been computed and are:

$$\tau_{ed} = \frac{L_d}{R} = 0.01 \text{ s}$$

$$\tau_{eq} = \frac{L_q}{R} = 0.0167 \text{ s}$$

$$\tau_m = \frac{J}{B} = 5 \text{ s}$$

$$\tau_{m1} = \frac{JR}{\frac{3}{2}(p\Lambda_{mg})^2} = 0.017 \text{ s}$$

These values highlight that the typical conditions of the time constants in PM electric machine are verified, that is to say:

$$\tau_{ed} \ll \tau_m$$

$$\tau_{eq} \ll \tau_m$$

$$\tau_{m1} \ll \tau_m$$

$$\tau_{m1} < 4\tau_{eq}$$

# Current regulator design

The first step in the current controller design is to define the current loops bandwidth ( $\nu_i$ ) and the pole position of the PI current controllers. Let's note that to ensure stability it is necessary that the current loop bandwidth should be lower than converter pole ( $1/\tau_c$ ) and higher than the frequency of the electric time constant ( $1/\tau_e$ ), in this example it is  $\nu_i = 1800 \text{ rad/s}$ . Instead the PI regulator pole ( $\tau_{rid}$  and  $\tau_{riq}$ ) is generally chosen equal to the electric time constant associated to it. The constraints chosen for the current controllers are therefore:

- $\nu_i \gg \frac{1}{\tau_{ed}} = 1800 \text{ rad/s}$
- $\tau_{rid} = \tau_{ed}$
- $\tau_{riq} = \tau_{eq}$

Referring to the block diagram of  $d$ -axis current loop after axes decoupling, i.e. that shown in Fig., it is chosen the value of  $K_{P_{id}}$  that guarantees the current bandwidth, its value is computed by:

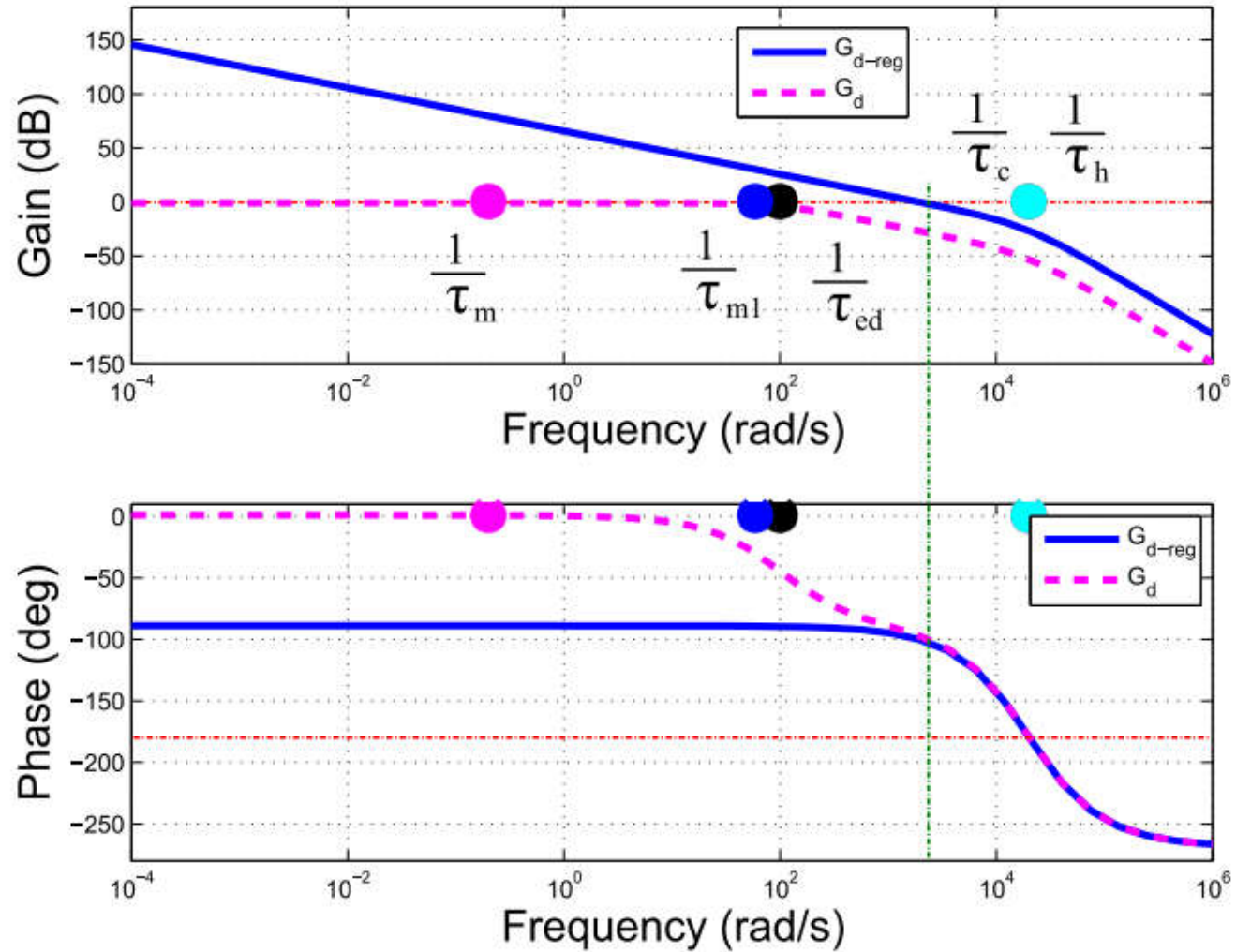
$$K_{P_{id}} \equiv \frac{1}{|G_d(j\nu_i)|}$$

therefore the value of  $K_{I_{id}}$  is:

$$K_{I_{id}} \equiv \frac{K_{P_{id}}}{\tau_{r_{id}}}$$

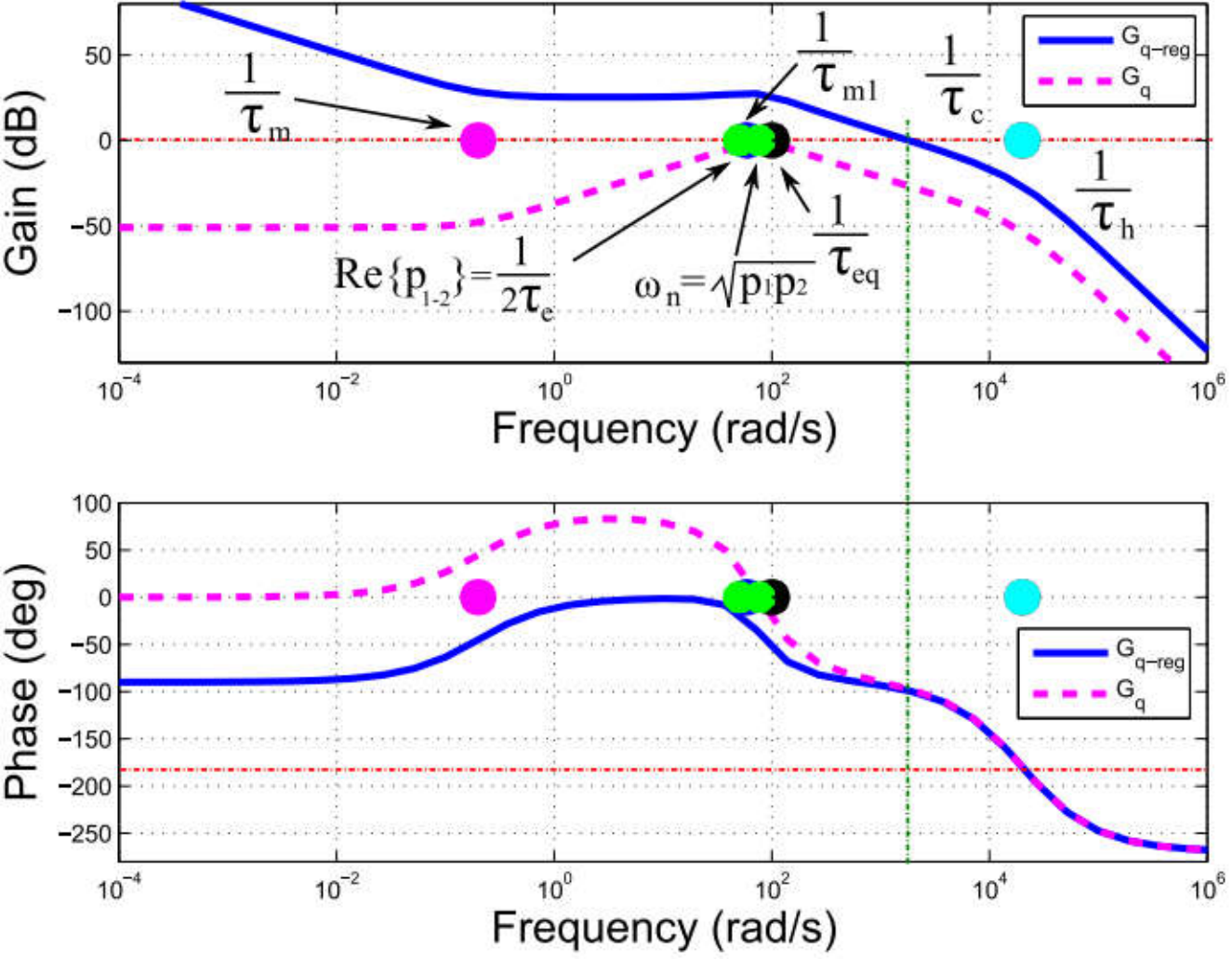
The same approach is used to design the  $q$ -axis current loop.

Frequency-response of d-axis loop without ( $G_d(s)$ ) and with the PI regulator ( $G_{d-reg}(s)$ ).





Frequency-response of  $q$ -axis loop without ( $G_q(s)$ ) and with the PI regulator ( $G_{q-reg}(s)$ )



# Speed regulator design

In order to design the speed loop controller the speed loop bandwidth ( $\nu_\omega$ ) and the pole position of the PI speed controller are at first defined. Let's note that to ensure stability it is necessary that the speed loop bandwidth should be lower than the current loop bandwidth ( $\nu_i$ ). The constraints chosen for the speed controllers are therefore:

- $\nu_\omega = \frac{\nu_i}{30} = 60 \text{ rad/s}$
- $\tau_{r_\omega} = \frac{2\sqrt{2}}{\nu_\omega}$

Referring to the block diagram of speed loop after axes decoupling it is chosen the value of  $K_{P_{\omega d}}$  that guarantees the current bandwidth, its value is computed by:

$$K_{P_{\omega q}} \equiv \frac{1}{|G_\omega(j\nu_i)|}$$

therefore the value of  $K_{I_\omega}$  is:  $K_{I_\omega} \equiv \frac{K_{P_\omega}}{\tau_{r_\omega}}$

Frequency-response of speed loop without ( $G_{\omega}(s)$ ) and with the PI regulator ( $G_{\omega-reg}(s)$ )

