$f(x) = |x-1|\, e^x$ $\qquad f: \mathbb{R} \to \mathbb{R}$ continua $\qquad f(x) = \begin{cases} (x-1)\, e^x & \text{se } x \geq 1 \\ (1-x)\, e^x & \text{se } x < 1 \end{cases}$

$f(x) \geq 0 \quad \forall x \qquad f(x) > 0 \quad \forall x \neq 1 \qquad f(1) = 0 \qquad \min f = 0$

$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = 0 \qquad \sup f = +\infty$

$$f'(x) = \begin{cases} e^x + (x-1)\, e^x = \underline{e^x x} & \text{se } x > 1 \\ -e^x + (1-x)\, e^x = -x\, e^x & \text{se } x < 1 \end{cases}$$

se $x = 1$? $\qquad f'_+(1) = \lim_{x \to 1^+} f'(x) = e \qquad f'_-(1) = \lim_{x \to 1^-} f'(x) = -e$

se esiste il limite (per il teorema sul limite della derivata) $\qquad$ se esiste

f non è derivabile in $x = 1$

$$f'(x) = \begin{cases} x e^x & x > 1 \\ -x e^x & x < 1 \end{cases}$$

$$f'(0) = 0$$

$f'(x) > 0$ se $x > 1$

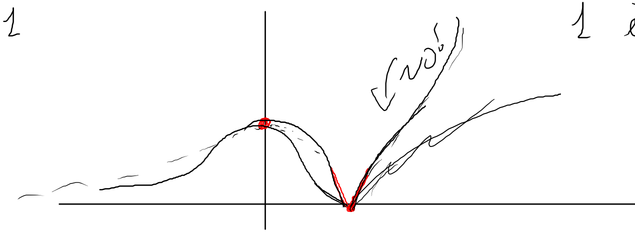
$f'(x) < 0$ se $x \in]0,1[$

$f'(x) > 0$ se $x \in]-\infty, 0[$

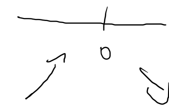
$$f(x) = |x-1| e^x$$

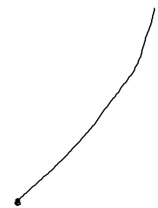
0 è un punto di massimo relativo per f

$f(0) = 1$



1 è un punto di minimo





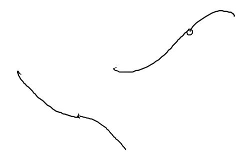
$$f''(x) = \begin{cases} e^x + x e^x \quad e^x(x+1) & \text{se } x > 1 \\ -e^x - x e^x = -e^x(x+1) & \text{se } x < 1 \end{cases}$$

$$f'(x) = \begin{cases} x e^x \\ -x e^x \end{cases}$$

se $x > 1$ $\quad f''(x) > 0$

se $x < 1$ $\quad f''(-1) = 0 \qquad f''(x) > 0$ se $x \in\,]-\infty, -1[$

$f''(x) < 0$ se $x \in\,]-1, 0[$



-1 è un punto di flesso discendente

f è convessa su $]-\infty, -1]$ e su $[1, +\infty[$

f è concava su $[-1, 1]$

$f''(x) > 0 \quad x \in\,]-\infty, -1[\, \cup\,]1, +\infty[$

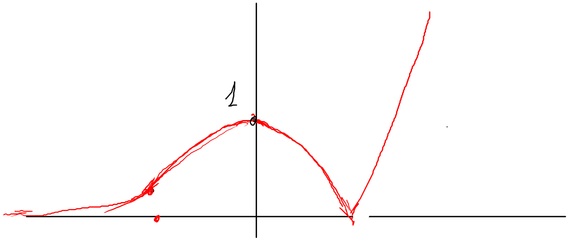
~~f convesso su]-∞,-1] ∪ [1,+∞[~~

$|x-1|\, e^{x}$

$f(-1) = 2\, e^{-1}$

numero delle soluzioni dell'equazione

$f(x) = \alpha \qquad \alpha \in \mathbb{R}$



$\alpha < 0 \qquad \emptyset$

$\alpha = 0 \qquad 1$ sol.

$0 < \alpha < 1 \qquad 3$ sol.

$\alpha = 1 \qquad 2$ sol.

$\alpha > 1 \qquad 1$ sol.

Es: si determini il numero delle soluzioni dell'equazione

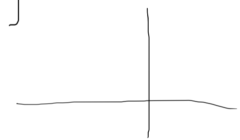
$$\log(1+x^2) = (x-1)^2$$

Inoltre si trovi una $\overset{\text{STIMA}}{\text{stima}}$ delle soluzioni con un errore $< 0{,}5$.

Consideriamo $f(x) = \log(1+x^2) - (x-1)^2 = (x-1)^2\left[\dfrac{\log(1+x^2)}{(x-1)^2} - 1\right]$

dom $f = \mathbb{R}$ $\quad f \in C^\infty(\mathbb{R})$ $\quad \lim_{x \to +\infty} f(x) = -\infty$

$\quad x \to -\infty \qquad = -\infty$



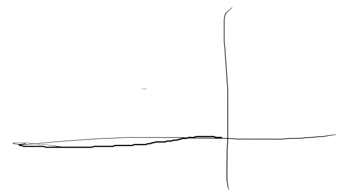
$$f'(x) = \frac{2x}{1+x^2} - 2(x-1) = \frac{2x - 2\overbrace{(x-1)(1+x^2)}^{x+x^3-1-x^2}}{1+x^2} = \frac{\cancel{2x} - \cancel{2x} - 2x^3 + 2 + 2x^2}{1+x^2} = \frac{-2x^3 + 2x^2 + 2}{1+x^2}$$

$x^3 - x^2 - 1 \overset{?}{=} 0 \qquad \varphi(x) = x^3 - x^2 - 1$

$$f''(x) = D\left[\frac{2x}{1+x^2} - 2(x-1)\right] = 2\left[\frac{1+x^2 - x\cdot 2x}{(1+x^2)^2} - 1\right] =$$

$$= 2\left[\frac{1-x^2-(1+x^2)^2}{(1+x^2)^2}\right] = \frac{2}{(1+x^2)^2}\cdot\left(\cancel{1}-x^2-\cancel{1}-2x^2-x^4\right) = -\frac{2}{(1+x^2)^2}\left(x^4+3x^2\right)$$

$$= -\frac{2x^2}{(1+x^2)^2}\left(x^2+3\right) < 0 \quad \forall x \neq 0 \qquad f''(0) = 0$$

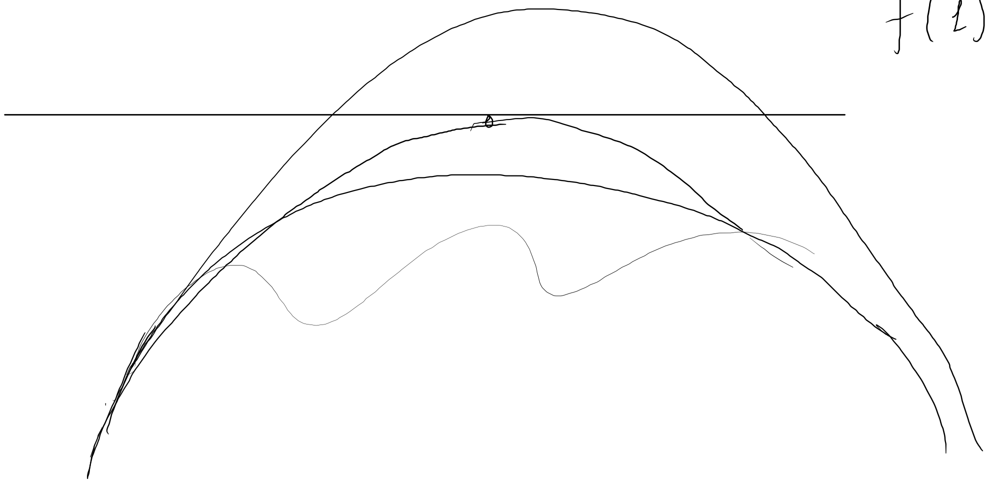


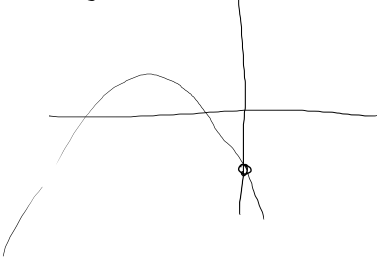
f è strettamente concava

$$f(x) = \log(1+x^2) - (x-1)^2$$

$$f(0) = -1$$

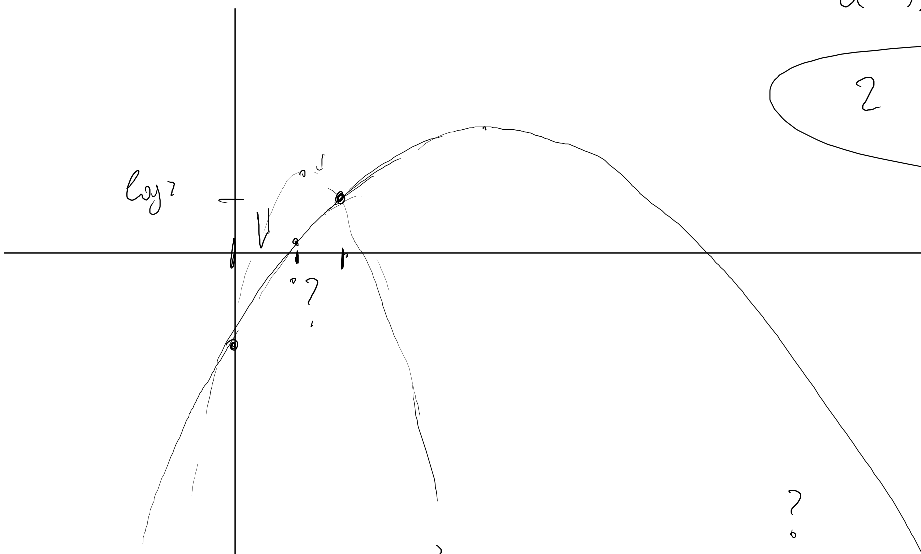
$$f(1) = \log 2 > 0$$





ci sono 2 zeri!

2 soluzioni α, β



$\alpha \in \,]0,1[$

$$f\left(\frac{1}{2}\right) = \log\left(1+\frac{1}{4}\right) - \left(\frac{1}{2}-1\right)^2 = \log\frac{5}{4} - \frac{1}{4} \overset{?}{>} 0$$

$$\log\frac{5}{4} \overset{?}{>} \frac{1}{4}$$

$\log \frac{5}{4} \overset{?}{>} \frac{1}{4}$

$\Downarrow$

$\frac{5}{4} \overset{?}{>} e^{\frac{1}{4}}$

NO

$\frac{5}{4} < e^{1/4} \Rightarrow$

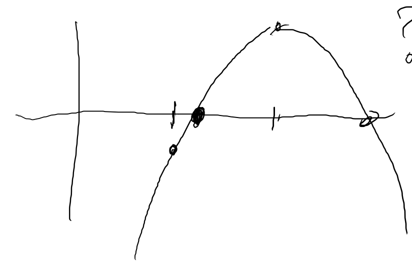
$\alpha \simeq \frac{1}{2}$

$\alpha \simeq 1$

$e^{\frac{1}{4}} \overset{?}{\simeq} 1 + \frac{1}{4} + \frac{1}{2} \frac{1}{4^2} + \dots$

$e^{1/4} > 1 + \frac{1}{4} = \frac{5}{4}$

$f\left(\frac{1}{2}\right) < 0$



$f(1) > 0$

$$f(2) = \log(1+4) - (2-1)^2 = \underbrace{\log 5 - 1}_{>0}$$

$$f(3) = \log(10) - 4 \overset{?}{>} 0$$

$$\beta \in \left]2, \frac{5}{2}\right[$$

$$\log(10) \overset{?}{>} 4$$

$$10 \overset{?}{>} e^4$$

$$e^4 > 2^4 = 16 > 10 \Rightarrow f(3) < 0$$

$$f\left(\frac{5}{2}\right) = \dots$$