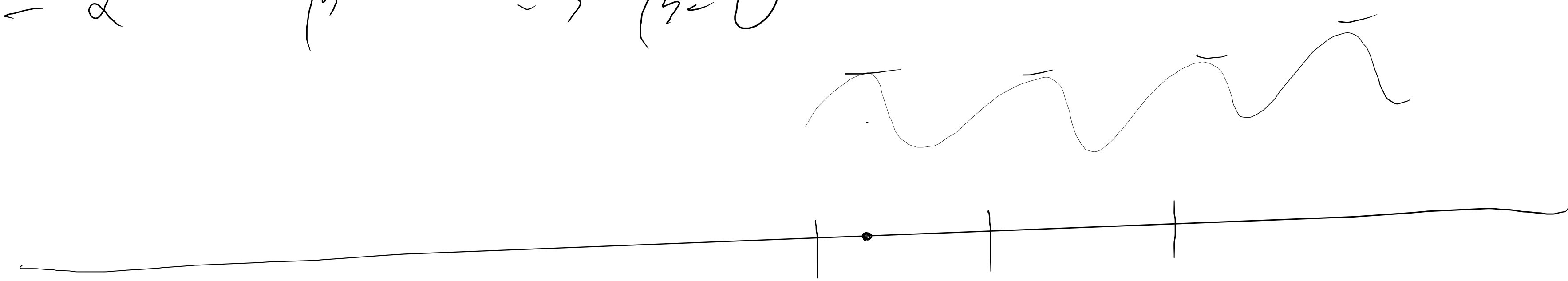
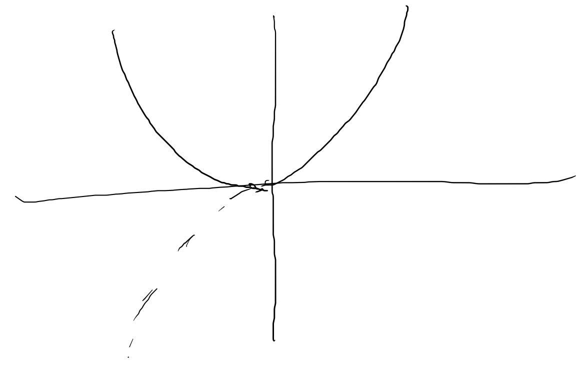
$\lim\limits_{x\to+\infty} f(x) = \alpha \in \mathbb{R}$ $\quad \exists \lim\limits_{x\to+\infty} f'(x) = \beta \quad \Rightarrow \quad \beta = 0$

$f(n+1) - f(n) = f'(\xi_n)$ Lagrange $\quad \xi_n \in]n, n+1[$

$$\begin{array}{ccccc} f(n+1) & - & f(n) & = & f'(\xi_n) \\ \downarrow & & \downarrow & & \downarrow \\ \alpha & - & \alpha & & \beta \end{array} \quad \Rightarrow \quad \beta = 0$$



$\lim_{x \to +\infty} \left(f(x) - x^3 \right) = 0$ f derivabile



$f(x) = |x|^3$ convessa quindi f può essere convessa

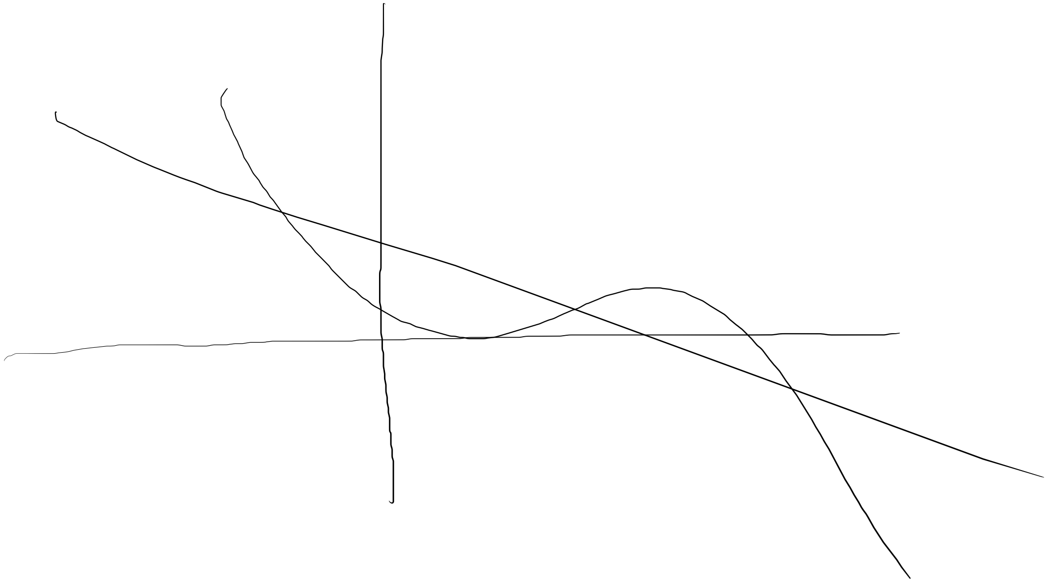
f può essere concava? f concava $\Rightarrow$ f' decrescente

$\Rightarrow \exists \lim_{x \to +\infty} f'(x) \in \mathbb{R} \cup \{-\infty\}$

$g(x) \to 0$

$g'(x) = f'(x) - \underbrace{3x^2}_{-\infty} = -\infty$ impossibile

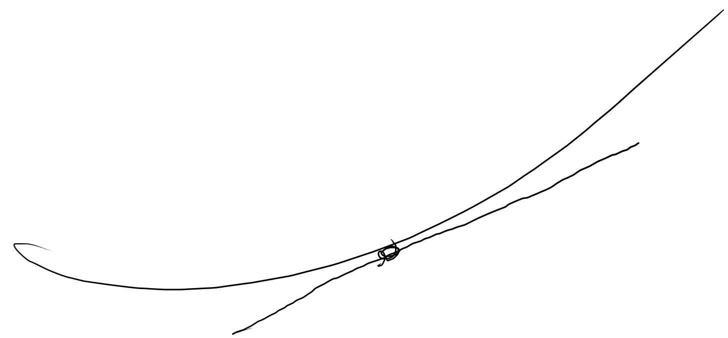
f' è superiormente limitata



$f \in C^2(\mathbb{R})$

$f''(x) = 0$

f ~~convessa~~



f ~~concava~~

$f \quad g$ con gli stessi P_n

$$\sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^n = \sum_{k=0}^{n} \frac{g^{(k)}(0)}{k!} x^n \Rightarrow f^{(k)}(0) = g^{(k)}(0) \quad \forall k$$

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f'(0) = \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}}}{x} = \lim_{x \to 0} \frac{1}{e^{\frac{1}{x^2}} x} = 0$$

$$f'(x) = e^{-\frac{1}{x^2}} \cdot \left(2 \frac{1}{x^3} \right)$$

$f : [a,b] \to \mathbb{R}$ derivabile $\overset{?}{\Rightarrow}$ f' limitata $\qquad [0,1]$

$$f(x) = \begin{cases} x^{3/2} \sin\frac{1}{x} & x \neq 0 \\ 0 & \end{cases} \qquad f'(x) = \begin{cases} \frac{3}{2} x^{\frac{1}{2}} \sin\frac{1}{x} - x^{3/2} \cos\left(\frac{1}{x}\right) \cdot \frac{1}{x^2} \\ 0 & x = 0 \end{cases}$$

$\frac{1}{\sqrt{x}}$

$D \frac{1}{x} \to -\frac{1}{x^2}$

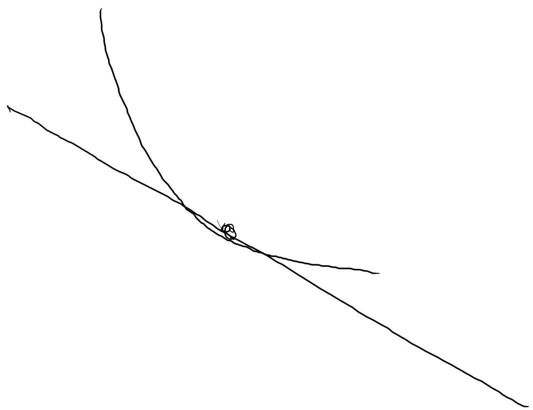
$$\begin{cases} x \sin\frac{1}{x} \\ 0 \end{cases} \qquad \lim_{x \to 0} \frac{f(x)}{x} = \nexists$$

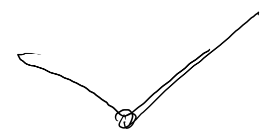
$$f'(0) = \lim_{x \to 0} \frac{f(x)}{x} = 0 \qquad \frac{x^{3/2} \sin\frac{1}{x}}{x} = \sqrt{x} \sin\frac{1}{x} \to 0$$

f strett. convessa $\Rightarrow$ f illimitata superiormente

f derivabile f' è crescente. Esiste $x_1 \in \mathbb{R}$: $f'(x_1) > 0$

$f'(x_1) < 0$





$$f(x) = 2\,\text{settsenh}(x) - \text{arcsen}(x)$$

$$\left[\log\left(1+\sqrt{x^2+1}\right)\right]$$

$$\text{dom} f = [-1,1]$$

f dispari $\quad f(-x) = -f(x)$

$$f(0) = 0$$

$$f(1) = 2\,\text{settsenh}(1) - \underbrace{\text{arcsen}(1)}_{=\pi/2}$$

$$\text{senh}(x) = 1 \qquad e^x - e^{-x} = 2$$

$$e^{-x}\left[e^{2x}-1\right] = 2$$

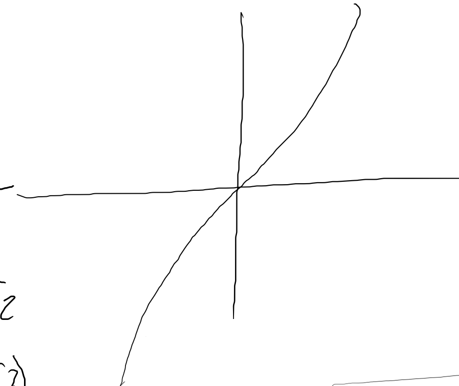
$$e^{2x} - 1 = 2e^x$$

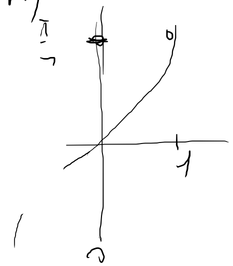
$$t^2 - 2t - 1 = 0$$

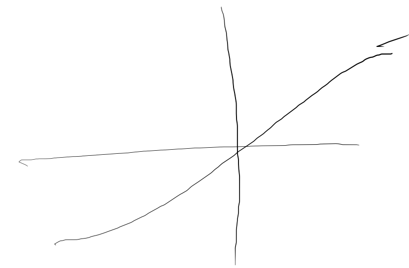
$$t = 1 \pm \sqrt{2}$$

$$e^x = 1+\sqrt{2}$$

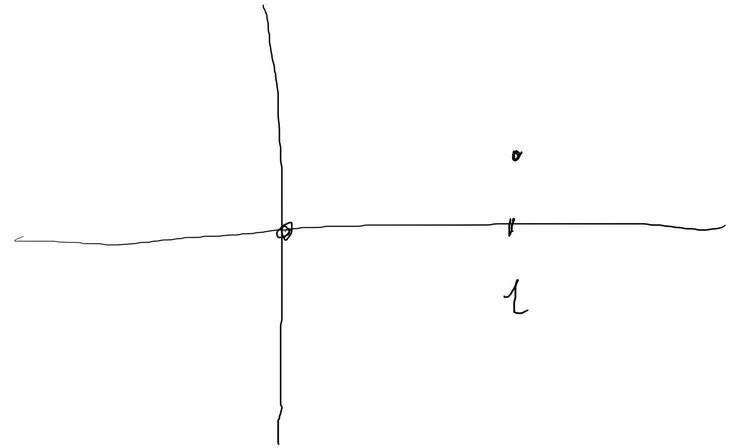
$$x = \log(1-\sqrt{2})$$







$$2\log(1+\sqrt{2}) - \frac{\pi}{2} \overset{?}{>} 0$$



$$4\log(1+\sqrt{2}) > \pi \quad \text{vero}$$

$$f'(x) = 2\underbrace{D(\text{settsinh}\, x)}_{} - D(\arcsin x)$$

$$D(\text{settsinh}\, x) = \frac{1}{\sqrt{1+x^2}}$$

$$D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{D\sinh(y)} = \frac{1}{\cosh(y)} = \frac{1}{\sqrt{1+x^2}}$$

$$y = \text{settsinh}\, x$$

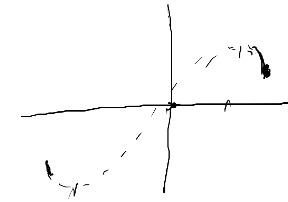
$$\cosh(y)^2 - \sinh(y)^2 = 1$$

$$\sqrt{\cosh(y)^2} = \sqrt{1+\sinh(y)^2}$$

$$\cosh y = \sqrt{1+\sinh^2(y)}$$

$$\boxed{f'(x) = \frac{2}{\sqrt{1+x^2}} - \frac{1}{\sqrt{1-x^2}}} = (1+x^2)^{-1/2} - (1-x^2)^{-1/2}$$

$$f'(1) = -\infty$$



$$f''(x) = 2\left[-\frac{1}{\cancel{2}}(1+x^2)^{-3/2}\cdot \cancel{2}x\right] - \left(-\frac{1}{\cancel{2}}\right)(1-x^2)^{-3/2}\cdot(-\cancel{2}x)$$

$$= -x\,\underbrace{\left[2(1+x^2)^{-3/2} + (1-x^2)^{-3/2}\right]}_{>0\ \forall x!}$$

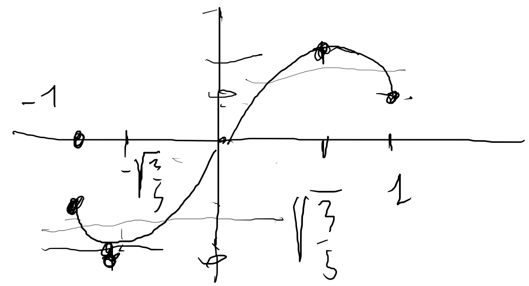
- <0 se $x>0$
- $=0$ se $x=0$
- >0 se $x<0$

$$f'(x) = \frac{2}{\sqrt{1+x^2}} - \frac{1}{\sqrt{1-x^2}} = \frac{2\sqrt{1-x^2} - \sqrt{1+x^2}}{(\quad)(\quad)}$$

$$\left[-1, -\sqrt{\tfrac{3}{5}}\right]$$

$$2\sqrt{1-x^2} - \sqrt{1+x^2} > 0$$

$$2\sqrt{1-x^2} > \sqrt{1+x^2}$$



$f(x) = 2$

$$4(1-x^2) > 1+x^2 \Rightarrow 4-4x^2 > 1+x^2 \qquad 5x^2 < 3$$

$$|x|^2 < \frac{3}{5} \qquad |x| < \sqrt{\frac{3}{5}}$$

6/9/2021 es 2 "Ghibli"

$f(x) = 2\arctan(x) + \frac{1-x}{1+x}$

1) dom f

dom $(2\arctan(x)) = \mathbb{R}$

dom $\left(\frac{1-x}{1+x}\right) = \mathbb{R} \setminus \{-1\}$ $x \neq -1$

dom $f = \mathbb{R} \setminus \{-1\}$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(2 \cdot \frac{\pi}{2} + \frac{x\left(\frac{1}{x} - 1\right)}{x\left(\frac{1}{x} + 1\right)} \right) = \pi - 1$

$y = \pi - 1$ è asintoto orizzontale

$\lim_{x \to -\infty} f(x) = -\pi - 1$

$y = -\pi - 1$ asintoto orizzontale

$\lim_{x \to -1^+} f(x) = +\infty$ $\lim_{x \to -1^-} f(x) = -\infty$

$x = -1$ è asintoto verticale

la $f(x)$ è illimitata in $\mathbb{R}$

II)

$$Df(x) = D\left(2\operatorname{arctg}(x) + \frac{1-x}{1+x}\right) = \frac{2}{1+x^2} + \left[\frac{-1-x-(1-x)}{(1+x)^2}\right] =$$

$$= \frac{2}{1+x^2} - \frac{2}{(1+x)^2} = \frac{4x}{(1+x^2)(1+x)^2}$$

III) Segno della derivata

$f'(x) = 0$ $\quad \dfrac{4x}{(1+x^2)(x+1)^2} = 0$ $\quad f'(0) = 0$ $\quad x = 0$ è candidato per essere

Segni della derivata

$f'(x) > 0$ se $x > 0$ $\quad f'(x) < 0$ se $x \in]-\infty, -1[\cup]-1, 0[$

f è crescente su $[0, +\infty[$ ed è decrescente su $]-\infty, -1[$ e $]-1, 0[$

$\Rightarrow x = 0$ è punto di min

$f(0)$ è min relativo

Segno delle funzioni

$f(x)$ tende a $-\pi-1$ se $x \to -\infty$ e $f(x)$ è decrescente su

$]-\infty, -1[$ $f(x) < 0$

$f(1)$ è positiva

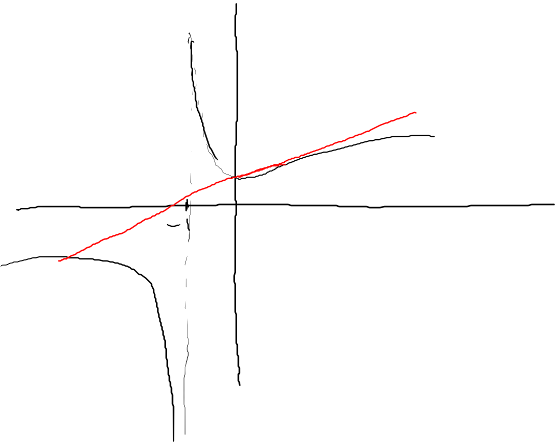
$f(x)$ in $x \to -1^+$ diverge a $+\infty$ e

$f(0)=1$ è minimo per $f \big|_{]-1, +\infty[}$

v)

$x=1$, $f(1) = \frac{\pi}{2}$ $f'(1) = \frac{1}{2}$

$y = f'(1)(x-1) + \frac{\pi}{2} = \frac{1}{2}x + \frac{\pi-1}{2}$



$$f(x) = \frac{1}{2}x + \frac{m-1}{2} \qquad \frac{m-1}{2} \overset{?}{>} 1$$

$$f(0) > 1 \qquad m-1 \overset{?}{>} 2$$

$\longrightarrow$ crescente $\qquad m > 3$

$$f(x) - f(x) = g(x) \longrightarrow \text{funzioni continue}$$

$$g(0) > 0$$

$$\lim_{x \to -1^+} g(x) = -\infty$$

$\exists$ uno zero (per Bolzano) in $]-1, 0[$

$$f(-1) = \frac{m-2}{2} > 0$$

$$\lim_{x \to -1^-} g(x) = +\infty$$

$$\lim_{x \to -\infty} g(x) = -\infty$$

$\exists$ zero in $]-\infty, -1[$ (per Bolzano)

ESERCIZIO 7 SETTEMBRE ES. 1

$$f(x) = \frac{\operatorname{arctg}(x) - \operatorname{arctg}(x^2)}{x}$$

(I) Si calcoli $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\operatorname{arctg}(x)}{x} - \frac{\operatorname{arctg}(x^2)}{x} \cdot \frac{x}{x} = \lim_{x\to 0}\left(\underbrace{\frac{\operatorname{arctg}(x)}{x}}_{\to 1} - \underbrace{\frac{\operatorname{arctg}(x^2)}{x^2}}_{\to 2} \cdot \underbrace{x}_{\to 0}\right) = 1$

(II) Si verifichi che la funzione f è infinitesima a $-\infty, +\infty$. Si calcolino gli ordini di infinitesimo:

$\lim_{x\to +\infty} \frac{\overbrace{\operatorname{arctg}(x)}^{\to \frac{\pi}{2}} - \overbrace{\operatorname{arctg}(x^2)}^{\to \frac{\pi}{2}}}{\underbrace{x}_{\to +\infty}} = 0$ $\qquad$ $\lim_{x\to -\infty} \frac{\overbrace{\operatorname{arctg}(x)}^{\to -\frac{\pi}{2}} - \overbrace{\operatorname{arctg}(x^2)}^{\to \frac{\pi}{2}}}{\underbrace{x}_{\to -\infty}} = 0$

$-\pi$

· $\operatorname{ord}_{-\infty} f = ?$ $\lim_{x\to -\infty} \left| \frac{\frac{\operatorname{arctg}(x) - \operatorname{arctg}(x^2)}{x}}{\frac{1}{|x|^\alpha}} \right| = \lim_{x\to -\infty} \left| |x|^\alpha \left(\frac{\operatorname{arctg}(x) - \operatorname{arctg}(x^2)}{x} \right) \right| = \lim_{x\to -\infty} \left| |x|^{\alpha-1} \overbrace{\left[\operatorname{arctg}(x) - \operatorname{arctg}(x^2)\right]}^{\pi} \right|$

$\Rightarrow$ Osserviamo che se vogliamo lim $\to$ numero reale $\neq 0$

· dobbiamo porre $\alpha - 1 = 0$ quindi $\operatorname{Ord}_{-\infty} f = 1$

· $\operatorname{ord}_{+\infty} f = ?$ $\lim_{x\to +\infty} \left| |x|^\alpha \frac{\operatorname{arctg}(x) - \operatorname{arctg}(x^2)}{x} \right| = \lim_{x\to +\infty} \left| |x|^{\alpha-1} \underbrace{\left(\operatorname{arctg}(x) - \operatorname{arctg}(x^2) \right)}_{\to 0} \right| = \lim_{x\to +\infty} \left| \frac{\operatorname{arctg}(x) - \operatorname{arctg}(x^2)}{\frac{1}{|x|^{\alpha-1}}} \right|$

- Usiamo il Teorema di De L'Hôpital $\Leftarrow \lim_{x\to +\infty} \frac{\frac{1}{x^2+1} - \frac{2x}{1+x^4}}{|(1-\alpha)x^{-\alpha}|} = \lim_{x\to +\infty} \left| |x|^\alpha \cdot \left(\frac{1}{1+x^2} - \underbrace{\frac{2x}{1+x^4}}_{\to 0} \right) \right| \cdot \frac{1}{|(1-\alpha)|}$

$\lim_{x\to +\infty} \left| \underbrace{\frac{x^\alpha}{1+x^2}}_{1 \text{ se } \alpha = 2} - \underbrace{\frac{2x^{\alpha+1}}{1+x^4}}_{0 \text{ se } \alpha = 2} \right| \cdot \frac{1}{|(1-\alpha)|} = 1$

. Osserviamo che se vogliamo il limite che tende ad un numero reale deve essere $\alpha = 2$

allora $\operatorname{ord}_{+\infty} f = 2$

27 GENNAIO 2020

$\log \cos 1$

1) $$\lim_{x \to 0} \frac{\sinh(2x) \log(\cos(e^{-x}))}{\arctan x}$$ $\to \simeq 0{,}5$

$$\lim_{x \to 0} \frac{\operatorname{senh} x}{x} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{senh}(2x)}{2x} \cdot \frac{2x}{\operatorname{arctg} x} \cdot \log(\cos(e^{-x})) \approx 2 \log(\cos(1))$$

$\sinh(2x) \to 0$, $\arctan x \to 0$, $\frac{\operatorname{senh}(2x)}{2x} \to 1$, $\frac{x}{\operatorname{arctg} x} \to 1$

$$\lim_{x\to+\infty} \frac{\operatorname{senh}(2x)\,\log(\cos(e^{-x}))}{\operatorname{arctg} x} = \lim_{x\to+\infty} \frac{\operatorname{senh}(2x)\,\log(\cos(e^{-x}))}{\frac{\pi}{2}}$$

$\operatorname{senh}(2x) \to +\infty$, $\log(\cos(e^{-x})) \to 0$, $\operatorname{arctg} x \to \frac{\pi}{2}$

$$\frac{2}{\pi}\lim_{x\to+\infty} \frac{\operatorname{senh}(2x)}{e^{2x}} \cdot \frac{\log(\cos(e^{-x}))}{e^{-2x}} = \frac{\cancel{2}}{\pi}\cdot\left(-\frac{1}{\cancel{4}_{2}}\right) = -\frac{1}{2\pi}$$

$\frac{\operatorname{senh}(2x)}{e^{2x}} \to \frac{1}{2}$, $\frac{\log(\cos(e^{-x}))}{e^{-2x}} \to -\frac{1}{2}$

$$\frac{1}{2}\left(e^{2x} - e^{-2x}\right)\frac{1}{e^{2x}} = \frac{1}{2} - \frac{e^{-4x}}{2} \to \frac{1}{2}$$

$$\frac{\log(\cos(e^{-x}))}{e^{-2x}} \overset{LH}{\Leftarrow} \frac{\frac{1}{\cos e^{-x}}\cdot(-\operatorname{sen} e^{-x})\cdot e^{-x}\cdot(-1)}{-2e^{-2x}} = \frac{\frac{1}{\cos e^{-x}}\cdot \operatorname{sen}(e^{-x})\cdot \cancel{e^{-x}}}{-2e^{-2x}}$$

$$= \frac{\operatorname{tg} e^{-x}}{-2e^{-x}}, \quad \frac{\operatorname{tg} e^{-x}}{e^{-x}} \to 1$$

$$f(x) = (x+1)\log|x+1| - x$$

$$\mathrm{dom}(f) = \mathbb{R} \setminus \{-1\}$$

$$\lim_{\substack{x \to -\infty \\ +\infty}} \underset{\downarrow}{\underset{-\infty}{(x+1)}}\ \underset{+\infty}{\log|x+1|}\ \underset{+\infty}{-x} = \lim_{\substack{x\to-\infty \\ +\infty}} \overset{\substack{+\infty \\ -\infty}}{(x+1)}\left[\overset{\substack{+\infty \\ +\infty}}{\log|x+1|} - \overset{1}{\frac{x}{x+1}}\right] = -\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -1} f(x) \qquad y = x+1 \qquad \lim_{y\to 0} \overset{0}{y\log|y|} - (y-1) = 1$$

$$f'(x) = \log|x+1| + \frac{1}{\cancel{(x+1)}}\cancel{(x+1)} - 1 = \log|x+1|$$

$$D\left[\log|x+1|\right] \begin{cases} D\left[\log(x+1)\right] & \text{se } x+1>0 \quad \dfrac{1}{x+1} \\ D\left[\log(-x-1)\right] & \text{se } x+1<0 \quad \dfrac{1}{-x-1}\cdot(-1) = \dfrac{1}{x+1} \end{cases}$$

$f'(x) > 0 \qquad \log|x+1| > 0 \qquad |x+1| > 1 \rightarrow$

$x+1 > 1 \qquad x > 0$

$-x-1 > 1 \qquad x < -2 \qquad \longrightarrow \qquad x < -2 \text{ e } x > 0$

$f'(x) > 0$ se $x < -2$ o se $x > 0$ $\qquad f'(-2) = f'(0) = 0$

$f'(x) < 0$ se $x \in \left]-2, 0\right[\setminus \{-1\}$

f CRESCENTE SU $]-\infty, -2]$ e $[0, +\infty[$

f DECRESCENTE SU $[-2, -1[$ e $]-1, 0] \to [-2, -1[\cup]-1, 0]$

$R(y) = x$ $\quad f'(x) = R(x)$ $\quad \log|x+1| = 1$ $\quad |x+1| = e$

$-x-1 = e \quad x = -e-1$

$x+1 = e \quad x = e-1$

$$f'(x_0)(x-x_0) + f(x_0) = y \qquad (x+e+1)+1 = x+2+e$$

$f' = 1$

$$f(-e-1) = -e\log|\underbrace{-e}_{1}| + e + 1 = 1$$

$$y = 1\cdot(x+e+1)+1 \leadsto y = x+e+2$$