Identificazione dei coeff. di trasporto

1) <u>Diffusione</u> : legge di Fick $\quad T = \text{cost} \quad$ diluito $\rightarrow \quad \bar{J}_N = -D\,\bar{\nabla} \rho_N \quad \nearrow D \approx \text{cost}$

$$\bar{J}_N = \underbrace{L_{EN}\,\bar{\nabla}\left(\frac{1}{T}\right)}_{=0} + L_{NN}\,\bar{\nabla}\left(-\frac{\mu}{T}\right) = -\frac{L_{NN}}{T}\,\bar{\nabla}\mu = -\underbrace{\frac{L_{NN}}{T}\,\frac{\partial \mu}{\partial \rho_N}\Big|_T}_{\text{coeff. diffusione}}\,\bar{\nabla}\rho_N = -\frac{k_B L_{NN}}{\rho_N}\,\bar{\nabla}\rho_N$$

$\uparrow$ $\mu = \mu_0 + k_B T \ln \rho_N$ (diluito)

<u>es</u>.

2) <u>Conduzione termica</u> : legge di Fourier $\quad$ solidi isolanti $\rightarrow \quad \bar{J}_E = -k_T\,\bar{\nabla} T$

Le Bellac : $\quad \bar{J}_N = 0$

$$\begin{cases} \bar{J}_N = L_{EN}\,\bar{\nabla}\left(\frac{1}{T}\right) + L_{NN}\,\bar{\nabla}\left(-\frac{\mu}{T}\right) = 0 & \Rightarrow \quad \bar{\nabla}\left(-\frac{\mu}{T}\right) = -\frac{L_{EN}}{L_{NN}}\,\bar{\nabla}\left(\frac{1}{T}\right) \\ \bar{J}_E = L_{EE}\,\bar{\nabla}\left(\frac{1}{T}\right) + L_{NE}\,\bar{\nabla}\left(-\frac{\mu}{T}\right) & \Rightarrow \quad \bar{J}_E = \left(L_{EE} - \frac{L_{EN}^2}{L_{NN}}\right)\bar{\nabla}\left(\frac{1}{T}\right) = -\underbrace{\frac{1}{T^2}\,\frac{L_{EE}L_{NN} - L_{EN}^2}{L_{NN}}}_{= k_T}\,\bar{\nabla} T \end{cases}$$

BH : $\quad \mu = \text{cost} \quad \Rightarrow \quad k_T = \dfrac{L_{EE}}{T^2}$

$$\bar{J}_E = \left(L_{EE} - \mu L_{NE}\right)\bar{\nabla}\left(\frac{1}{T}\right) = -\frac{L_{EE} - \mu L_{NE}}{T^2}\,\bar{\nabla} T$$

<u>Es.</u> : $\quad \bar{\nabla} \rho_N = 0$

Equazioni di trasporto

Eq. continuità + eq. costitutive $\Rightarrow$ eq. del moto $T(\bar{r},t)$, $\rho_N(\bar{r},t)$, ...

1) Eq. diffusione : $T = \text{cost}$ diluito

$$\frac{\partial \rho_N}{\partial t} = -\vec{\nabla}\cdot\vec{J}_N \underset{\text{Fick}}{=} -\vec{\nabla}\cdot\left(-D\vec{\nabla}\rho_N\right) \underset{D\,\approx\,\text{cost}}{=} D\nabla^2\rho_N$$

2) Eq. calore : solido isolante $E = C_V T$

$$\frac{\partial \rho_E}{\partial t} = -\overline{\nabla}\cdot\overline{J}_E \underset{\text{Fourier}}{=} -\vec{\nabla}\cdot\left(-K_T\overline{\nabla}T\right) = K_T\nabla^2 T \qquad \rho_E = \rho\, c_v\, T$$

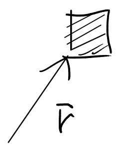
$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho C_v}\nabla^2 T = \underbrace{D_T}_{\text{diffusività termica}}\nabla^2 T$$

Campo esterno

$T = \text{cost}$

Esempi: campo elettrico in un conduttore

campo gravitazionale su sospensione colloidale



$\phi(\vec r)$ = energia potenziale per particella $\qquad \phi = q\phi_{el}$; $\phi = mgz$

$\Phi = N\phi$

$S(E, N, \phi) = S(E - N\phi, N, 0)$

$dS = \frac{1}{T}dE - \frac{\phi}{T}dN - \frac{\mu}{T}dN$

$$dS = \frac{1}{T}dE - \frac{\phi}{T}dN - \frac{\mu}{T}dN = \frac{1}{T}dE + \left(-\frac{\phi}{T} - \frac{\mu}{T}\right)dN$$

$$Y_N = \frac{\partial S}{\partial N} = \underbrace{-\frac{\phi}{T}}_{\text{potenziale esterno}} - \underbrace{\frac{\mu}{T}}_{\text{potenziale chimico}}$$

⚠ in assenza di campo

Legge di Ohm: $T = \text{cost}$

$\vec J_e = -\sigma \vec\nabla \phi_e \qquad \vec\nabla \rho_e = 0$

$\vec J_e = \frac{L_{ee}}{T} q \vec\nabla\phi_e \Rightarrow \sigma = -\frac{L_{ee}\, q}{T}$

$$\vec J_N = L_{NN}\vec\nabla\left(-\frac{\phi}{T} - \frac{\mu}{T}\right) = -\frac{L_{NN}}{T}\vec\nabla\phi - \frac{L_{NN}}{T}\frac{\partial\mu}{\partial\rho_N}\Big|_T \vec\nabla\rho_N$$

$\sim \rho_N$

$$\frac{\partial\rho_N}{\partial t} = -\vec\nabla\cdot\vec J_N = -\vec\nabla\cdot\left(-\underbrace{\frac{L_{NN}}{T}\frac{\partial\mu}{\partial\rho_N}\Big|_T}_{D}\vec\nabla\rho_N - \frac{L_{NN}}{T}\underbrace{\vec\nabla\phi}_{-\vec F}\right) = \vec\nabla\cdot\left(D\vec\nabla\rho_N + \frac{L_{NN}}{T}\vec F\right)$$

$$\frac{\partial\rho_N}{\partial t} = \vec\nabla\cdot\left(D\vec\nabla\rho_N + \alpha\rho_N\vec F\right)$$

$\alpha \equiv$ mobilità $\qquad \longrightarrow$ eq. deriva-diffusione (cf. Smoluchowski) regime diluito: $\rho_N(\bar{r},t) \sim p(\bar{r},t)$ $\lambda \to \frac{1}{\varepsilon}$

Esempio: diluito, equilibrio, campo esterno ϕ $\to$ $\lambda \leftrightarrow D$?

$\bar{J}_N = 0 \quad \to \quad -D\bar{\nabla}\rho_N - \lambda\rho_N\bar{\nabla}\phi = 0$

$\rho_N \sim p \sim \exp\left(-\frac{\phi(\bar{r})}{K_B T}\right)$

$-D\left(-\frac{1}{K_BT}\right)\cancel{\bar{\nabla}}\phi \cdot \cancel{\exp(\;)} - \lambda\cancel{\exp(\;)}\cancel{\bar{\nabla}}\phi = 0 \quad \Rightarrow \quad \frac{D}{K_BT} = \lambda \quad \Rightarrow \quad D = K_BT \cdot \lambda$

$D = \frac{K_BT}{\varepsilon}$