3 Dicembre

Espansioni di Hermite per $R(z) = \dfrac{P(z)}{Q(z)}$ con

grado $P(z) <$ grado $Q(z) = n$

$Q(z) = a_n z^n + a_{n-1} z^{n-1} + \dots + a_0$, ha radici distinte

$z_1, \dots, z_k$

$m_1, \dots, m_k$

$$Q(z) = a_n (z - z_1)^{m_1} \dots (z - z_k)^{m_k}$$

Teor Assumendo quanto indicato sopra esistono delle costanti

$$
\begin{array}{ll}
z_1 & A_{11}, \dots, A_{1m_1} \\
\vdots & \\
z_j & A_{j1}, \dots, A_{jm_j} \\
z_k & A_{k1}, \dots, A_{km_k}
\end{array}
$$

tali che

$$
R(z) = \frac{A_{11}}{z-z_1} + \frac{A_{12}}{(z-z_1)^2} + \dots + \frac{A_{1m_1}}{(z-z_1)^{m_1}}
$$

$$
\vdots
$$

$$
+ \frac{A_{k1}}{z-z_k} + \dots + \frac{A_{km_k}}{(z-z_k)^{m_k}} = \sum_{j=1}^{k} \sum_{l=1}^{m_j} \frac{A_{jl}}{(z-z_j)^l}
$$

e vale inoltre la seguente formula:

$$A_{jl} = \frac{1}{(m_j - l)!} \left(\frac{d}{dz} \right)^{m_j - l} \left(R(z)(z - z_j)^{m_j} \right) \Big|_{z = z_j}$$

$\forall\ j = 1, \dots, k$ e $\forall\ l = 1, \dots, m_j$

$$R(x) = \frac{1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

$$A = R(x)\, x \Big|_{x=0}, \qquad B = R(x)(x-1) \Big|_{x=1}$$

$$C = R(x)(x-2) \Big|_{x=2}$$

$$R(x)=\frac{1}{x^2(x-1)(x-2)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}+\frac{D}{x-2}$$

$$C=R(x)(x-1)\Big|_{x=1}=\frac{1}{x^2(x-2)}\Big|_{x=1}=-1$$

$$A_{jl}=\frac{1}{(m_j-l)!}\left(\frac{d}{dz}\right)^{m_j-l}R(z)(z-z_j)^{m_j}\Big|_{z=z_j}$$

$$D=R(x)(x-2)\Big|_{x=2}=\frac{1}{x^2(x-1)}\Big|_{x=2}=\frac{1}{4}$$

$$B=R(x)\,x^2\Big|_{x=0}=\frac{1}{(x-1)(x-2)}\Big|_{x=0}=\frac{1}{2}$$

$m_j=2,\ l=2$

$m_j=2,\ l=1$

$$A=\frac{1}{1!}\frac{d}{dx}R(x)\,x^2\Big|_{x=0}=\left(\frac{1}{(x-1)(x-2)}\right)'\Big|_{x=0}=\left(\frac{1}{x^2-3x+2}\right)'\Big|_{x=0}$$

$$=-\frac{2x-3}{(x^2-3x+2)^2}\Big|_{x=0}=\frac{3}{4}$$

$$R(x) = \frac{1}{x^2(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x-2} \quad \cdot x$$

$$\frac{1}{x(x-1)(x-2)} = A + \frac{B}{x} + C\,\frac{x}{x-1} + D\,\frac{x}{x-2}$$

$$\Big\downarrow\; x \to +\infty$$

$$0 = A + 0 + C + D$$