7 Dicembre

2015–2016 luglio

$$\lim_{x\to+\infty} \frac{\int_x^{2x} \sin(t^{-a})\,dt + \frac{3}{4}x^{-2} - th(x)\,lg(2)}{\cancel{lg\left(1+\frac{1}{x^3}\right)}\ \frac{1}{x^5}} \qquad a>0$$

$$\lim_{y\to 0} \frac{lg(1+y)}{y} = 1 \implies \lim_{x\to+\infty} \frac{lg\left(1+\frac{1}{x^5}\right)}{\frac{1}{x^5}} = 1$$

$$th(x) = 1 - 2e^{-2x}(1+o(1))$$

$$\frac{\int_x^{2x} \sin(t^{-a})\,dt + \frac{3}{4}x^{-2} - lg2}{1/x^5} + 2x^5 e^{-2x}(1+o(1)) \to 0$$

$$\lim_{x\to+\infty} \frac{x^5}{e^{+2x}} \overset{H}{=} \lim_{x\to+\infty} \frac{5x^4}{2e^{2x}} = \dots\dots = \lim_{x\to+\infty} \frac{5!}{2^5 e^{2x}} = 0$$

$$\int_x^{2x} \sin(t^{-a})\,dt = \int_x^{2x} t^{-a}(1+o(1))\,dt \qquad a\neq 1 \qquad 0<a<1$$

$$\sin x = x + o(x) = x(1+o(1)) \qquad x = t^{-a}$$

$$\int_x^{2x} t^{-a}\,dt = \left.\frac{t^{-a+1}}{-a+1}\right]_x^{2x} = \frac{2^{1-a}x^{1-a}}{1-a} - \frac{x^{1-a}}{1-a} =$$

$$= \frac{x^{1-a}}{1-a}\left(2^{1-a}-1\right) \xrightarrow{x\to+\infty} +\infty \quad \text{se} \quad 0<a<1$$

Se ad esem $0<a<1$

$$\int_x^{2x} t^{-a}\underbrace{(1+o(1))}_{\geq\frac{1}{2}}\,dt \geq \frac{1}{2}\int_x^{2x} t^{-a}\,dt = \frac{1}{2}\underbrace{\frac{x^{1-a}}{1-a}(2^{1-a}-1)}_{\downarrow\, x\to+\infty\quad +\infty}$$

$0<a<1$ il nostro limite coincide col limite di

$$\lim_{x\to+\infty} x^5\left(\int_x^{2x} t^{-a}(1+o(1))\,dt + \frac{3}{4}x^{-2} - lg2\right) =$$

$$\lim_{x\to+\infty} x^5 \int_x^{2x} t^{-a}(1+o(1))\,dt = +\infty \qquad 0<a<1$$

$a=1$

$$x^5\left(\int_x^{2x} \sin(t^{-1})\,dt + \frac{3}{4}x^{-2} - lg2\right) = x^5\left(\left(\frac{3}{4}-\frac{1}{16}\right)x^{-2} + o(x^{-2})\right)$$

$$= \underbrace{x^3\left(\left(\frac{3}{4}-\frac{1}{16}\right)+o(1)\right)}_{\downarrow\ +\infty\ \frac{11}{16}} = +\infty$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{5!} + o(x^6)$$

$$\sin(t^{-1}) = t^{-1} - \frac{t^{-3}}{6} + \frac{t^{-5}}{5!} + o(t^{-6})$$

$$\int_x^{2x} \sin(t^{-1})\,dt + \frac{3}{4}x^{-2} - lg2 = \underbrace{\int_x^{2x} t^{-1}\,dt}_{=lg2} - \frac{1}{6}\int_x^{2x} t^{-3}\,dt + \underbrace{\frac{1}{5!}\int_x^{2x} t^{-5} + \int_x^{2x} o(t^{-6})\,dt}_{o(x^{-2})} + \frac{3}{4}x^{-2} - \cancel{lg2}$$

$$-\frac{1}{6}\int_x^{2x} t^{-3}\,dt = \frac{1}{6}\left.\frac{t^{-2}}{2}\right]_x^{2x} = \frac{1}{12}\left(2^{-2}x^{-2} - x^{-2}\right) = -\frac{1}{12}\,\frac{3}{4}\,x^{-2}$$

$$= -\frac{1}{16}x^{-2}$$

$$= \left(\frac{3}{4}-\frac{1}{16}\right)x^{-2} + o(x^{-2})$$

$a>1$

$$x^5\left(\int_x^{2x} \sin(t^{-a})\,dt + \frac{3}{4}x^{-2} - lg2\right)$$

$$\sin(t^{-a}) = t^{-a}(1+o(1))$$

$$\int_x^{2x} t^{-a}\,dt = \left.\frac{t^{-a+1}}{-a+1}\right]_x^{2x} = x^{-a+1}\,\frac{(2^{-a+1}-1)}{-a+1} \xrightarrow{x\to+\infty} 0$$

$$0 < \underbrace{\int_x^{2x} t^{-a}(1+o(1))\,dt}_{\downarrow\ 0} < \int_x^{2x} t^{-a}\,2\,dt = 2x^{-a+1}\,\frac{2^{-a+1}-1}{-a+1} \xrightarrow{x\to+\infty} 0$$

$$= x^5\left(-lg2 + o(1)\right) \longrightarrow +\infty\,(-lg2) = -\infty$$

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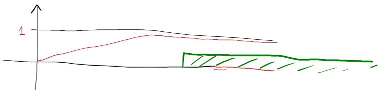
$$f(x) = \begin{cases} \displaystyle\int_0^x \frac{t}{\sqrt{t^2+t}+1}\,dt & x \geq 0 \\ \displaystyle\int_0^x 3^{[t]}\,dt & x \leq 0 \end{cases}$$

$$\lim_{x\to+\infty} f(x) = +\infty$$

$$\lim_{t\to+\infty} \overbrace{\frac{t}{\sqrt{t^2+t}+1}}^{g(t)} = 1$$

$$= \lim_{t\to+\infty} \frac{t}{\sqrt{t^2+t}} = \lim_{t\to+\infty} \frac{t}{t} = 1$$



Asintoto a $+\infty$?

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\int_0^x \frac{t}{\sqrt{t^2+t}+1}\,dt}{x} \overset{\frac{\infty}{\infty}}{\overset{H}{=}}$$

$$= \lim_{x\to+\infty} \frac{x}{\sqrt{x^2+x}+1} = 1 = m$$

$$\lim_{x\to+\infty} (f(x) - x) = ?$$

$$\int_0^x \frac{t}{\sqrt{t^2+t}+1}\,dt - x = \int_0^x \left(\frac{t}{\sqrt{t^2+t}+1} - 1\right) dt$$

$$1 - \frac{t}{\sqrt{t^2+t}+1} = \frac{\sqrt{t^2+t}+1-t}{\sqrt{t^2+t}+1} = \frac{\sqrt{t^2+t}-(t-1)}{\sqrt{t^2+t}+1}$$

$$= \frac{\sqrt{t^2+t}-(t-1)}{\sqrt{t^2+t}+1} \cdot \frac{\sqrt{t^2+t}+(t-1)}{\sqrt{t^2+t}+(t-1)} =$$

$$= \frac{\cancel{t^2}+t-(\cancel{t^2}-2t+1)}{\left(\sqrt{t^2+t}+1\right)\left(\sqrt{t^2+t}+(t-1)\right)} = \frac{3t-1}{\left(\sqrt{t^2+t}+1\right)\left(\sqrt{t^2+t}+(t-1)\right)}$$

$$= \frac{3t-1}{t(1+o(1))\;2t(1+o(1))} = \frac{3t}{2t^2}(1+o(1)) =$$

$$= \frac{3}{2}\,\frac{1}{t}\,(1+o(1))$$

Non è integrabile in $[0,+\infty)$

$$\Rightarrow \lim_{x\to+} \int_0^x \left(1 - \frac{t}{\sqrt{t^2+t}+1}\right) dt = +\infty$$

Non esiste retta asintotica

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \int_0^x 3^{[t]}\,dt =$$

$$= -\lim_{x\to-\infty} \int_x^0 3^{[t]}\,dt = -\int_{-\infty}^0 3^{[t]}\,dt$$

$[t] \le t \qquad 3^{[t]} \le 3^t$

$\int_{-\infty}^0 3^t\,dt = \frac{1}{\lg 3}$ esiste finito e quindi per confronto esiste finito $\int_{-\infty}^0 3^{[t]}dt$

$$\int_{-\infty}^0 3^t\,dt = \int_{-\infty}^0 e^{t\lg 3}\,dt = \lim_{R\to+\infty} \int_{-R}^0 e^{t\lg 3}\,dt$$

$$= \lim_{R\to+\infty} \left[\frac{e^{t\lg 3}}{\lg 3}\right]_{-R}^0 = \lim_{R\to+\infty}\left(\frac{1}{\lg 3} - \underbrace{\frac{e^{-R\lg 3}}{\lg 3}}_{\to 0}\right)$$

$$= \frac{1}{\lg 3}$$

$$\int_{-\infty}^0 3^{[t]}\,dt = \lim_{R\to+\infty} \int_{-R}^0 3^{[t]}\,dt = \lim_{n\to+\infty} \int_{-n}^0 3^{[t]}\,dt =$$

$$= \lim_{n\to+\infty} \sum_{j=1}^n \int_{-j}^{-(j-1)} 3^{[t]}\,dt =$$

$$= \lim_{n\to+\infty} \sum_{j=1}^n \int_{-j}^{-(j-1)} 3^{-j}\,dt = \lim_{n\to+\infty} \sum_{j=1}^n 3^{-j} =$$

$$= \lim_{n\to+\infty} \left(\sum_{j=0}^n 3^{-j} - 1\right) = \lim_{n\to+\infty}\left(\frac{1-\left(\frac13\right)^{n+1}}{1-\frac13} - 1\right) = \frac12$$

$$= \frac{1}{\frac23} - 1 = \frac32 - 1 = \frac12$$

$$\lim_{x\to-\infty} f(x) = -\frac12$$

Per $x>0$ $\qquad f'(x) = \frac{x}{\sqrt{x^2+x}+1} \ge 0 \qquad f'_d(0)=0$

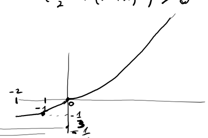
per $x>0$

$$f''(x) = \frac{\sqrt{x^2+x}+1-\frac12 x\,(x^2+x)^{-\frac12}\,(2x+1)}{\left(\sqrt{x^2+x}+1\right)^2} =$$

$$= \frac{1}{\left(\sqrt{x^2+x}+1\right)^2}\left((x^2+x)^{\frac12}+1-(x^2+\tfrac12 x)\,(x^2+x)^{-\frac12}\right)$$

$$= \frac{1}{\left(\sqrt{x^2+x}+1\right)^2\sqrt{x^2+x}}\left(\cancel{x^2}+x+(x^2+x)^{\frac12} - \cancel{x^2} - \tfrac12 x\right)$$

$$= \quad '' \quad \left(\tfrac12 x + (x^2+x)^{\frac12}\right) > 0$$



Per $x<0$ $\qquad f(x) = \int_0^x 3^{[t]}\,dt$

per $x \notin \mathbb{Z}_{\le 0}$ $\qquad f'(x) = 3^{[x]}$

$$f'_s(0) = \frac13$$