10 Dicembre

Espansioni di Hermite

$$R(x) = \frac{3x^2+4x+2}{(x+1)(x^2+x+1)}$$

<u>Cuccagna</u> $\quad x^2+x+1=0 \qquad x_\pm = -\frac{1}{2} \pm \sqrt{-3} = -\frac{1}{2} \pm i\sqrt{3}$

$$R(x) = \frac{3x^2+4x+2}{(x+1)\underbrace{(x+\frac{1}{2}-i\sqrt{3})}_{(x-x_+)}\underbrace{(x+\frac{1}{2}+i\sqrt{3})}_{(x-x_-)}} = \frac{A}{x+1} + \frac{B}{x-x_+} + \frac{C}{x-x_-}$$

$$A = (x+1)R(x)\Big|_{x=-1} \qquad C = (x-x_-)R(x)\Big|_{x=x_-=-\frac{1}{2}-i\sqrt{3}}$$

$$B = (x-x_+)R(x)\Big|_{x=x_+=-\frac{1}{2}+i\sqrt{3}}$$

$$R(x) = \frac{3x^2+4x+2}{\cancel{(x+1)(x^2+x+1)}} = \frac{A(x-x_+)(x-x_-) + B(x+1)(x-x_-) + C(x+1)(x-x_+)}{\cancel{(x+1)(x^2+x+1)}}$$

$$3x^2+4x+2 = (A+B+C)x^2 + (-Ax_+ - Ax_- + B - Bx_- + C - Cx_+)x +$$
$$+ (Ax_+x_- - Bx_- - Cx_+)$$

$$\begin{cases} A+B+C = 3 \\ -(x_+ + x_-)A + (1-x_-)B + (1-x_+)C = 4 \\ Ax_+x_- - Bx_- - Cx_+ = 2 \end{cases} \qquad \text{ecc.}$$

Graffa
$$R(x) = \frac{3x^2+4x+2}{(x+1)(x^2+x+1)} = \frac{\alpha}{x+1} + \frac{\beta x+\gamma}{x^2+x+1}$$

Cuccagna $\quad R(x) = \dfrac{3x^2+4x+2}{(x+1)(x-x_-)(x-x_+)} = \dfrac{A}{x+1} + \dfrac{B}{x-x_+} + \dfrac{C}{x-x_-}$

uguali $\alpha = A$

$x^2+x+1 = (x-x_-)(x-x_+)$

$$\frac{B}{x-x_+} + \frac{C}{x-x_-} = \frac{B(x-x_-)+C(x-x_+)}{(x-x_+)(x-x_-)} = \frac{(B+C)x - Bx_- - Cx_+}{x^2+x+1}$$
$$= \frac{\beta x+\gamma}{x^2+x+1}$$

$$\begin{cases} \beta = B+C \\ \gamma = -Bx_- - Cx_+ \end{cases}$$

$$R(x) = \frac{P(x)}{Q(x)} \qquad \text{grado } P < \text{grado } Q = n$$

$$Q(x) = a_n (x-x_1)^{m_1} \cdots (x-x_k)^{m_k} \qquad m_1 + \cdots m_k = n$$

$$= \frac{A_1}{x-x_1} + \cdots + \frac{A_k}{x-x_k} + \frac{d}{dx} S(x)$$

dove $S(x)$ e' una funzione razionale.

Espansione di Hermite per $R(x) = \dfrac{P(x)}{Q(x)}$ con grado $P \geq$ grado Q

Si divide P per Q, cioè si trovano un polinomio $q(x)$ (il quoziente) ed $r(x)$ (il resto) con grado $r <$ grado Q

con $\quad P(x) = Q(x)\,q(x) + r(x)$

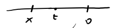
$$R(x) = \frac{P(x)}{Q(x)} = \frac{Q(x)q(x)+r(x)}{Q(x)} = q(x) + \frac{r(x)}{Q(x)}$$

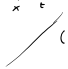
dove $\dfrac{r(x)}{Q(x)}$, essendo grado $r <$ grado Q, rientra nella casistica precedente.

14 giugno 21

$g:\mathbb{R}\to\mathbb{R}$ è l'inversa di $x\to(x+2x^3+4x^7)$

$$f(x)=\begin{cases}\displaystyle\int_x^{x+\arctan(x)}\frac{1}{g(t)}\,dt & \text{se } x>0\\ \displaystyle\int_0^x\frac{t}{(t-1)(t-2)^2}\,dt & \text{se } x\le 0\end{cases}$$





$(x+2x^3+4x^7)'=$

$1+6x^2+28x^6>0$

>0

$$\frac{1}{x+2x^3+4x^7}$$

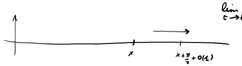
$y=4x^7+2x^3+x \iff x=g(y)$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\int_x^{x+\arctan x}\frac{1}{g(t)}\,dt$$

$$\int_x^{x+\arctan x}\frac{1}{g(t)}\,dt=\int_x^{x+\frac{\pi}{2}+o(1)}\frac{1}{g(t)}\,dt$$

$\lim_{t\to+\infty}g(t)=+\infty$

$\lim_{t\to+\infty}\frac{1}{g(t)}=0$



$$0<\int_x^{x+\frac{\pi}{2}+o(1)}\frac{1}{g(t)}\,dt<\int_x^{x+\frac{\pi}{2}}\frac{1}{g(t)}\,dt<\int_x^{x+\frac{\pi}{2}}\frac{1}{g(x)}\,dt$$

$$=\frac{\pi}{2}\,\frac{1}{g(x)}\xrightarrow{x\to+\infty}0$$

$$\Rightarrow \lim_{x\to+\infty}f(x)=0$$

$$\lim_{x\to-\infty}f(x)=?\qquad \lim_{x\to-\infty}\int_0^x\frac{t}{(t-1)(t-2)^2}\,dt$$

$$\frac{t}{(t-1)(t-2)^2}=\frac{A}{t-1}+\frac{B}{t-2}+\frac{C}{(t-2)^2}$$

$$A=\frac{t}{(t-2)^2}\Big|_{t=1}=1\ ,\quad C=\frac{t}{(t-1)}\Big|_{t=2}=2$$

So che $A+B=0$

$$\frac{t^2}{(t-1)(t-2)^2}=A\frac{t}{t-1}+B\frac{t}{t-2}+C\frac{t}{(t-2)^2}\qquad t\to+\infty$$

$$0=A+B\qquad \boxed{B=-1}$$

$$\frac{t}{(t-1)(t-2)^2}=\frac{1}{t-1}-\frac{1}{t-2}+\frac{2}{(t-2)^2}$$

$$\int_0^x\frac{t}{(t-1)(t-2)^2}\,dt=\int_0^x\frac{1}{t-1}\,dt-\int_0^x\frac{1}{t-2}\,dt+2\int_0^x\frac{1}{(t-2)^2}\,dt$$

$$=\lg|t-1|\Big]_0^x-\lg|t-2|\Big]_0^x-2\,\frac{1}{t-2}\Big]_0^x$$

$$=\lg\left|\frac{t-1}{t-2}\right|\Big]_0^x-2\,\frac{1}{t-2}\Big]_0^x$$

$$=\lg\left|\frac{x-1}{x-2}\right|-\lg\frac{1}{2}-\frac{2}{x-2}+2\,\frac{1}{-2}$$

$$=\lg\frac{1-x}{2-x}-\frac{2}{x-2}+\lg 2-1\xrightarrow{x\to-\infty}\lg 2-1<0$$

$\downarrow$
0

Per $x<0$ $\quad f(x)=\displaystyle\int_0^x\frac{t}{(t-1)(t-2)^2}\,dt=-\int_x^0\frac{t}{(t-1)(t-2)^2}\,dt$

≤ 0