Geometria oggi alle 14 – 15:30

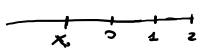
13 Dicembre

$g(t)$ è l'inversa di $t \to (t + 2t^3 + 4t^7)$

$$f(x) = \begin{cases} \int_x^{x+\arctan x} \frac{1}{g(t)}\, dt & x > 0 \\ \int_0^x \frac{t}{(t-1)(t-2)^2}\, dt & x \le 0 \end{cases}$$

Continua in $\mathbb{R}$? L'unica eventuale discontinuità si ha in $x = 0$.

$$\lim_{x\to 0^-} f(x) = \lim_{x \to 0^-} \int_0^x \frac{t}{(t-1)(t-2)^2}\, dt$$



$$= \int_0^0 \frac{t}{(t-1)(t-2)^2}\, dt = 0 = f(0)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \int_x^{x+\arctan x} \frac{1}{g(t)}\, dt$$

Consideriamo il polinomio di McLaurin di $g(x)$

$P_1(x) = \underbrace{g(0)}_{0} + \underbrace{g'(0)}_{1}\, x$. Sappiamo che $g(x)$ è la funzione inversa di $h(x) = x + 2x^3 + 4x^7$. Se calcolo $g(0)$ e $g'(0)$. $h'(x) = 1 + 6x^2 + 28x^6$, $h'(0) = 1$

Ricordiamo $y = h(x) \iff x = g(y)$

$0 = h(0) \iff 0 = g(0)$ $\qquad g'(y) = \frac{1}{h'(x)}$

$g'(0) = \frac{1}{h'(0)} = 1$

$P_1(x) = x$

$g(x) = x + o(x) = x(1 + o(1))$

$g(t) = t(1 + o(1))$

$\arctan x = x + o(x)$

$$f(x) = \int_x^{x+\arctan(x)} \frac{1}{g(t)}\, dt =$$

$$= \int_x^{x+\arctan x} \frac{1}{t} \underbrace{\frac{1}{1+o(1)}}_{1+o(1)} dt = \int_x^{x+\arctan x} \frac{1}{t}(1 + o(1))\, dt =$$

$$= \int_x^{2x+o(x)} \frac{1}{t}\, dt + \int_x^{2x+o(x)} \frac{o(1)}{t}\, dt$$

$$= \lg \frac{2x+o(x)}{x} + \int_x^{2x+o(x)} \frac{o(1)}{t}\, dt$$

$$\lim_{x\to 0^+} \lg \frac{2x+o(x)}{x} = \lim_{x\to 0^+} \lg(2 + o(1)) = \lg 2$$

$$\int_x^{2x+o(x)} \frac{o(1)}{t}\, dt = (x + o(x)) \left.\frac{o(1)}{t}\right|_{c_x}$$

$f \in C^0([a,b])$ allora $\int_a^b f(x)\,dx = (b-a) f(c)$ per un opportuno $c \in [a,b]$

$x \le c_x \le 2x + o(x)$

$$\left| (x + o(x)) \frac{o(1)(c_x)}{c_x} \right| = \frac{x}{c_x} \underbrace{(1 + o(1))}_{1} |o(1)(c_x)| \le$$

$$\le \underbrace{(1 + o(1))}_{1} |o(1)(c_x)| \xrightarrow{x\to 0^+} 0$$

$$\lim_{x\to 0} c_x = 0 \implies \lim_{x\to 0^+} |o(1)(c_x)| = \lim_{c\to 0^+} |o(1)(c)| = 0$$

$$\lim_{x\to 0^+} f(x) = \lg 2 \ne f(0) = 0$$

$$f(x) = \begin{cases} \int_x^{x+\arctan x} \frac{1}{g(t)}\, dt & \text{se } x > 0 \\ \int_0^x \frac{t}{(t-1)(t-2)^2}\, dt & x \le 0 \end{cases}$$

Se $x < 0$ per il teorema fondamentale del calcolo

$f'(x) = \frac{x}{(x-1)(x-2)^2}$

$f'_s(0) = 0$

Per $x > 0$

$$f'(x) = \left(\int_x^{x+\arctan x} \frac{1}{g(t)}\, dt \right)' =$$

$$= (x + \arctan x)' \frac{1}{g(x + \arctan x)} - \frac{1}{g(x)}$$

$$= \left(1 + \frac{1}{1+x^2}\right) \frac{1}{g(x + \arctan x)} - \frac{1}{g(x)}$$

$$\boxed{F(x) = \int_{x_0}^{x} f(t)\,dt} \quad \text{e se} \quad f \text{ è continua in } x$$

$$\Rightarrow F'(x) = f(x)$$

$$G(x) = \int_{x}^{x+\arctan x} f(t)\,dt = \int_{x}^{x_0} f(t)\,dt + \int_{x_0}^{x+\arctan x} f(t)\,dt$$

$$= \int_{x_0}^{x+\arctan x} f(t) - \int_{x_0}^{x} f(t)\,dt = F(x+\arctan x) - F(x)$$

$$G'(x) = \left(F(x+\arctan x) - F(x)\right)' =$$

$$= F'(x+\arctan x)\,(x+\arctan x)' - F'(x)$$

$$= f(x+\arctan x)\left(1 + \frac{1}{1+x^2}\right) - f(x)$$

11 febbraio 2019 $\qquad a>0$

$$\lim_{x\to+\infty}\frac{\lg\left(\lg(6+2e^{x^a}+2x^5+x^a)\right)+5x^{-a}}{\arctan x+\int_1^x \operatorname{th}\left(\frac{\pi}{[t]}\right)dt}=L_a$$

Vicino a 0 $\qquad \operatorname{th} y = y(1+o(1)) = y+o(y^2)$

$$\operatorname{th}\left(\frac{\pi}{[t]}\right)=\frac{\pi}{[t]}+\left(\frac{\pi}{[t]}\right)^2 o(1) = \frac{\pi}{[t]}+\frac{1}{t^2}\underbrace{\left[\left(\pi\frac{t}{[t]}\right)^2 o(1)\right]}_{o(1)} \in L[1,+\infty)$$

$$\Rightarrow \lim_{x\to+\infty}\int_1^x \frac{1}{t^2}\,o(1)\,dt \in \mathbb{R}$$

$$\int_1^x \frac{\pi}{[t]}dt = \pi\int_1^x \frac{1}{t}dt + \pi\int_1^x\left(\frac{1}{[t]}-\frac{1}{t}\right)dt$$

$$=\pi\lg x + \pi\int_1^x\left(\frac{1}{[t]}-\frac{1}{t}\right)dt$$

$$\lim_{t\to+\infty}\frac{t}{[t]}=1$$

$$0\le \frac{1}{[t]}-\frac{1}{t} = \frac{1}{[t]\,t}(t-[t]) \le \frac{1}{[t]\,t} \in L[1,+\infty)$$

$$\Rightarrow \frac{1}{[t]}-\frac{1}{t}\in L[1,+\infty) \Rightarrow \lim_{x\to+\infty}\int_1^x\left(\frac{1}{[t]}-\frac{1}{t}\right)dt\in\mathbb{R}$$

Ricapitolando

$$\text{denominatore} = \int_1^x \operatorname{th}\left(\frac{\pi}{[t]}dt\right)+\arctan x =$$

$$=\pi\lg x + o(\lg x) = \pi\lg x\,(1+o(1))$$

$$\text{num} = \lg\left(\lg(6+2e^{x^a}+2x^5+x^a)\right)+5x^{-a}$$

$$=\lg\left[\lg 2e^{x^a}\left(1+\frac{6}{2}e^{-x^a}+x^5e^{-x^a}+\frac{1}{2}x^a e^{-x^a}\right)\right]+5x^{-a}$$

$$=\lg\left[x^a+\lg 2+\lg\left(1+3x^{-a}+x^5e^{-x^a}+\frac{1}{2}x^a e^{-x^a}\right)\right]+5x^{-a}$$

$$=\lg\left[x^a\left(1+\underbrace{\frac{\lg 2}{x^a}+\frac{1}{x^a}\lg\left(1+3x^{-a}+x^5e^{-x^a}+\frac{1}{2}x^a e^{-x^a}\right)}_{o(1)}\right)\right]+5x^{-a}$$

$$=a\lg x+\lg(1+o(1))+5x^{-a} = a\lg x+o(1) = \text{num}$$

$$\frac{a\lg(x)+o(1)}{\pi\lg(x)(1+o(1))}=\frac{a\,\cancel{\lg(x)}\,(1+o(1))}{\pi\,\cancel{\lg(x)}\,(1+o(1))}\xrightarrow{x\to+\infty}\frac{a}{\pi}$$

$P_6(x) = ?$ $\qquad f(x) = \int_2^x \frac{1}{1+t+t^3}\,dt$

$$P_6(x) = \sum_{j=0}^{6} \frac{f^{(j)}(0)}{j!} x^j$$

$$f(x) = \underbrace{\int_2^0 \frac{1}{1+t+t^3}\,dt}_{f(0)} + \int_0^x \frac{1}{1+t+t^3}\,dt$$

$$\frac{1}{1+y} = \sum_{j=0}^{6} (-1)^j y^j + o(y^6)$$

$$\frac{1}{1+t+t^3} = 1 - (t+t^3) + (t+t^3)^2 - (t+t^3)^3 + (t+t^3)^4 - (t+t^3)^5 + o(t^5)$$

qui $o((t+t^3)^5) = o(t^5)$

$$= 1 - t - t^3 + t^2 + 2t^4 - (t^3 + 3t^5) + t^4 - t^5 + o(t^5)$$

$$= 1 - t + t^2 - 2t^3 + 3t^4 - 4t^5 + o(t^5)$$

$$\int_0^x \frac{1}{1+t+t^3}\,dt = \int_0^x \left(1 - t + t^2 - 2t^3 + 3t^4 - 4t^5 + o(t^5)\right) dt$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{2} + \frac{3}{5}x^5 - \frac{2}{3}x^6 + o(x^6)$$

$$\lim_{x \to 0^+} \frac{\lg(1 + x^a + x^{2a}) - \operatorname{th} x}{\int_0^{x^2} \sin\left(\frac{1}{t}\right) dt + 1 - \cos x}$$

denom $= F(x^2) + 1 - \cos x$ $\qquad F(x) = \int_0^x \sin\left(\frac{1}{t}\right) dt$

$$\cos x = 1 - \frac{x^2}{2} + o(x^3)$$

$$1 - \cos x = \frac{x^2}{2} + o(x^3)$$

$F(0) = 0$, $\quad F'(0) = 0$ $\qquad F'(x) = \sin\left(\frac{1}{x}\right)$ se $x \neq 0$

$F(y) = y + o(y)$ $\qquad F(x^2) = x^2 + o(x^2)$

denom $= F(x^2) + 1 - \cos x = x^2 + o(x^2) + \frac{x^2}{2} + o(x^3) = \frac{3}{2}x^2 + o(x^2)$

$$= \frac{3}{2}x^2 (1 + o(1))$$