Esame 18/06/2018

i) $f(x) = \dfrac{x}{(x-1)^2(x+1)} = \quad p(x) \prec q(x)$

$= \dfrac{x}{(x-1)(x-1)(x+1)} =$

$\Rightarrow \dfrac{A}{x+1} + \dfrac{B}{x-1} + \underbrace{\dfrac{d}{dx} \cdot \dfrac{C}{x-1}}_{\frac{-C}{(x-1)^2}} =$

$= \dfrac{A(x-1)^2 + B(x^2-1) - C(x+1)}{(x-1)^2(x+1)} =$

$= \dfrac{Ax^2 - 2Ax + A + Bx^2 - B - Cx - C}{(x-1)^2(x+1)} =$

$\Rightarrow \begin{cases} A+B=0 \\ -2A-C=1 \\ A-B-C=0 \end{cases} \Rightarrow \begin{cases} B=-A \\ \\ C=A-B \end{cases} \Rightarrow \begin{cases} B=-A \\ -2A-2A=1 \\ C=2A \end{cases} \Rightarrow \begin{matrix} B=1/4 \\ A=-1/4 \\ C=-1/2 \end{matrix}$

$\dfrac{-1/4}{x+1} + \dfrac{1/4}{x-1} + \dfrac{d}{dx} \dfrac{-1/2}{x-1}$

$$\int_2^3 \frac{x}{(x-1)^2(x+1)}\,dx =$$

$$= \int_2^3 \left[\frac{1}{4}\left(\frac{1}{x-1} - \frac{1}{x+1}\right) - \frac{1}{2}\,\frac{d}{dx}\left(\frac{1}{x-1}\right)\right] dx =$$

$$= \frac{1}{4}\int_2^3 \frac{1}{x-1}\,dx - \frac{1}{4}\int_2^3 \frac{1}{x+1}\,dx - \frac{1}{2}\int_2^3 \left(\frac{d}{dx}\,\frac{1}{x-1}\right) \cdot dx =$$

$$= \frac{1}{4}\Big[\log(x-1)\Big]_2^3 - \frac{1}{4}\Big[\log(x+1)\Big]_2^3 - \frac{1}{2}\left[\frac{1}{x-1}\right]_2^3 =$$

$$= \frac{1}{4}\left(\log(2) - \cancel{\log(1)}\right) - \frac{1}{4}\log(4) + \frac{1}{4}\log(3) - \frac{1}{4} + \frac{1}{2} =$$

$$= \frac{1}{4}\left(\log(2) - \log(4) + \log(3) +\right.$$

$$+ \frac{1}{4} =$$

$$= \frac{1}{4}\left[\log\left(\frac{3}{2}\right) + 1\right]$$

$$\int_2^3 \frac{\log(x^2-1)}{(x-1)^2}\,dx = \left[\log(x^2-1)\left(\frac{-1}{x-1}\right)\right]_2^3 - \int_2^3 -\frac{1}{x-1}\cdot\frac{1}{x^2-1}\cdot 2x\,dx =$$

$\log(x^2-1) \to g(x)$

$(x-1)^2 \to f'(x)$

$$= \left[-\log(x^2-1)\cdot\frac{1}{x-1}\right]_2^3 + 2\int_2^3 \frac{x}{\underbrace{(x-1)(x^2-1)}_{(x-1)(x+1)(x-1)=(x-1)^2(x+1)}}\,dx =$$

$$= -\frac{\log(8)}{2} + \frac{\log(3)}{1} + \cancel{2}\left(\frac{1}{\cancel{4}_{2}}\log\left(\frac{3}{2}\right) + \frac{1}{\cancel{4}_{2}}\right) =$$

$$= \frac{-\log(8) + \log(3)^2}{2} + \frac{1}{2}\log\left(\frac{3}{2}\right) + \frac{1}{2} =$$

$$= \frac{-\log(8) + \log(9) + \log\left(\frac{3}{2}\right) + 1}{2} = \frac{1}{2}\left(-\log(8) + \log(9) + \log\left(\frac{3}{2}\right) + 1\right) =$$

$$= \frac{1}{2}\left[\log\left(\frac{9}{8}\right) + \log\left(\frac{3}{2}\right) + 1\right] = \frac{1}{2}\left[\log\left(\frac{27}{16}\right) + 1\right]$$

Es 1

$$f(x) = \begin{cases} x \neq 0 & \dfrac{\int_{-x^2}^{x^2} \cos(t^2)\, dt}{2^{x^2}-1} \\ x = 0 & a \end{cases}$$

$$\lim_{x\to 0} \frac{2\int_0^{x^2} \cos(t^2)\,dt}{x^2} \cdot \frac{x^2}{2^{x^2}-1} \to \frac{1}{\ln 2}$$

$$\lim_{x\to 0} \frac{2\cos(x^4)\cancel{2x}}{\cancel{2x}} \cdot \frac{1}{\ln 2} = \frac{2}{\ln 2}$$

$$\lim_{x\to +\infty} \frac{\int_{-x^2}^{x^2} \cos(t^2)\,dt}{2^{x^2}-1} \leq \lim_{x\to +\infty} \frac{2x^2}{2^{x^2}-1} = \emptyset$$

$$\int_{-x^2}^{x^2} 1\, dt = [t]_{-x^2}^{x^2} = 2x^2$$

$$-1 \leq \cos t^2 \leq 1$$

$$-2x^2 \leq \int_{-x^2}^{x^2} \cos(t^2)\,dt \leq 2x^2$$

$$\lim_{x\to +\infty} \frac{2\int_0^{x^2} \cos(t^2)\,dt}{2^{x^2}-1} \leq \lim_{x\to +\infty} \frac{2x^2}{2^{x^2}-1}$$

$$\int_0^{x^2} \cos(t^2)\, dt = \cos(y^2)\cdot x^2 \leq 1 \cdot x^2$$

$$0 \leq y \leq x^2$$

Siccome $f(x)$ pari, considero $I: [0, +\infty[$

$\lim_{x\to +\infty} f(x) = \emptyset$ $\quad \varepsilon > 0 \quad \exists K > 0 : \forall x \in I$ e $x > K \Rightarrow f(x) < \varepsilon$

$[-K; K]$ per W. $\exists$ max

Esame 22/01/18

$$f(x) = x^3 e^{-|x|}$$

I) simmetrie $\longrightarrow$ dispari

$$f(-x) = (-x)^3 e^{-|-x|} = -x^3 e^{-|x|} = -f(x)$$

$$\lim_{x \to +\infty} x^3 e^{-|x|} = \lim_{x \to +\infty} \frac{x^3}{e^x} = 0$$

II) $x^3 e^{-x} = 0 \iff x = 0$ $\qquad f(0) = 0$

$f(x) > 0$ se $x > 0$

$$f'(x) = 3x^2 e^{-x} - x^3 e^{-x} \qquad x > 0$$

$$f'(0) = 0$$

$$f''(x) = e^{-x}(3-x)\,2x - e^{-x}x^2 - e^{-x}(3-x)x^2 =$$

$$= e^{-x}(x^3 - 6x^2 + 6x)$$

IV)
$$f'''(x) = \begin{cases} e^{-x}(-x^3 + 9x^2 - 18x + 6) & x > 0 \\ e^{x}(x^3 + 9x^2 + 18x + 6) & x < 0 \end{cases}$$

$$f'''(0) = 6$$

$$\left.\begin{aligned} f^{IV}_{+}(0) &= -18 - 6 = -24 \\ f^{IV}_{-}(0) &= +18 + 6 = +24 \end{aligned}\right\} \text{Sono diversi} \Rightarrow n = 3$$

$$f \in C^3(\mathbb{R})$$

$$P_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f'''(1)}{6}(x-1)^3 =$$

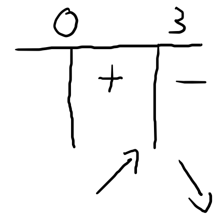
$$f(1) = e^{-1}$$

$$f'(1) = 2e^{-1} \qquad = e^{-1}\left(1 + 2(x-1) + \frac{1}{2}(x-1)^2 - \frac{2}{3}(x-1)^3\right)$$

$$f''(1) = e^{-1}$$

$$f'''(1) = 4e^{-1}$$

VI) $e^{-x}x^2(3-x)>0$



per simmetria

Punti critici $3,\ 0,\ -3$

VII) $f'(x)>0 \quad]-3,0[\ \cup\ ]0,3[$

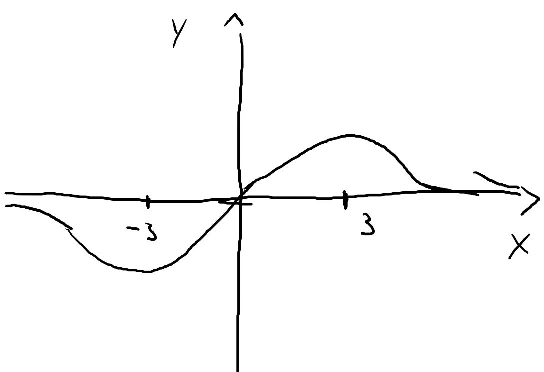
$f'(x)<0$ se $x\in\ ]-\infty;-3[\ \cup\ ]3;+\infty[$

f crescente in $[-3;3]$

f decrescente in $]-\infty,-3]$ e in $[3,+\infty[$.

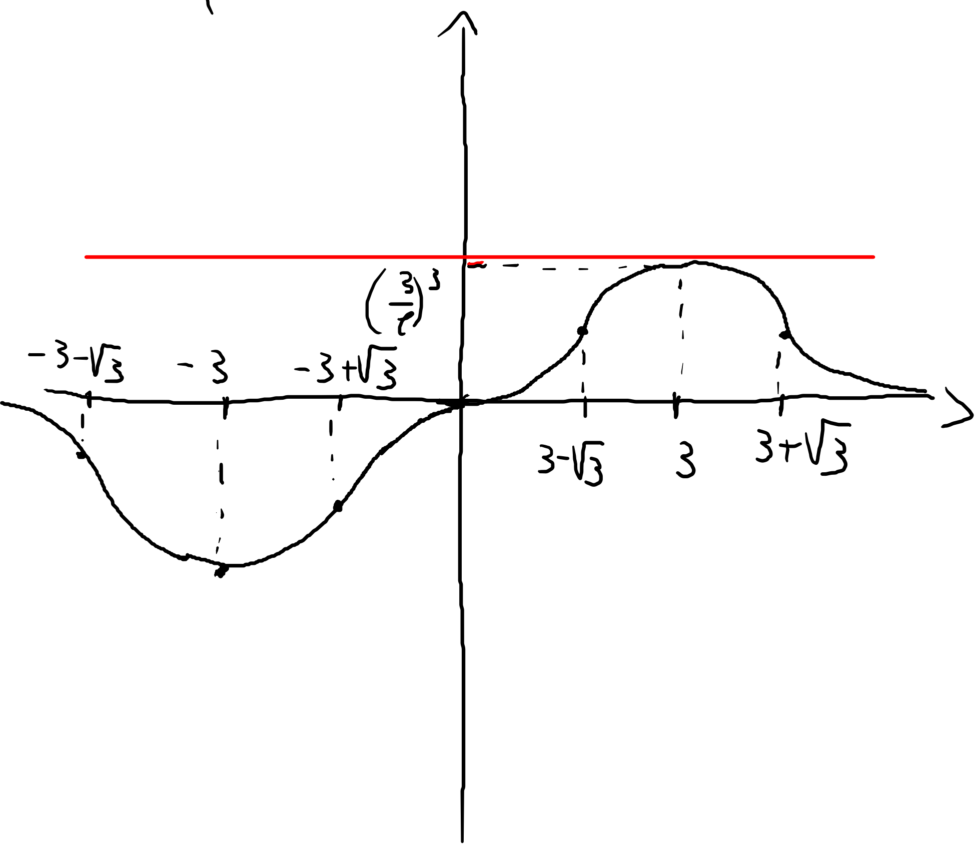
minimo assoluto $\quad f(-3) = -\left(\frac{3}{e}\right)^3$

massimo assoluto $\quad f(3) = \left(\frac{3}{e}\right)^3$



VIII) $\quad x e^{-x}(x^2 - 6x + 6) > 0$

zeri f'' $\qquad -3 \pm \sqrt{3}, \quad 0, \quad 3 \pm \sqrt{3}$



X1)

$$g(x) = \sqrt{n - f(x)} \longrightarrow n - f(x) \geq 0$$

$$n - x^3 e^{-|x|} \geq 0$$

$$n \geq x^3 e^{-|x|} \quad \forall x$$

$$n \geq \max f \rightarrow \quad n \geq \left(\frac{3}{e}\right)^3 \quad ; \ n \in \mathbb{N}$$

$$2 \cdot e^3 > 2 \cdot \left(2 + \frac{1}{2}\right)^3 = 2 \cdot \left(8 + 6 + \frac{3}{2} + \frac{1}{8}\right) > 30 > 3^3$$

$$2 > \left(\frac{3}{e}\right)^3 \qquad 1 < \left(\frac{3}{e}\right)^3 < 2$$

$$n = 2$$

X11)

$$\int_0^{+\infty} x^3 e^{-x}\,dx = \lim_{b \to +\infty} \int_0^b x^3 e^{-x}\,dx =$$

$$= \lim_{b \to +\infty} \left[-e^{-x} x^3\right]_0^b + \int_0^b 3x^2 e^{-x}\,dx =$$

$$= \lim_{b \to +\infty} \left[-e^{-x} x^3\right]_0^b + \left[-3x^2 e^{-x}\right]_0^b + \left[-6x e^{-x}\right]_0^b + \left[-6e^{-x}\right]_0^b =$$

$$= +6 \cdot e^0 = +6$$

$$f(x) = \frac{\left((1+x)^{\alpha} - \log_{\alpha}(1+x) - 1\right.}{x^2}$$

i) $\alpha_0 \cdot \log(\alpha_0) = 1$

$\alpha_0 = 1$ $\qquad g(\alpha) = \alpha \cdot \log \alpha \qquad g(1) = 0$

$g(2) = 2 \log 2 > 1$

$[1,2]$ il teorema di connessione $\exists\, \alpha_0 \in [1,2]$ t.c. $g(\alpha_0) = 1$

$g(\alpha) < 0$ se $\alpha \in]0,1[$ $\quad g'(\alpha) = \log \alpha + 1 > 0$ se $\alpha > 1$