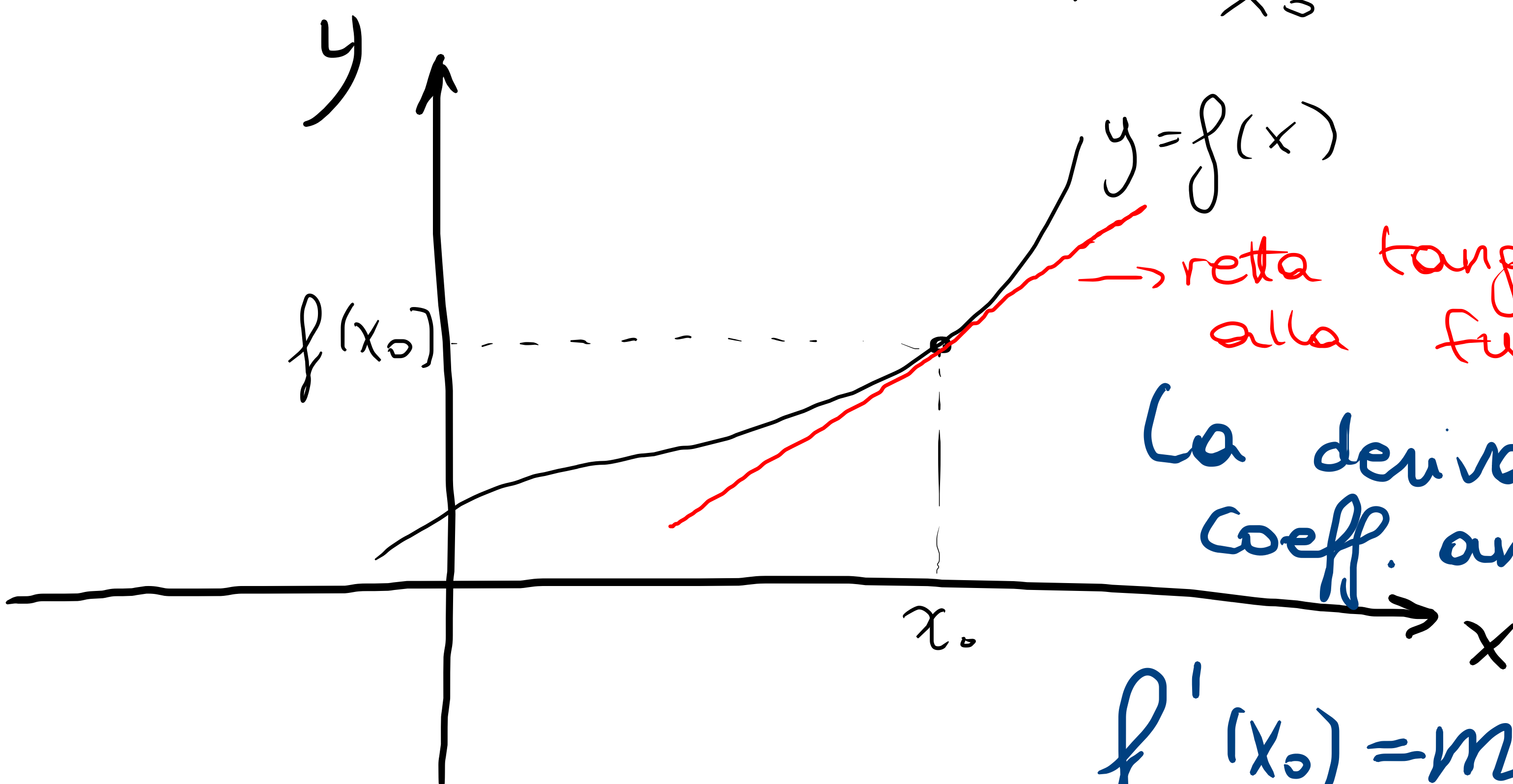
DERIVATE

Definizione

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

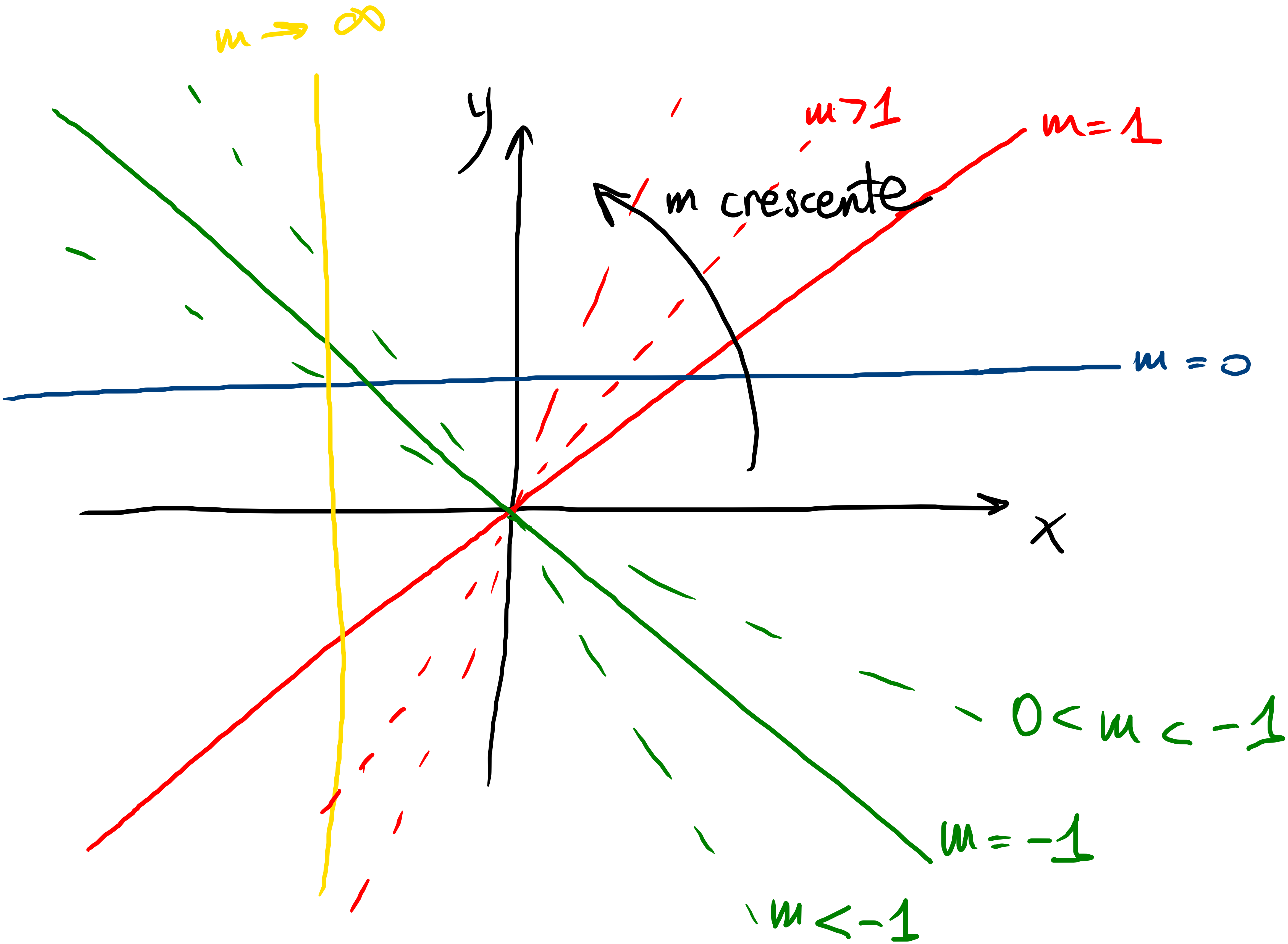
$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$



→ retta tangente in $x = x_0$ alla funzione $(y = mx + q)$

La derivata prima è il coeff. angolare della retta tangente

$f'(x_0) = m$



m → ∞
y
m crescente
m>1
m=1
m = 0
x
0<m<-1
m=-1
m<-1

1) $$\boxed{f(x) = a x^{n} \Longrightarrow f'(x) = n \cdot a\, x^{n-1} \qquad n \neq 0}$$

Se $n = 0$ $\quad f(x) = a \Longrightarrow f'(x) = 0$

$$f(x) = \frac{1}{x} = x^{-1} \Longrightarrow f'(x) = (-1) \cdot x^{-2} = -\frac{1}{x^2}$$

$$f(x) = \frac{2}{x^3} = 2x^{-3} \Longrightarrow f'(x) = 2 \cdot (-3) \cdot x^{-4} = -\frac{6}{x^4}$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \Longrightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \Longrightarrow f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

2) $f(x) = e^x \implies f'(x) = e^x$

3) $f(x) = \ln x \implies f'(x) = \frac{1}{x}$

4) $f(x) = \sin x \implies f'(x) = \cos x$

5) $f(x) = \cos x \implies f'(x) = -\sin x$

Proprietà delle derivate

- $(f+g)' = f' + g'$
- $(f \cdot g)' \neq f' \cdot g' \qquad (f \cdot g)' = f'g + f \cdot g'$
- $\left(\frac{f}{g}\right) = \frac{f'g - f \cdot g'}{g^2}$

$$f(x) = \overbrace{x}^{f(x)} \cdot \overbrace{e^x}^{g(x)} + 3$$

$$f'(x) = \underbrace{1}_{f'(x)} \cdot \underbrace{e^x}_{g(x)} + \underbrace{x}_{f(x)} \underbrace{e^x}_{g'(x)} = e^x(x+1)$$

$$f(x) = \frac{\overbrace{5x+4}^{f(x)}}{\underbrace{x^2+3x+2}_{g(x)}}$$

$$f'(x) = \frac{\overbrace{5}^{f'(x)} \cdot (x^2+3x+2) - (5x+4) \cdot \overbrace{(2x+3)}^{g'(x)}}{(x^2+3x+2)^2} =$$

$$= \frac{5x^2 + \cancel{15x} + 10 - 10x^2 - \cancel{15x} - 8x - 12}{(x^2+3x+2)^2} = -\frac{5x^2+8x+2}{(x^2+3x+2)^2}$$

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x$$

$$= \frac{1}{\cos^2 x}$$

Derivata della funzione composta

$$
\begin{tikzcd}
x \arrow[r, bend left, "y=f(x)"] \arrow[rr, bend right, "(g\circ f)(x)"'] & y \arrow[r, bend left, "z=g(y)"] & z
\end{tikzcd}
$$

$$g(f(x))$$

$$z = \cos(5x^3)$$

$$g(y) = \cos(y)$$

$$f(x) = 5x^3$$

$$(g\circ f)'(x) = g'(f(x)) \cdot f'(x)$$

derivo l'argomento del coseno

$$z' = \underbrace{-\sin(5x^3)}_{\text{Derivo il coseno senza modificare il suo argomento}} \cdot 5 \cdot 3x^2 = -15x^2 \sin(5x^3)$$

$$f(x) = e^{(x^2+5x)}$$

$$f'(x) = \underbrace{e^{x^2+5x}}_{\text{derivata dell'esponenziale}} \cdot \underbrace{(2x+5)}_{\text{derivata dell'argomento}}$$

$$f(x) = \tan(\sqrt{x})$$

derivata di $\sqrt{x}$

$$f'(x) = \frac{1}{\cos^2(\sqrt{x})} \cdot \frac{1}{2\sqrt{x}}$$

derivata della tangente

$$f(x) = \sqrt{\ln(x)+1} = (\ln x + 1)^{\frac{1}{2}}$$

$$f(x) = x^2 \Longrightarrow f'(x) = 2x$$

derivata argomento

$$f'(x) = \frac{1}{2\sqrt{\ln(x)+1}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln(x)+1}}$$

$$f(x) = 3\sin^2 x = 3[\sin x]^2$$

derivata argomento

$$f'(x) = 3 \cdot \underbrace{2 \cdot \sin x}_{\text{derivata della potenza}} \cdot \cos x$$

$$= 6 \sin x \cdot \cos x$$

$$\ln^2 x = [\ln x]^2$$

$$\ln x^2 = \ln(x^2)$$

$$\sin^2 x = [\sin x]^2$$

$$\sin x^2 = \sin(x^2)$$

$$f(x) = 3\sin(x^2)$$

deriv l'argomento

$$f'(x) = 3\cos(x^2)\cdot 2x = 6x\cos(x^2)$$

$$f(x) = [\log(\sin x)]^4$$

deriv l'argomento

$$f'(x) = 4\,[\log(\sin x)]^3 \cdot \frac{1}{\sin(x)} \cdot \cos(x)$$

$$= \frac{4\log^3(\sin x)}{\tan x}$$

$$\frac{\sin x}{\cos x} = \tan x \qquad \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$