$$f(x) = (1+x^3)^5$$

$$f'(x) = 5(1+x^3)^4 \cdot \underbrace{3x^2}_{\text{derivate di } (1+x^3)} = 15x^2(1+x^3)^4$$

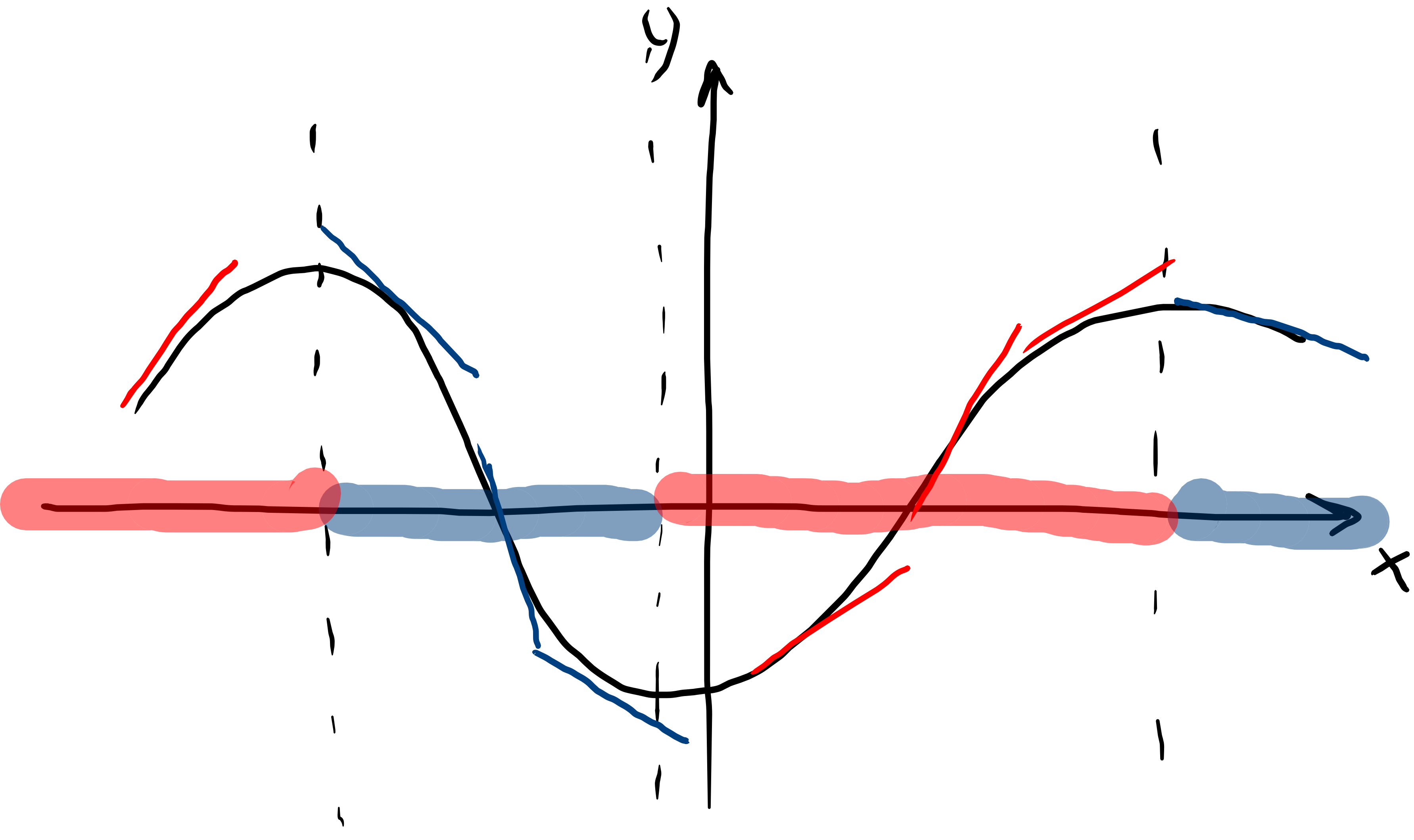
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$f(x) = \frac{1+\cos x}{x - \sin x}$$

$$f'(x) = \frac{-\sin x(x - \sin x) - (1-\cos x)(1+\cos x)}{(x-\sin x)^2}$$

$$= \frac{-x\sin x + \cancel{\sin^2 x} - \overbrace{(\cancel{1-\cos^2 x})}^{\sin^2 x}}{x^2 - 2x\sin x + \sin^2 x} = \frac{-x\sin x}{x^2 - 2x\sin x + \sin^2 x}$$

STUDIO della DERIVATA PRIMA



FUNZIONE CRESCENTE → $f'(x) > 0$

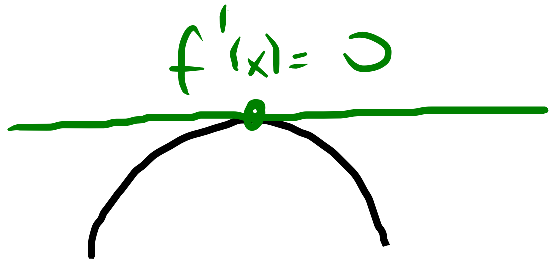
rette tg con $m > 0$

FUNZIONE DECRESCENTE → $f'(x) < 0$

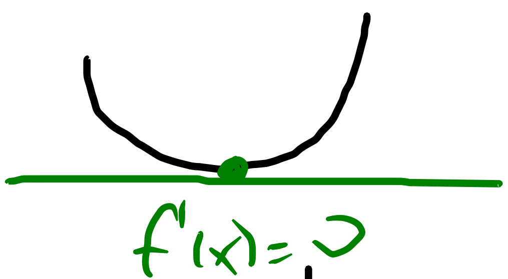
rette tg con $m < 0$

Se $f'(x)=0$ ho un PUNTO CRITICO

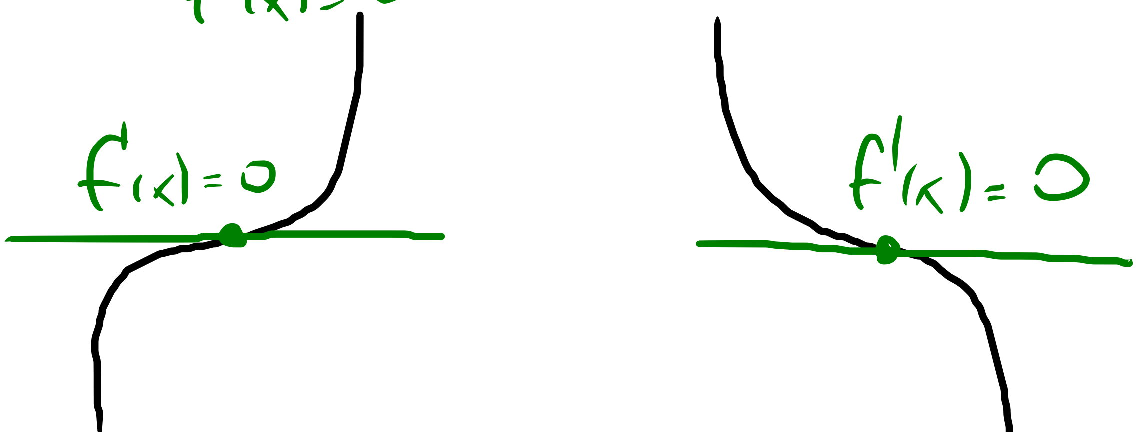
3 casi:

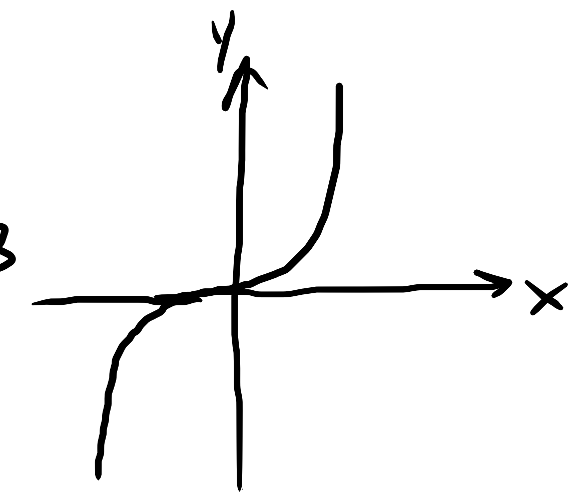
① 

MASSIMO RELATIVO

② 

MINIMO RELATIVO

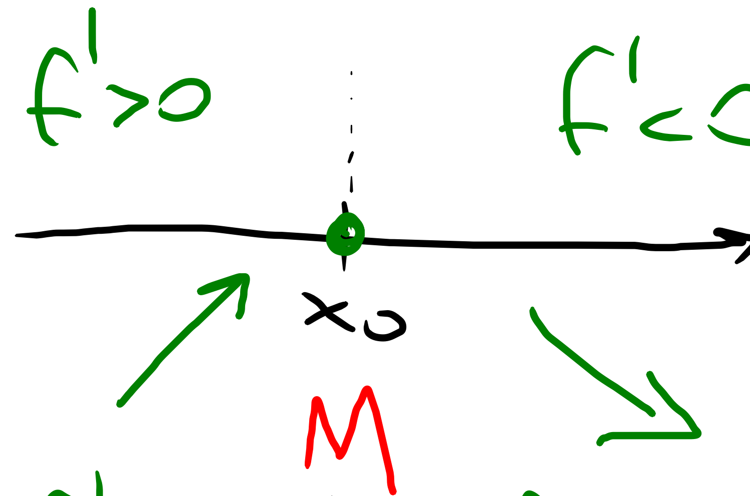
③ 

$f(x)=x^3$ 

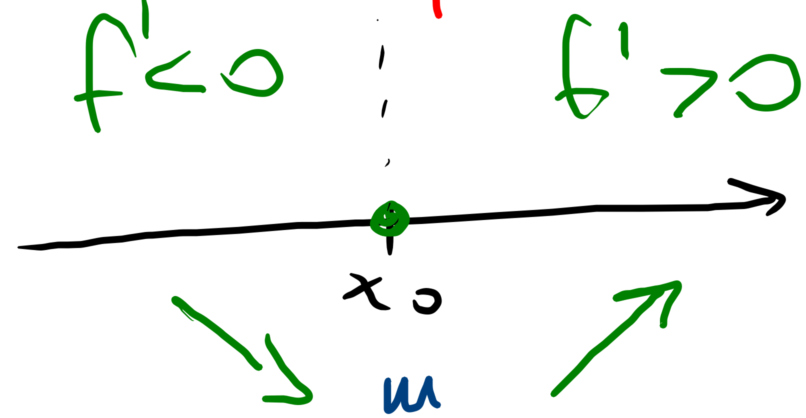
PUNTO DI FLESSO A TANGENTE ORIZZONTALE

Per classificare i punti studio come varia il segno di $f'(x)$ intorno al punto critico x_0

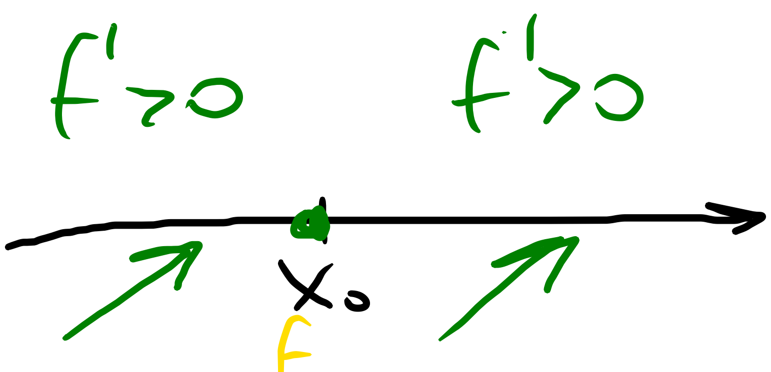
① MASSIMO RELATIVO

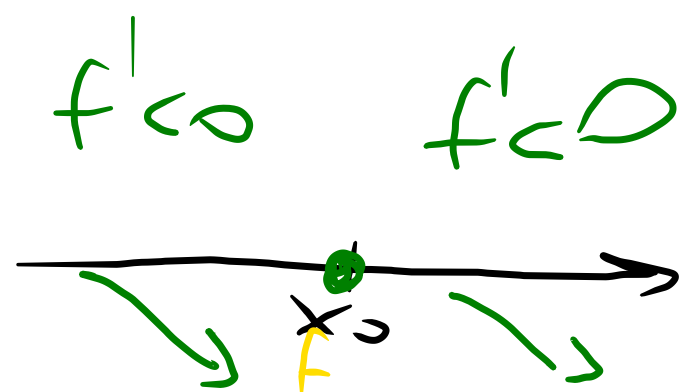


② MINIMO RELATIVO



③ PUNTO DI FLESSO A TG ORIZZONTALE





$$f(x) = x(x^2-1)^2$$

① Dominio: $D_f = \mathbb{R}$

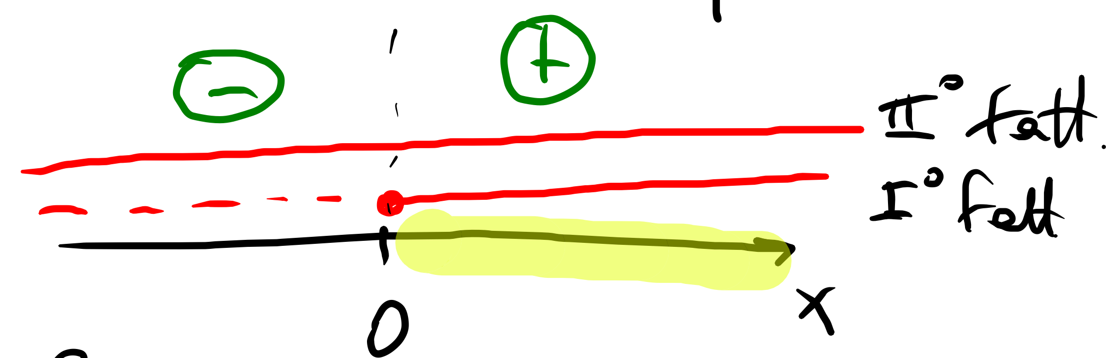
② Positività $\Longrightarrow$ vedere dove la funzione è positiva

Quindi risolvo $f(x) \geq 0$

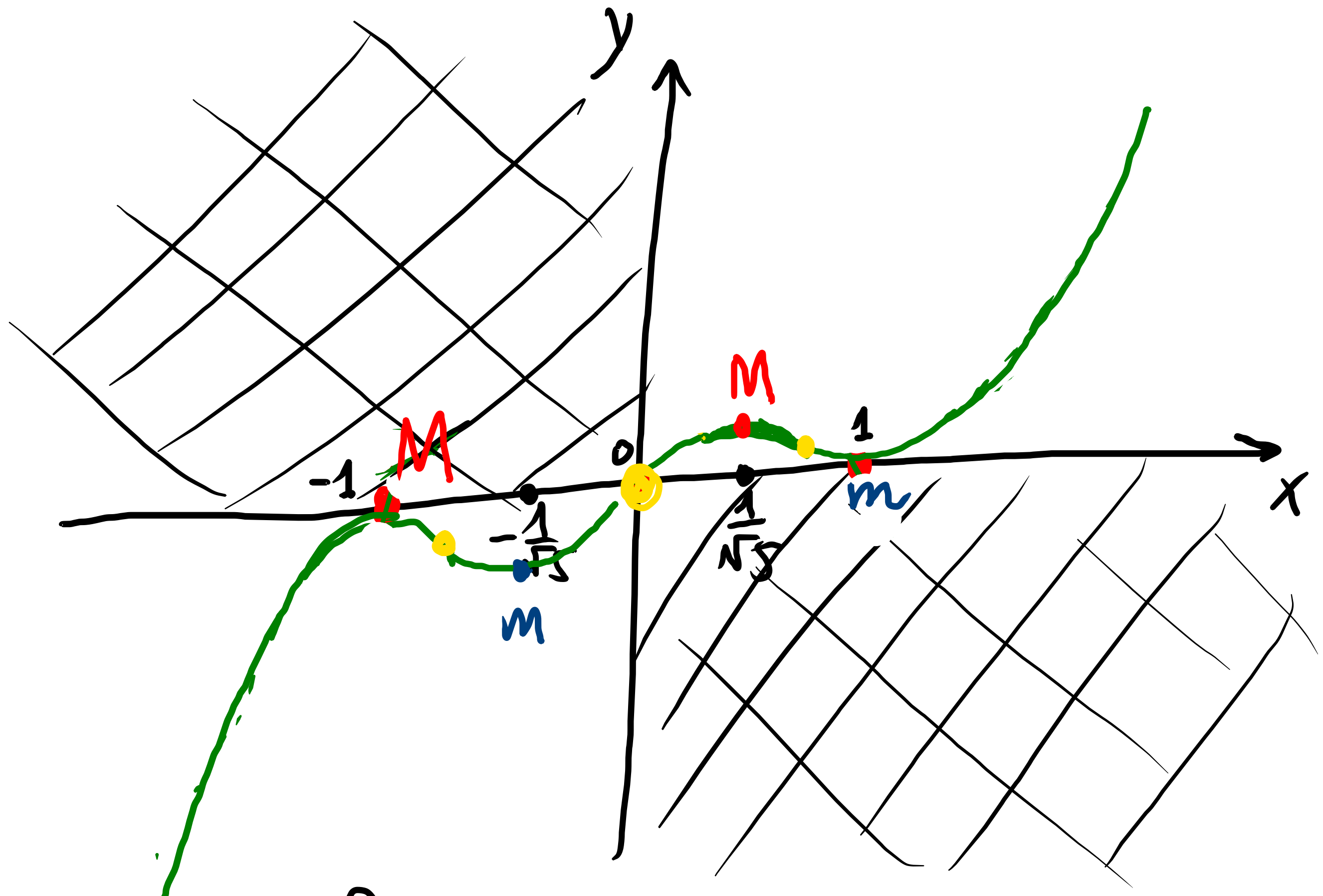
$$x(x^2-1)^2 \geq 0$$

I° fattore $\quad x \geq 0$

II° fattore $\quad (x^2-1)^2 \geq 0 \quad \underline{\forall x \in \mathbb{R}}$



$f(x) \geq 0$ se $x \geq 0$



③ Int. con gli assi:

– asse y: $\begin{cases} y = f(x) \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = f(0) = 0 \\ x = 0 \end{cases}$

– asse x $\begin{cases} y = f(x) \\ y = 0 \end{cases} \Rightarrow \begin{cases} 0 = f(x) \\ y = 0 \end{cases}$

$x\underbrace{(x^2-1)}_{x^2=1}{}^2 = 0$ — $x = 0$, $x = \pm 1$

④ Studio derivata prima

$$f'(x) = 1 \cdot (x^2-1)^2 + 2x(x^2-1)\cdot 2x$$

$$\overset{!}{=} (x^2-1)\left[(x^2-1)+4x^2\right] =$$

$$= (x^2-1)(5x^2-1)$$

- $f'(x)=0 \iff (x^2-1)(5x^2-1)=0$
  - $x^2-1=0 \rightarrow x=\pm 1$
  - $5x^2-1=0 - x=\pm\frac{1}{\sqrt{5}}$

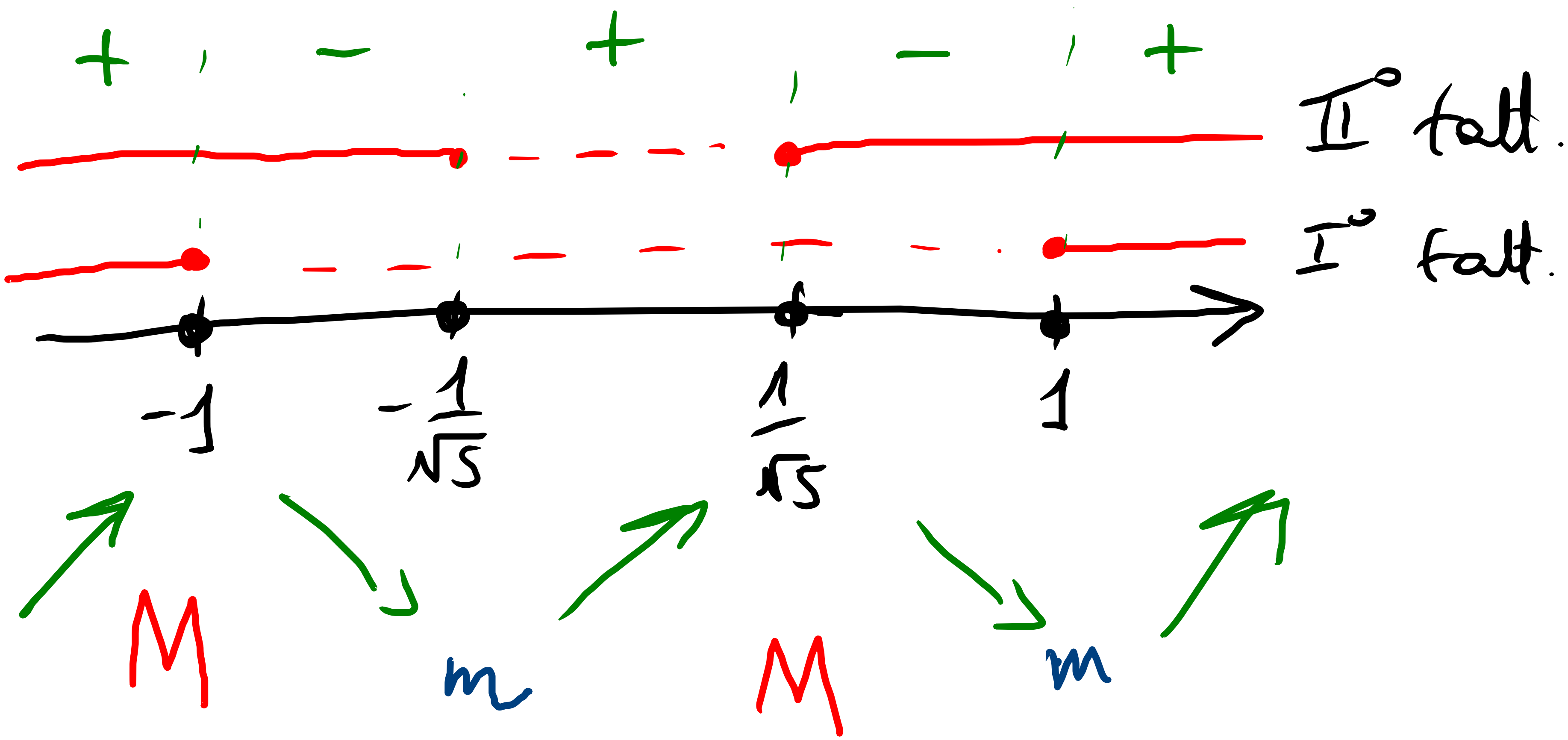
4 punti critici

$x_1 = 1 \quad x_2 = -1 \quad x_3 = \frac{1}{\sqrt{5}} \quad x_4 = -\frac{1}{\sqrt{5}}$

• $f'(x) \geq 0$ $\quad (x^2-1)(5x^2-1) \geq 0$

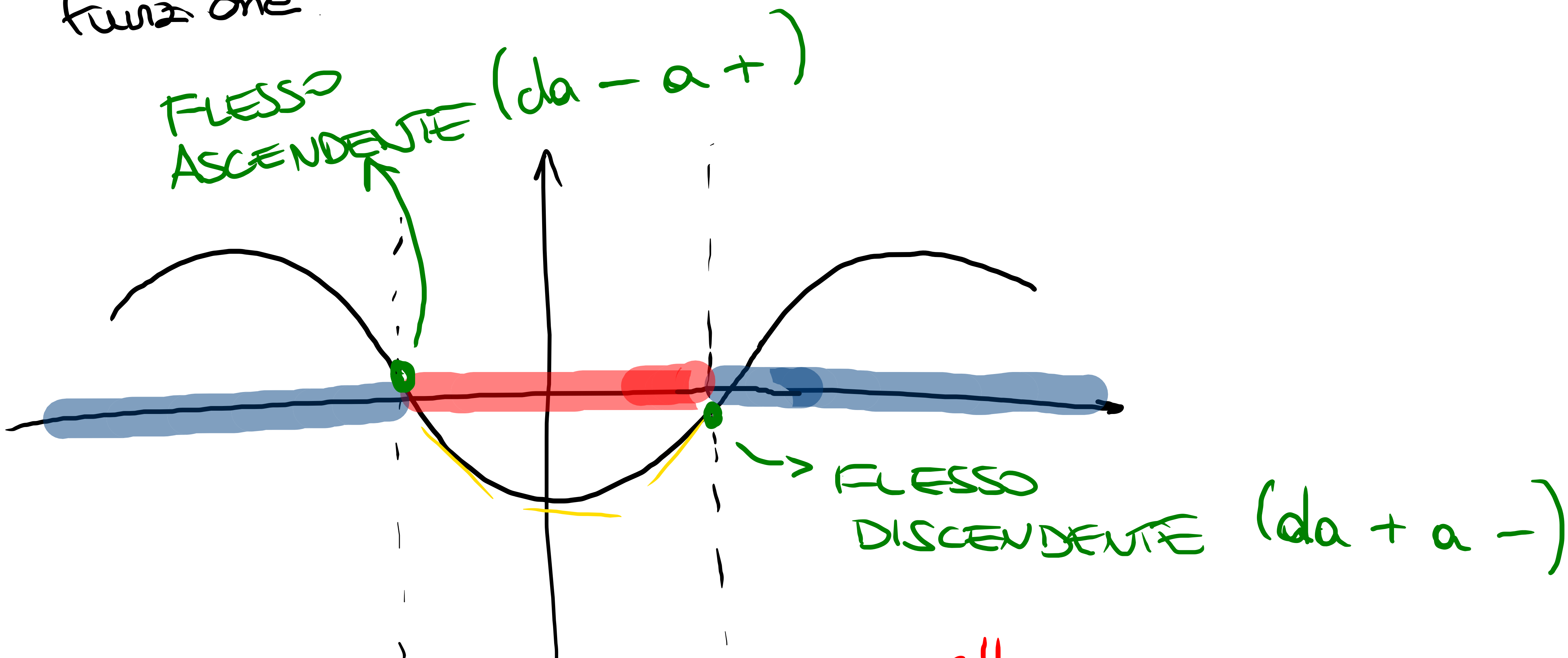
I° fatt. $\quad x^2-1 \geq 0 \quad$ eq. ass. $\quad x = \pm 1$

II° fatt. $\quad 5x^2-1 \geq 0 \quad$ eq. ass. $\quad x = \pm \frac{1}{\sqrt{5}}$



STUDIO della DERIVATA SECONDA

Necessario per determinare flessi e concavità della funzione



FUNZIONE CONVESSA $f''(x) > 0$

FUNZIONE CONCAVA $f''(x) < 0$

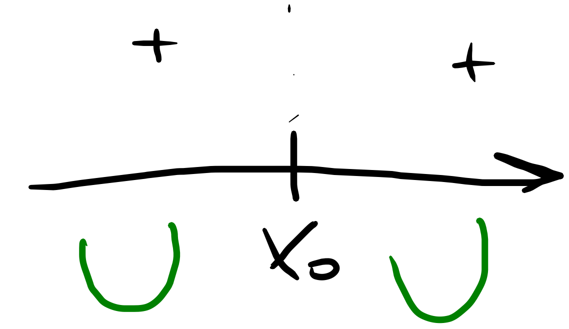
Punti di flesso: sono quelli dove cambia la concavità

Tornando all'esempio di prima...

$$f'(x) = (x^2-1)(5x^2-1)$$

$$f''(x) = 2x(5x^2-1) + (x^2-1)(10x)$$
$$= 10x^3 - 2x + 10x^3 - 10x$$
$$= 20x^3 - 12x = 4x(5x^2-3)$$

N.B. Anche se $f''(x_0)=0$



x_0 non è un flesso

• $f''(x)=0 \iff 4x(5x^2-3)=0$

$\left.\begin{array}{l} x=0 \\ x=\pm\sqrt{\frac{3}{5}} \end{array}\right\}$ 3 punti dove $f''(x)=0$

• $f''(x)\geq 0 \iff 4x(5x^2-3)\geq 0$

I° fatt. $4x \geq 0$ $\quad x\geq 0$

II° fatt. $5x^2-3\geq 0$ eq ass $x=\pm\sqrt{\frac{3}{5}}$