Studio FUNZIONE

$$f(x) = x^2 \left(\ln|x| - 1 \right)$$

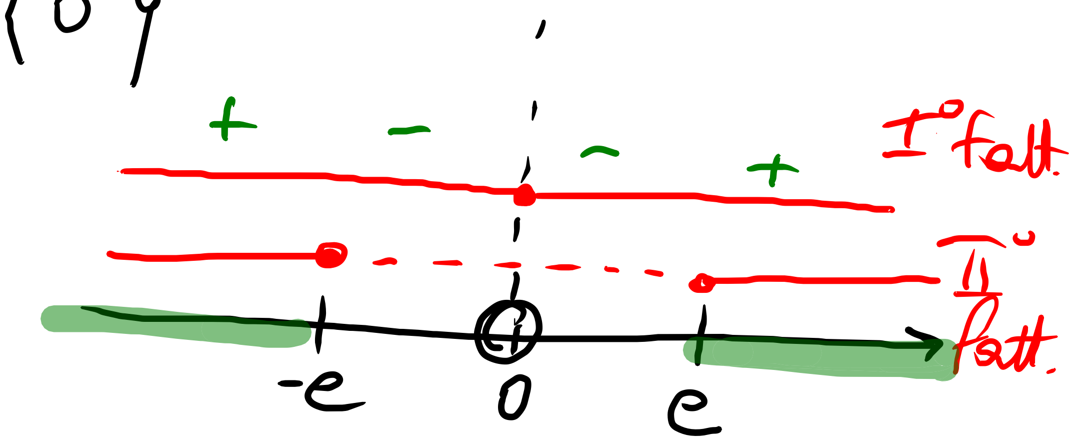
$$a = a \cdot \frac{b}{b} \qquad a = a + b - b$$

$$a = e^{\ln a}$$

Dominio: $|x| > 0 \rightarrow x \neq 0$

$D = \{ \forall x \in \mathbb{R}, x \neq 0 \}$ $D: \mathbb{R} \setminus \{0\}$

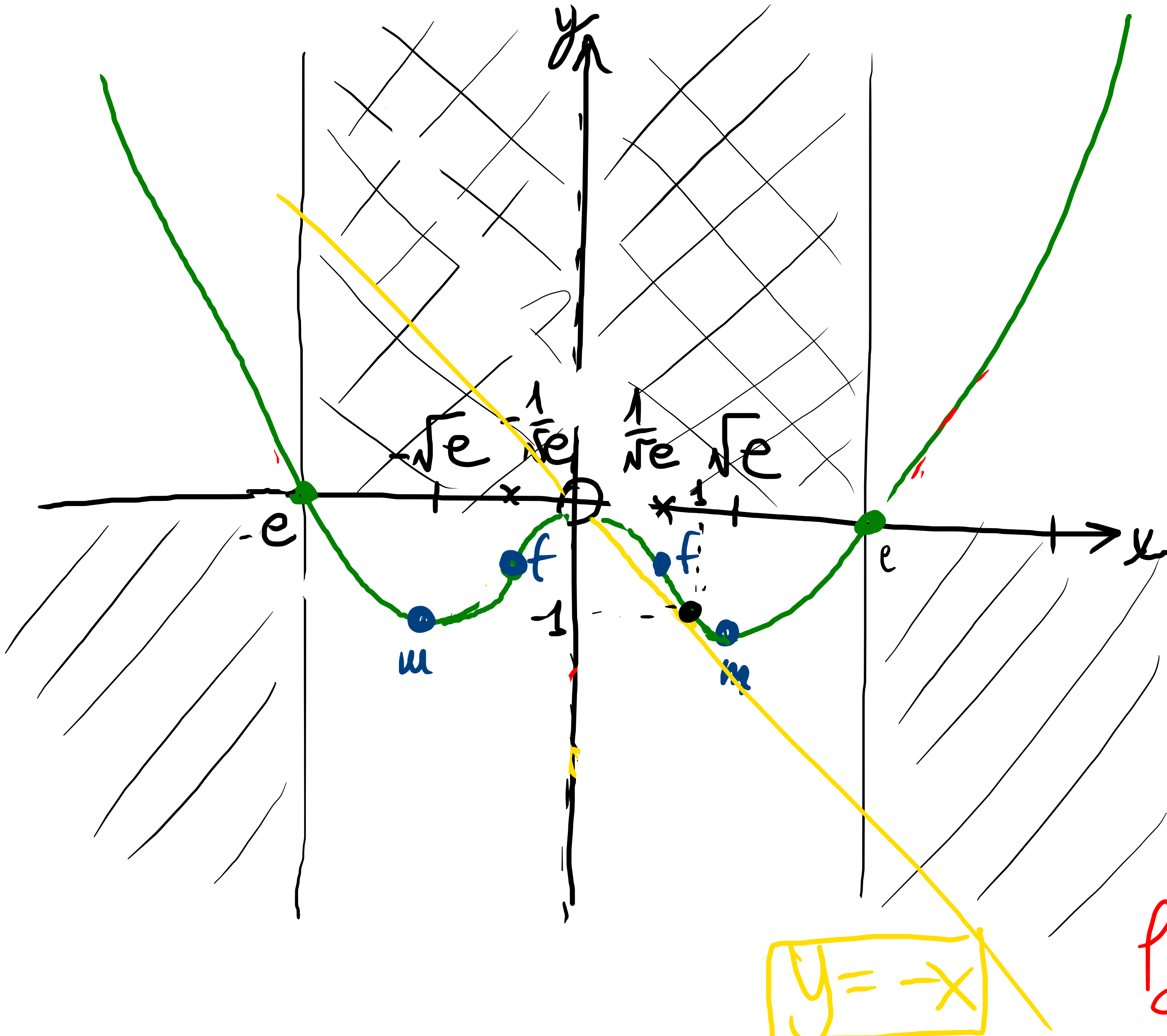
Segno: $x^2 \left(\ln|x| - 1 \right) \geq 0$



I° fattore $x^2 \geq 0 \quad \forall x \in \mathbb{R}$

II° fattore $\ln|x| - 1 \geq 0 \quad e^{\ln|x|} \geq e^{1}$

$|x| \geq e \longrightarrow x \geq e \vee x \leq -e$



Int. asse y
($x=0$ è fuori dominio)

Int asse x

$$\begin{cases} y = x^2(\ln|x|-1) \\ y=0 \end{cases}$$

$\Downarrow$

$$0 = x^2(\ln|x|-1)$$

$\cancel{x=0}$ fuori dominio

$x=\pm e$ ($\ln|x|=1$)

$(e,0)$ $(-e,0)$

$$\lim_{x \to \pm\infty} f(x) = \ldots$$

$\lim_{x \to x_0} f(x)$ nei punti x_0 particolari

In questo caso $-\infty$, $+\infty$, 0

$$\lim_{x \to +\infty} x^2 (\ln|x| - 1) = +\infty \qquad (\text{vince } x^2)$$

$$\lim_{x \to -\infty} x^2 (\ln|x| - 1) = +\infty$$

$$\lim_{x \to 0} x^2 (\ln|x| - 1) = \lim_{x \to 0} \overbrace{\underbrace{x^2 \ln|x|}_{0 \cdot (-\infty)}}^{0} - \cancel{\lim_{x \to 0} x^2}^{\,0} = 0$$

FORMA INDETERMINATA

Derivata I^a

$$f'(x) = 2x\left(\ln|x| - 1\right) + x^{\cancel{2}}\left(\frac{1}{\cancel{x}}\right) =$$

$$= x\left(2\ln|x| - 2 + 1\right) = x\left(2\ln|x| - 1\right)$$

$$f'(x) = 0 \iff x\left(2\ln|x| - 1\right) = 0$$

$\cancel{x = 0}$ FUORI DOMINIO

$2\ln|x| = 1 \rightarrow \ln|x| = \frac{1}{2}$

$|x| = e^{\frac{1}{2}} \rightarrow x = \pm\sqrt{e}$

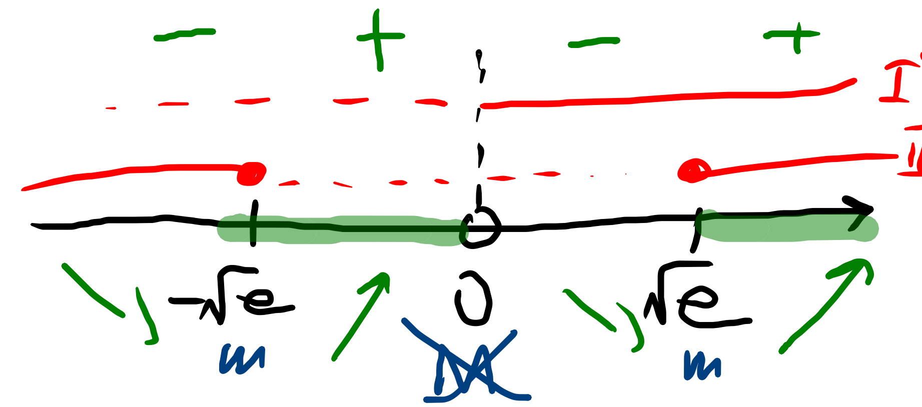
$$f'(x) \geq 0 \iff x\left(2\ln|x| - 1\right) \geq 0$$

I° fatt. $x \geq 0$

II° fatt. $2\ln|x| - 1 \geq 0 \rightarrow |x| \geq \sqrt{e}$

$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x} \qquad x > 0$

$f'(x) = \ln|x| \Rightarrow f'(x) = \frac{1}{x}$



f è crescente in $\left]-\sqrt{e}, 0\right[\cup \left]\sqrt{e}, +\infty\right[$

f è decrescente in $\left]-\infty, -\sqrt{e}\right[\cup \left]0, +\sqrt{e}\right[$

minimo in $+\sqrt{e}$, $-\sqrt{e}$

Trovare l'equazione della retta tangente in $P(+1, -1)$

Equazione retta passante per un punto

$$y - y_P = m(x - x_P)$$

$$y - y_P = f'(x_P)(x - x_P)$$

$$f'(x) = x(2\ln|x| - 1)$$

$$f'(x_P) = 1 \cdot (2\ln|1| - 1) = -1$$

$$y - (-1) = (-1)(x - 1)$$

$$y = -x - 5$$

$$y = -x + 1 - 1 \Rightarrow \boxed{y = -x}$$

Derivata II $\qquad f'(x) = x(2\ln|x|-1)$

$$f''(x) = (2\ln|x| - 1) + \cancel{x}\left(\frac{2}{\cancel{x}}\right)$$

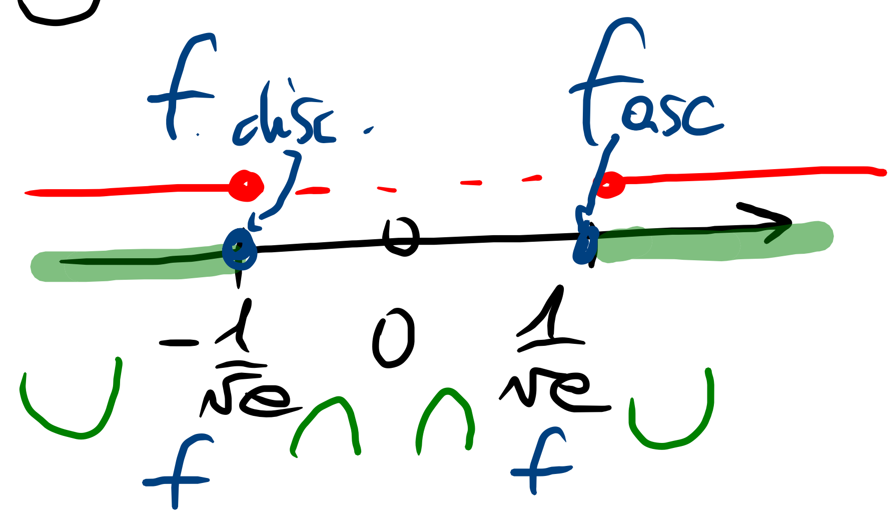
$$= 2\ln|x| + 1$$

$$f''(x) = 0 \iff 2\ln|x| + 1 = 0 \qquad \ln|x| = -\frac{1}{2}$$

$$|x| = \pm\frac{1}{\sqrt{e}} \simeq 0{,}6$$

$$f''(x) \geq 0 \iff 2\ln|x| + 1 \geq 0$$

$$|x| \geq \frac{1}{\sqrt{e}}$$



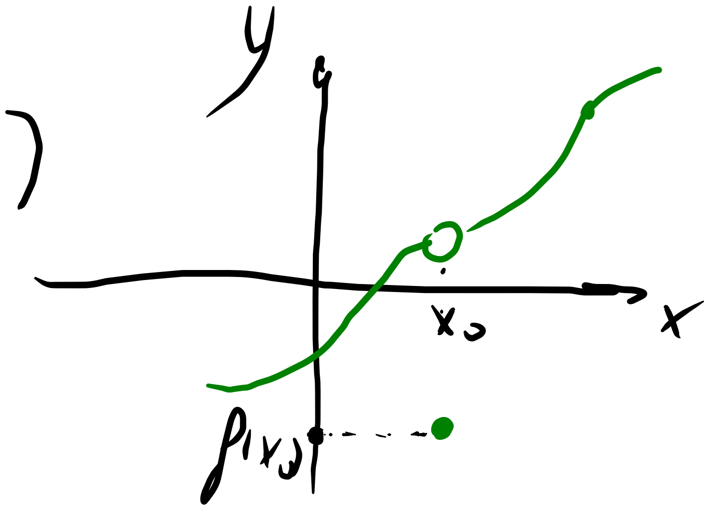
$$f(x) = \begin{cases} \dfrac{\ln(1+5x)}{e^{2x}-1} & x > 0 \\ a\,2^{x} + 7 & x \leq 0 \end{cases}$$

PARAMETRO DA DETERMINARE

Determino a in modo tale che $f(x)$ sia continua in 0

Definizione di continuità in un punto

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) = f(x_0)$$



In questo caso

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\boxed{\begin{aligned} &\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1 \\ &\lim_{x\to 0} \frac{e^x-1}{x} = 1 \end{aligned}}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\ln(1+5x)}{e^{2x}-1} =$$

$$= \lim_{x\to 0^+} \frac{\ln(1+5x)}{e^{2x}-1} \cdot \frac{5x}{5x} \cdot \frac{2x}{2x} =$$

$$= \lim_{x\to 0^+} \underbrace{\frac{\ln(1+5x)}{5x}}_{\to 1} \cdot \underbrace{\frac{2x}{e^{2x}-1}}_{\to 1} \cdot \frac{5\cancel{x}}{2\cancel{x}} = \frac{5}{2}$$

$$f(0) = a\,2^{0} + 7 = a + 7$$

$$f(0) = a + 7 = \frac{5}{2} = \lim_{x \to 0^+} f(x)$$

$$\boxed{a = \frac{5}{2} - 7 = -\frac{9}{2}}$$

Studio FUNZIONE

$$f(x) = \frac{\ln(x^2)}{x}$$

Dominio