LIMITI

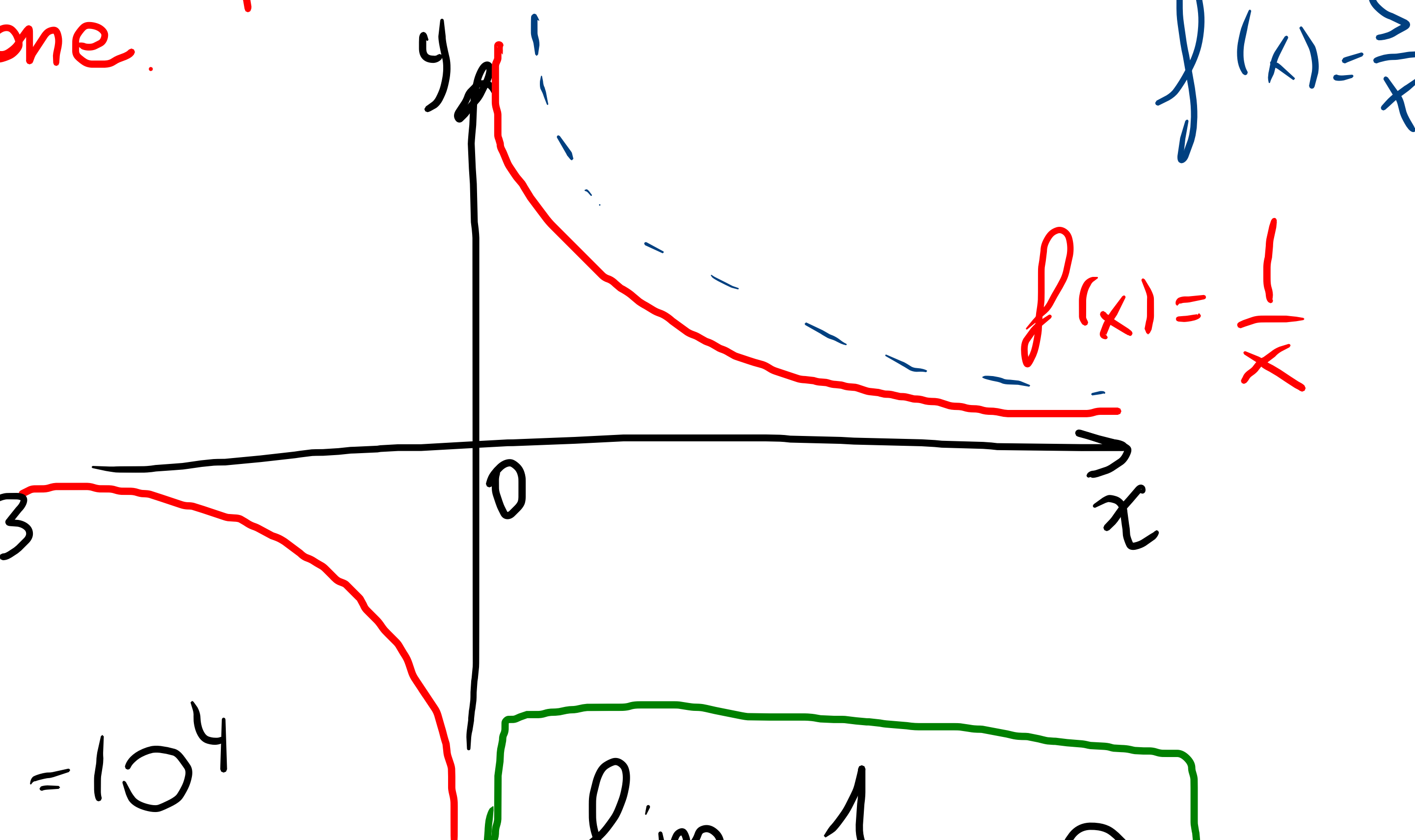
$$\lim_{x\to 2}(x^2+5)=9$$

→ Per prima cosa provo a sostituire nella funzione

$$\boxed{\lim_{x\to 0^+}\frac{1}{x}=+\infty}$$

$$\frac{1}{0{,}001}=1000 \Rightarrow \frac{1}{10^{-3}}=10^3$$

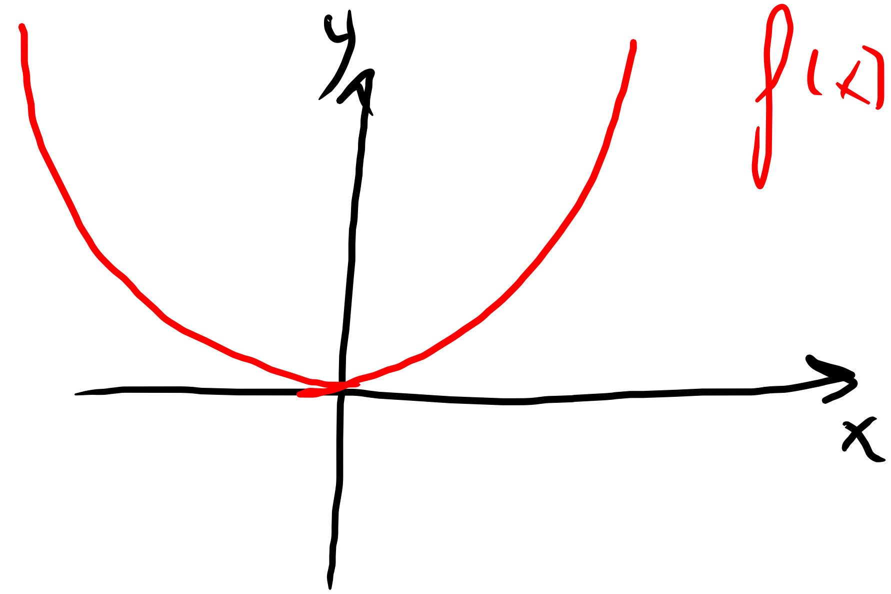
$$\frac{1}{0{,}0001}=10.000 \Rightarrow \frac{1}{10^{-4}}=10^4$$



$$\boxed{\lim_{x\to \pm\infty}\frac{1}{x}=0}$$

$$\lim_{x \to +\infty} x^2 + 3x = +\infty$$

$$\lim_{x \to -\infty} x^2 = +\infty$$



$f(x) = +x^2$

$f(x)$	$g(x)$	$f(x)+g(x)$
$+\infty$	$+\infty$	$+\infty$
$-\infty$	$-\infty$	$-\infty$
$+\infty$	$-\infty$	
$-\infty$	$+\infty$	

} FORME INDETERMINATE

$f(x)$	$g(x)$	$f(x)\cdot g(x)$
∞	∞	∞
0	0	0
0	∞	F. INDETERMINATA

$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
l	∞	0
∞	l	∞
∞	∞	FORME INDETERMINATE
0	0	

ALTRE F. INDETERMINATE

1^{∞}, ...

$$\lim_{x \to +\infty} \frac{x^3+5x-3}{x+4} = \lim_{x \to +\infty} \frac{x^{\cancel{3}\,2}\left(1+\frac{5}{x^2}-\frac{3}{x^3}\right)}{\cancel{x}\left(1+\frac{4}{x}\right)} =$$

FORMA $\frac{\infty}{\infty}$

$$= \lim_{x \to +\infty} x^2 \frac{\left(1+\frac{5}{x^2}-\frac{3}{x^3}\right)}{\left(1+\frac{4}{x}\right)} = +\infty$$

0 0 0

$$\lim_{x \to +\infty} \frac{x^2+2x-3}{x^4+5x^2+9} = \lim_{x \to +\infty} \frac{\cancel{x^2}\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}{x^{\cancel{4}\,2}\left(1+\frac{5}{x^2}+\frac{9}{x^4}\right)} = \lim_{x \to +\infty} \frac{1}{x^2} = 0$$

FORMA $\frac{\infty}{\infty}$

0 0 0 0

$$\lim_{x\to-\infty} \frac{\overset{+\infty\ -\infty}{3x^2+5x+3}}{2x^2-3x+2} = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(3+\frac{5}{x}+\frac{3}{x^2}\right)}{\cancel{x^2}\left(2-\frac{3}{x}+\frac{2}{x^2}\right)} =$$

$\frac{5}{x}\to 0$, $\frac{3}{x^2}\to 0$, $\frac{3}{x}\to 0$, $\frac{2}{x^2}\to 0$

FORMA $\frac{+\infty}{+\infty}$

$$= \frac{3}{2}$$

Quindi se $\lim_{x\to\infty} f(x) = \infty$ e $\lim_{x\to\infty} g(x) = \infty$ $\quad f(x), g(x)$ polinomi

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \begin{cases} \infty & \deg[f(x)] > \deg[g(x)] \\ \ell \neq 0 & \deg[f(x)] = \deg[g(x)] \\ 0 & \deg[f(x)] < \deg[g(x)] \end{cases}$$

$$\lim_{x \to +\infty} \underbrace{\sqrt{x^2 - 5x}}_{+\infty} \underbrace{- \sqrt{x+3}}_{-\infty}$$

$(a+b)(a-b) = a^2 - b^2$

FORMA INDET. $+\infty - \infty$

$$\lim_{x \to +\infty} \left(\sqrt{x^2 - 5x} - \sqrt{x+3}\right) \cdot \frac{\sqrt{x^2-5x} + \sqrt{x+3}}{\sqrt{x^2-5x} + \sqrt{x+3}} =$$

$$\lim_{x \to +\infty} \frac{(x^2 - 5x) - (x+3)}{\sqrt{x^2-5x} + \sqrt{x+3}} = \lim_{x \to +\infty} \frac{x^2 - 6x - 3}{\sqrt{x^2-5x} + \sqrt{x+3}} = +\infty$$

LIMITI NOTEVOLI

$\sin 0 = 0$
$\tan 0 = 0$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{\tan x}{x} = 1$$

FORME INDET. $\frac{0}{0}$

$$\lim_{x\to +\infty} \left(1 + \frac{1}{x}\right)^x = e \simeq 2{,}7$$

FORME INDET. 1^∞

$$\lim_{x\to 0} \frac{e^x - 1}{x}$$

FORME INDET. $\frac{0}{0}$

$e^0 = 1$

$$\lim_{x\to 0}\frac{\sin(5x)}{x} = \lim_{x\to 0}\frac{\sin(5x)}{x}\cdot\frac{5x}{5x} =$$

FORMA INDETERMINATA $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{f(x)\to 0}\frac{\sin(f(x))}{(f(x))} = 1$$

$$\left[\lim_{x\to 0}\frac{x}{\sin x} = 1\right]$$

$$= \lim_{x\to 0}\underbrace{\frac{\sin(5x)}{5x}}_{\to 1}\cdot\frac{5\cancel{x}}{\cancel{x}} = 5$$

$$\lim_{x\to 0}\frac{\sin(\sin x)}{x}\cdot\frac{\sin x}{\sin x} = 1$$

$\to 1$

$\to 1$

$$\lim_{x\to+\infty}\left(1+\frac{3}{x}\right)^x =$$

FORMA INDET. 1^∞

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e$$

Sost. $\frac{3}{x} = \frac{1}{t} \implies \frac{x}{3} = t$ $\underbrace{x = 3t}$

se $x \to +\infty$ $t \to +\infty$

$$\lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^{3t} = \lim_{t\to+\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^3 = e^3$$

e limite notevole

In generale $a \in \mathbb{R}$

$$\lim_{x\to+\infty}\left(1+\frac{a}{x}\right)^x = e^a$$

$$\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\frac{1}{x}}$$

FORMA 1^{∞}

$$\left\{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e\right.$$

Sost $\frac{x}{2}=\frac{1}{t} \Rightarrow \frac{2}{x}=t \Rightarrow \frac{1}{x}=\frac{t}{2}$ se $x\to 0$ $t\to+\infty$

$$=\lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^{t/2}=\lim_{t\to+\infty}\left[\left(1+\frac{1}{t}\right)^{t}\right]^{\frac{1}{2}}=\sqrt{e}$$

$\to e$

$$\frac{t}{2}=t\cdot\frac{1}{2}$$