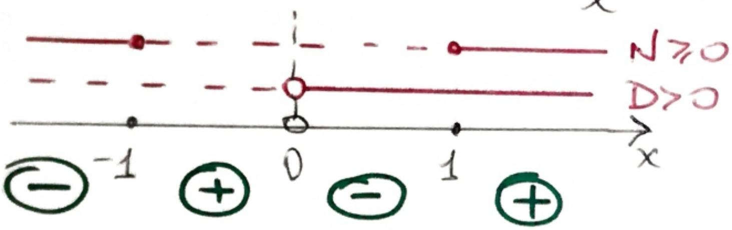
$$f(x) = \frac{\ln(x^2)}{x}$$

Dominio $\quad x^2 > 0 \quad \rightarrow \quad x \neq 0$

$$D = \{\forall x \in \mathbb{R},\ x \neq 0\}$$

Positività: $\quad f(x) \geq 0 \quad \frac{\ln(x^2)}{x} \geq 0 \qquad N \geq 0 \quad \ln(x^2) \geq 0 \quad x^2 \geq 1$

$D > 0 \quad x > 0$



Intersezioni con gli assi

Asse x $\begin{cases} y = f(x) \\ y = 0 \end{cases} \rightarrow 0 = \frac{\ln(x^2)}{x} \Rightarrow \ln(x^2) = 0 \quad x^2 = 1 \quad x = \pm 1$

Intersezione con asse x in $(1, 0)$ e $(-1, 0)$

Asse y $\begin{cases} y = f(x) \\ x = 0 \end{cases} \quad x = 0$ fuori dominio

Studio nei punti particolari

$\lim_{x \to 0} \frac{\ln(x^2)}{x} = \infty \qquad \lim_{x \to \infty} \frac{\ln(x^2)}{x} = 0 \qquad$ FORMA $\frac{\infty}{\infty}$, provo con de l'Hopital

$$\lim_{x \to \infty} \frac{\left(\frac{2x}{x^2}\right)}{1} = \lim_{x \to \infty} \frac{2}{x} = 0 \Rightarrow \lim_{x \to \infty} \frac{\ln(x^2)}{x} = 0$$

$\rightarrow$ derivata numeratore

$\rightarrow$ derivata denominatore

Studio derivata prima

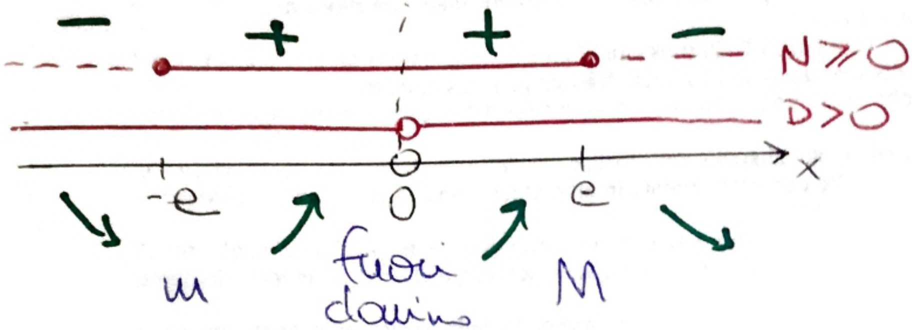
$$f'(x) = \frac{\frac{2x}{x^2} \cdot x - \ln(x^2) \cdot 1}{x^2} = \frac{2-\ln(x^2)}{x^2}$$

$$f'(x) = 0 \iff \frac{2-\ln(x^2)}{x^2} = 0 \qquad 2-\ln(x^2)=0 \qquad \ln(x^2)=2 \qquad x^2 = e^2$$

$$x = \pm e$$

$$f'(x) \geq 0 \iff \frac{2-\ln(x^2)}{x^2} \geq 0 \qquad N \geq 0 \quad 2-\ln(x^2) \geq 0 \quad x^2 \leq e^2$$

$$D > 0 \quad x^2 > 0 \quad x \neq 0$$



$\boxed{\begin{array}{l} \text{MAX in } x = e \\ \text{MIN in } x = -e \end{array}}$

Studio derivata seconda

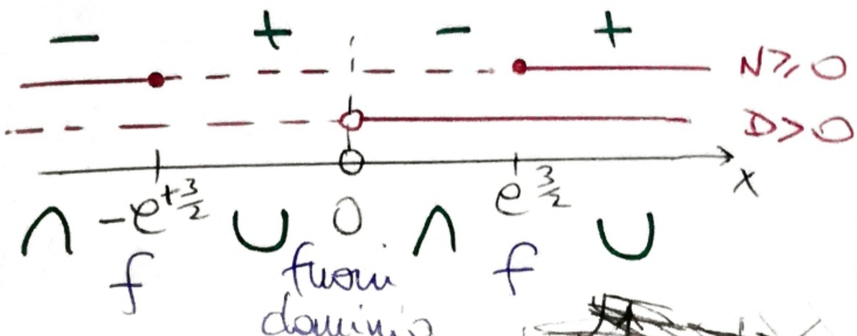
$$f''(x) = \frac{-\frac{2x}{\cancel{x^2}} \cdot \cancel{x^2} - 2x\left(2-\ln(x^2)\right)}{x^4} = \frac{-2x(3-\ln(x^2))}{x^{\cancel{4}\,3}} = \frac{2(\ln(x^2)-3)}{x^3}$$

$$f''(x) = 0 \iff \frac{2(\ln(x^2)-3)}{x^3} = 0 \qquad 2(\ln(x^2)-3)=0$$

$$\ln(x^2)=3 \qquad x^2 = e^3 \qquad x = \pm e^{\frac{3}{2}}$$

$$f''(x) \geq 0 \iff \frac{2(\ln(x^2)-3)}{x^3} \geq 0 \qquad N \geq 0 \quad 2(\ln(x^2)-3) \geq 0 \quad x^2 \geq e^3$$

$$D > 0 \quad x^3 > 0 \quad x > 0$$



$\boxed{\begin{array}{l} \text{FLESSO ASCENDENTE in } x = -e^{\frac{3}{2}} \\ \text{FLESSO ASCENDENTE in } x = e^{\frac{3}{2}} \end{array}}$