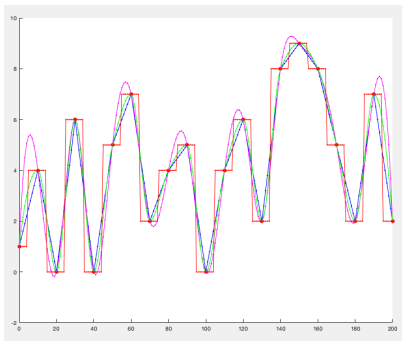


L'interpolazione in MATLAB

Eugenio G. Omodeo



Trieste, 30.11.2021

CHE SIGNIFICA *interpolare* ?

Interpolation is a method by which we can use data points to estimate the unknown value of a function at a given location. This is a common problem in the geosciences; for example, if we have a collection of radiocarbon ages through a sediment core, how can we estimate the age of the sediment at any given depth? **MATLAB** has inbuilt functions to perform interpolation in any number ¹ of dimensions, they are easy to use but it is important to appreciate that the various options you can select from strongly influence your results.

(Heslop 2012, pag.91)

¹Dim. 1, 2, 3, n

CHE SIGNIFICA *interpolare* ?

```
>> help interp1
```

```
interp1 1-D interpolation (table lookup)
```

Vq = **interp1**(X,V,Xq) interpolates to find Vq, the values of the underlying function $V=F(X)$ at the query points Xq.

X must be a vector. The length of X is equal to N.

If V is a vector, V must have length N, and Vq is the same size as Xq.

If V is an array of size [N,D1,D2,...,Dk], then the interpolation is performed for each D1-by-D2-by-...-Dk value in V(i,:,:,...,:). If Xq is a vector of length M, then Vq has size [M,D1,D2,...,Dk]. If Xq is an array of size [M1,M2,...,Mj], then Vq is of size [M1,M2,...,Mj,D1,D2,...,Dk].

Vq = **interp1**(V,Xq) assumes X = 1:N, where N is LENGTH(V) for vector V or SIZE(V,1) for array V.

CHE SIGNIFICA *interpolare* ?

```
>> help interp1
```

```
interp1 1-D interpolation (table lookup)
```

$V_q = \text{interp1}(X,V,X_q)$ interpolates to find V_q , the values of the underlying function $V=F(X)$ at the query points X_q .

X must be a vector. The length of X is equal to N .

If V is a vector, V must have length N , and V_q is the same size as X_q .

If V is an array of size $[N,D_1,D_2,\dots,D_k]$, then the interpolation is performed for each D_1 -by- D_2 -by-...- D_k value in $V(i, :, :, \dots, :)$. If X_q is a vector of length M , then V_q has size $[M,D_1,D_2,\dots,D_k]$. If X_q is an array of size $[M_1,M_2,\dots,M_j]$, then V_q is of size $[M_1,M_2,\dots,M_j,D_1,D_2,\dots,D_k]$.

$V_q = \text{interp1}(V,X_q)$ assumes $X = 1:N$, where N is $\text{LENGTH}(V)$ for vector V or $\text{SIZE}(V,1)$ for array V .

$V_q = \text{interp1}(X,V,X_q,\text{METHOD})$ specifies the interpolation method. The available methods are:

- 'linear' - (default) linear interpolation
- 'nearest' - nearest neighbor interpolation
- 'next' - next neighbor interpolation
- 'previous' - previous neighbor interpolation
- 'spline' - piecewise cubic spline interpolation (SPLINE)
- 'pchip' - shape-preserving piecewise cubic interpolation
- 'cubic' - same as 'pchip'
- 'v5cubic' - the cubic interpolation from MATLAB 5, which does not extrapolate and uses 'spline' if X is not equally spaced.
- 'makima' - modified Akima cubic interpolation

$V_q = \text{interp1}(X,V,X_q,\text{METHOD},\text{'extrap'})$ uses the interpolation algorithm specified by METHOD to perform extrapolation for elements of X_q outside the interval spanned by X .

ESEMPIO: CHE VALORE y' ASPETTARCI PER $x' = 41$?

Evidenziati in rosso, pochi dati misurati; come possiamo avanzare ipotesi circa i dati mancanti?

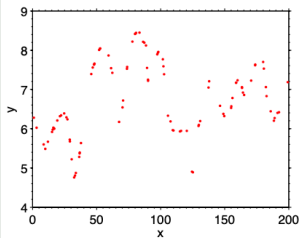


Figure 8.1: *plot of the bedf data set.*

ESEMPIO: CHE VALORE y' ASPETTARCI PER $x' = 41$?

Evidenziati in rosso, pochi dati misurati; come possiamo avanzare ipotesi circa i dati mancanti?

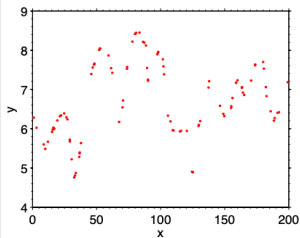


Figure 8.1: *plot of the bedf data set.*

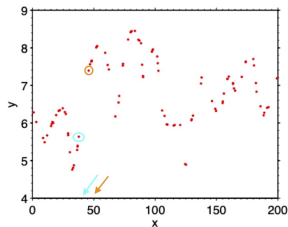


Figure 8.1: *plot of the bedf data set.*

ESEMPIO: CHE VALORE y' ASPETTARCI PER $x' = 41$?

Evidenziati in rosso, pochi dati misurati; come possiamo avanzare ipotesi circa i dati mancanti?

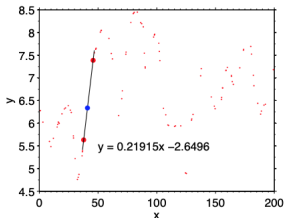


Figure 8.2: If we are interested in finding the value of y' at $x = 41$, we simply determine the straight line between the two data points which span the neighborhood of interest (large red symbols). Once we have the equation; $y = 0.21915x - 2.6496$ we can estimate that when $x' = 41$, $y' = 6.34$ (blue circle).

ESEMPIO: CHE VALORE y' ASPETTARCI PER $x' = 41$?

Evidenziati in rosso, pochi dati misurati; come possiamo avanzare ipotesi circa i dati mancanti?

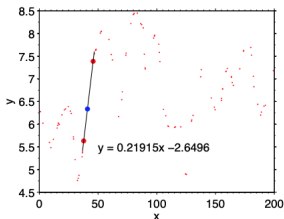
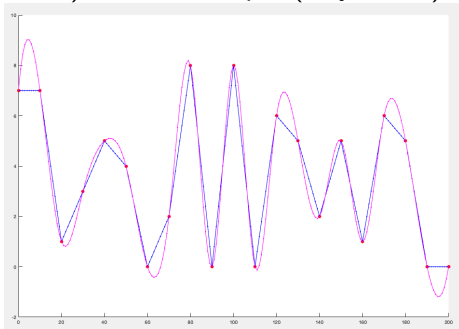


Figure 8.2: If we are interested in finding the value of y' at $x = 41$, we simply determine the straight line between the two data points which span the neighborhood of interest (large red symbols). Once we have the equation; $y = 0.21915x - 2.6496$ we can estimate that when $x' = 41$, $y' = 6.34$ (blue circle).

» $y_0 = \text{interp1}(x, y, 41, 'linear')$ % chiediamo, per $x_0 = 41$, una stima della
 $y_0 =$ % sua y_0 desunta 'linearmente' dai dati (x, y)
 6.3357

INTERPOLAZIONE: LINEARE *versus* TRAMITE *spline*

- ```
» close all, x=0:10:200; y=floor(rand(1,21)*10);
x0=[0:1:200]';
figure, hold on, plot(x,y,'or','markerfacecolor','r'),
y0=interp1(x,y,x0,'linear'); y1=interp1(x,y,x0,'spline');
» plot(x0,y0,'.b-'), plot(x0,y1,'.m-')
```



## ESERCIZIO

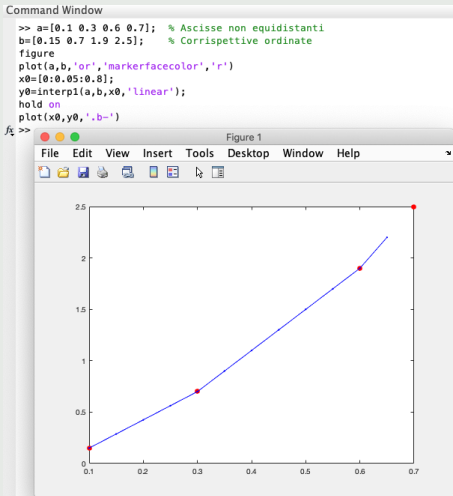
Complete la fig. con le interpolazioni 'pchip' e 'nearest'

*When performing interpolation with the `interp1` function, we don't need to limit ourselves to determining single values. If we supply the function with a vector of  $x'$  values, it will perform the interpolation for each one. This is useful for a lot of time series analysis techniques which assume that you have equally spaced data. If your measured data isn't equally spaced, then you simply interpolate it to produce a suitable new data series.*

(Heslop 2012, pag.93)

# ESEMPIO MOTIVANTE ALL'ESTRAPOLAZIONE

INTERPOLARE, FUORI DALLA FINESTRA-DATI, NON SI PUÒ. . .



Quanto varrà  $y_0$  in corrispondenza ad

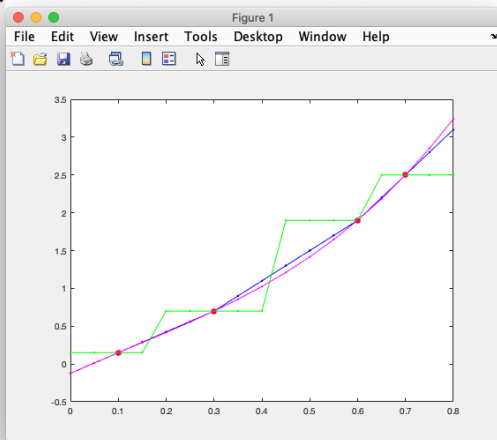
$x_0 = 0, 0.05, 0.7, 0.75, 0.8$  ?

# ESEMPIO MOTIVANTE ALL'ESTRAPOLAZIONE

... E ALLORA ESTRAPOLIAMO

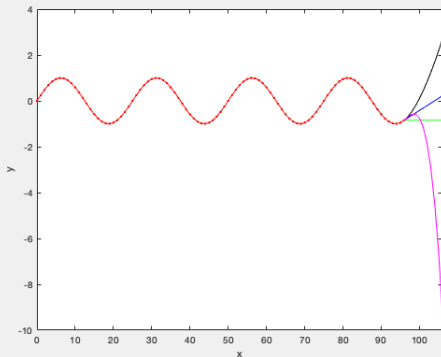
Command Window

```
>> %...e allora estrapoliamo
y0=interp1(a,b,x0,'linear','extrap');
plot(x0,y0,'.b-');
y1=interp1(a,b,x0,'nearest','extrap');
plot(x0,y1,'.g-');
y2=interp1(a,b,x0,'spline','extrap');
plot(x0,y2,'.m-');
fx>>
```



## ESEMPIO: PROLUNGHIAMO UNA CURVA SINUISODALE

```
>> x=0:1:96; % selection of x data sites
y=sin(2*pi*x/25); %sine function with a period of 25 units
plot(x,y,'r-') %plot the data
set(gca,'xlim',[0 106]) %extend the graphic window
xlabel('x'), ylabel('y'),
hold on
>> x0=[96:1:110]'; %x locations for extrapolation
y0=interp1(x,y,x0,'linear','extrap'); %linear extrapolation
plot(x0,y0,'b') %add linear extrapolation to the figure
y0=interp1(x,y,x0,'nn','extrap'); %nn extrapolation
plot(x0,y0,'g') %add nn extrapolation to the figure
y0=interp1(x,y,x0,'spline','extrap'); %cubic spline extrapo
>> plot(x0,y0,'k') %add spline extrapolation to the figure
y0=interp1(x,y,x0,'pchip','extrap'); %pchip extrapolation
plot(x0,y0,'m') %add pchip extrapolation to the figure
>>
```



## Estrapolazione

---

Da Wikipedia, l'enciclopedia libera.

In [matematica](#), il termine **estrapolazione** indica il processo che permette di calcolare il valore di informazioni esterne ad un insieme discreto di dati noti. In sostanza, dato un [piano cartesiano](#) sul quale siano stati tracciati i punti  $(x_i, y_i)$  corrispondenti all'insieme di valori noti, si vuole trovare il valore  $y_w$  corrispondente ad un valore  $x_w$  maggiore (o minore) di ciascun  $x_i$ . L'estrapolazione è simile al processo di [interpolazione](#), che costruisce nuovi punti all'interno di un insieme di punti noti, ma i suoi risultati hanno spesso minor significato e sono soggetti ad un abbondante grado di incertezza.

## Estrapolazione

Da Wikipedia, l'enciclopedia libera.

In [matematica](#), il termine **estrapolazione** indica il processo che permette di calcolare il valore di informazioni esterne ad un insieme discreto di dati noti. In sostanza, dato un [piano cartesiano](#) sul quale siano stati tracciati i punti  $(x_i, y_i)$  corrispondenti all'insieme di valori noti, si vuole trovare il valore  $y_w$  corrispondente ad un valore  $x_w$  maggiore (o minore) di ciascun  $x_i$ . L'estrapolazione è simile al processo di [interpolazione](#), che costruisce nuovi punti all'interno di un insieme di punti noti, ma i suoi risultati hanno spesso minor significato e sono soggetti ad un abbondante grado di incertezza.

### Metodi di estrapolazione [ [modifica](#) | [modifica wikitesto](#) ]

La scelta di quale metodo di estrapolazione applicare deve essere determinata da una conoscenza a priori del processo che ha creato i dati esistenti. È fondamentale sapere se ad esempio i dati possono essere considerati continui, periodici, alternati ecc.

## Extrapolation

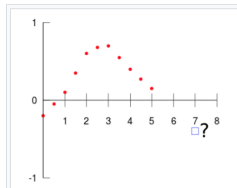
From Wikipedia, the free encyclopedia

*For the journal of speculative fiction, see [Extrapolation \(journal\)](#). For the John McLaughlin album, see [Extrapolation \(album\)](#).*

In **mathematics**, **extrapolation** is a type of **estimation**, beyond the original observation range, of the value of a variable on the basis of its relationship with another variable. It is similar to **interpolation**, which produces estimates between known observations, but extrapolation is subject to greater **uncertainty** and a higher risk of producing meaningless results. Extrapolation may also mean extension of a **method**, assuming similar methods will be applicable. Extrapolation may also apply to human **experience** to project, extend, or expand known experience into an area not known or previously experienced so as to arrive at a (usually conjectural) knowledge of the unknown <sup>[1]</sup> (e.g. a driver extrapolates road conditions beyond his sight while driving). The extrapolation method can be applied in the **interior reconstruction** problem.

### Contents [hide]

- 1 [Methods](#)
  - 1.1 [Linear](#)
  - 1.2 [Polynomial](#)
  - 1.3 [Conic](#)
  - 1.4 [French curve](#)
- 2 [Quality](#)
- 3 [In the complex plane](#)
- 4 [Fast](#)
- 5 [Extrapolation arguments](#)
- 6 [See also](#)
- 7 [Notes](#)



Example illustration of the extrapolation problem, consisting of assigning a meaningful value at the blue box, at  $x = 7$ , given the red data points.





## Methods [\[ edit \]](#)

A sound choice of which extrapolation method to apply relies on *a priori knowledge* of the process that created the existing data points. Some experts have proposed the use of causal forces in the evaluation of extrapolation methods.<sup>[1]</sup> Crucial questions are, for example, if the data can be assumed to be continuous, smooth, possibly periodic etc.

## French curve [\[ edit \]](#)

**French curve** extrapolation is a method suitable for any distribution that has a tendency to be exponential, but with accelerating or decelerating factors.<sup>[3]</sup> This method has been used successfully in providing forecast projections of the growth of HIV/AIDS in the UK since 1987 and variant CJD in the UK for a number of years. Another study has shown that extrapolation can produce the same quality of forecasting results as more complex forecasting strategies.<sup>[4]</sup>